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A SIMPLE SOLUTION TO THE FINITE–HORIZON LQ PROBLEM WITH ZERO TERMINAL STATE

LORENZO NTOGRAMATZIDIS

This short paper deals with the classical finite-horizon linear-quadratic regulator problem with the terminal state constrained to be zero, for both continuous and discrete-time systems. Closed-form expressions for the optimal state and costate trajectories of the Hamiltonian system, as well as the corresponding control law, are derived through the solutions of two infinite-horizon LQ problems, thus avoiding the use of the Riccati differential equation. The computation of the optimal value of the performance index, as a function of the initial state, is also presented.

Keywords: finite-horizon LQ problems, Hamiltonian system, Riccati differential equation, algebraic Riccati equation, optimal value of the quadratic cost

AMS Subject Classification: 93C15

1. INTRODUCTION

One of the traditional problems in control theory is the finite-horizon linear-quadratic regulator, treated in many textbooks mainly in a Riccati framework (see for example [9], [1], [8]). In recent years these problems, with constraints on the terminal state, have received a renewed attention.

Affine constraints have been considered in [2], and solved by approximating the solution of the constrained problem by that of a suitable unconstrained problem.

Analytical expressions for the closed-loop state trajectory and feedback gain are obtained in [6] in the case of equality constraints on the terminal state. The fact that the final value of the state is not penalized through a terminal cost in the performance index is of great interest. The expression of the optimal control law is given as a function of the solutions of three coupled Riccati-like differential equations. A solution to the same problem, with completely fixed terminal state, has been presented in [4] in the complex-frequency domain: the solution of the infinite-time problem is included as a special case. For discrete-time systems, the finite-horizon LQ problem with sharply assigned final state has been recently solved in [12], where the optimal cost has been determined as a quadratic form in the initial and final state.

In this paper, the finite-horizon LQ problem with zero terminal state is considered. The difficulty of solving that problem with the traditional techniques based on the Riccati differential equation arises since this constraint prevents the terminal condition, that enables the differential problem to be solved, from being expressed. On the other hand, solutions based on two-point boundary value conditions in the Hamiltonian system lead to heavy computational burden, since that system is unstable and the transition matrix of the state-costate equations becomes ill-conditioned as the terminal time increases.

The interest of this work is due to the simplicity of the solution presented, that consists of solving the infinite-horizon LQ problem twice, once for the original system and once for its reverse-time representation, thus avoiding the need for terminal conditions and taking advantage of very efficient and currently available software routines for the solution of the algebraic Riccati equation.

The analytic expressions of the state and costate functions allow the computation of the time-varying matrix that solves the Riccati differential equation, which is used to compute the optimal value of the cost; it is shown that the latter can be expressed as a quadratic form in the initial state, like the standard infinite-horizon LQ problem.

The results presented are easily extended to discrete-time systems.

2. STATEMENT OF THE PROBLEM

Consider the linear time-invariant state differential equation

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad x(0) = x_0 \tag{1}$$

where, for all $t \geq 0$, $x(t) \in I\!R^n$ is the state, $u(t) \in I\!R^m$ is the control input, $A \in I\!R^{n\times n}$ and $B \in I\!R^{n\times m}$.

Consider the performance index

$$J\{x(t), u(t)\} = \int_0^{t_f} [\,x^T(t)\,Q\,x(t) + 2\,x^T(t)\,S\,u(t) + u^T(t)\,R\,u(t)\,]\,\mathrm{dt} \tag{2}$$

where $t_f > 0$ is the terminal time, $R > 0$ and $Q - S\,R^{-1}\,S^T \geq 0$ are symmetric. The set of matrices $(A; B; Q, S, R)$ is often referred to as a *Popov triplet*.

Assumptions. In this paper it is assumed that

1. the pair $(A,\ B)$ is controllable
2. H has no eigenvalues on the imaginary axis

where H is defined as

$$H = \begin{bmatrix} A - B\,R^{-1}\,S^T & -B\,R^{-1}\,B^T \\ -Q + S\,R^{-1}\,S^T & -A^T + S\,R^{-1}\,B^T \end{bmatrix} \tag{3}$$

The optimal control problem concerned is defined as follows.

Problem 1. Find the feasible control law $u|_{[0,t_f]}$ that minimizes the performance index $J\{x(t), u(t)\}$ with the constraint on the terminal state

$$x(t_f) = 0$$

3. THE MAIN RESULT

The analytical expressions of the optimal state and costate functions and control law referred to Problem 1 are presented in Theorem 1. We first recall some important results concerning the infinite-horizon LQ problem.

Problem 1 can be solved through the Hamiltonian system, whose state is obtained by extending the state x of system (1) with the costate λ:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} A - B\,R^{-1}\,S^T & -B\,R^{-1}\,B^T \\ -Q + S\,R^{-1}\,S^T & -A^T + S\,R^{-1}\,B^T \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} \tag{4}$$

The state and costate trajectories, in order to be optimal for Problem 1, must satisfy (4). Since the initial and the final states $x(0)$ and $x(t_f)$ are fixed, the value of the costate in the same instants is free.

The reverse-time system associated to (1) is described by

$$\dot{z}(t) = -A\,z(t) - B\,v(t) \tag{5}$$

i. e., it is a system where the pair (A, B) is replaced by $(-A, -B)$.
Denote by P_1 and P_2 the symmetric positive semidefinite solutions of the following CAREs (continuous-time algebraic Riccati equations), referred to systems (1) and (5) respectively:

$$P_1\,A + A^T\,P_1 - (S + P_1\,B)\,R^{-1}\,(S + P_1\,B)^T + Q = 0 \tag{6}$$

$$-P_2\,A - A^T\,P_2 - (S - P_2\,B)\,R^{-1}\,(S - P_2\,B)^T + Q = 0 \tag{7}$$

and by K_1 and K_2 the infinite-horizon optimal gain matrices referred to (1) and (5) respectively[1]:

$$K_1 = R^{-1}\,(S^T + B^T\,P_1) \tag{8}$$

$$K_2 = R^{-1}\,(S^T - B^T\,P_2) \tag{9}$$

Lemma 1. The $n \times n$ matrix

$$X(t) := e^{(A - B\,K_1)\,t} - e^{(A - B\,K_2)\,(t - t_f)}\,e^{(A - B\,K_1)\,t_f} \tag{10}$$

[1]In the MATLAB Control System Toolbox a function *care* is available that provides the matrices P_1, P_2, K_1 and K_2 as follows:

$$[P_1, L_1, K_1] = care(A, B, Q, R, S)$$

$$[P_2, L_2, K_2] = care(-A, -B, Q, R, S)$$

is non-singular for all $t \in [0, t_f)$.

Proof. Suppose, by way of contradiction, that for a fixed $t^* \in [0, t_f)$ there exists a non-null vector $\bar{x} \in \ker X(t^*)$. Now consider a state function whose equation is

$$x(t) = X(t)\,\bar{x} = e^{(A-B\,K_1)\,t}\,\bar{x} - e^{(A-B\,K_2)\,(t-t_f)}\,e^{(A-B\,K_1)\,t_f}\,\bar{x}$$

Note that $x(t^*) = x(t_f) = 0$. This trajectory satisfies the Hamiltonian system (4), together with $\lambda(t) = P_1\,e^{(A-B\,K_1)\,t}\,\bar{x} + P_2\,e^{(A-B\,K_2)\,(t-t_f)}\,e^{(A-B\,K_1)\,t_f}\,\bar{x}$. In fact, define

$$e_1(t) := e^{(A-B\,K_1)\,t}\,\bar{x} \quad \text{and} \quad e_2(t) := e^{(A-B\,K_2)\,(t-t_f)}\,e^{(A-B\,K_1)\,t_f}\,\bar{x}$$

and note that, owing to (6)–(9), the equation

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} (A-B\,K_1)\,e_1(t) - (A-B\,K_2)\,e_2(t) \\ P_1\,(A-B\,K_1)\,e_1(t) + P_2\,(A-B\,K_2)\,e_2(t) \end{bmatrix}$$

equals the right side of (4), expressed by the matrix

$$\begin{bmatrix} (A-B\,R^{-1}\,S^T - B\,R^{-1}\,B^T\,P_1)\,e_1(t) - (A-B\,R^{-1}\,S^T + B\,R^{-1}\,B^T\,P_2)\,e_2(t) \\ (-Q' - A^T P_1 + SR^{-1}B^T P_1)\,e_1(t) + (Q' - A^T P_2 + SR^{-1}B^T P_2)\,e_2(t) \end{bmatrix}$$

where $Q' := Q - S\,R^{-1}\,S^T$. But this trajectory is not optimal as it differs from a state function that is zero for each $t \in [t^*, t_f)$, which is still feasible for the Hamiltonian system since the costate is not fixed in t^* and t_f, and whose cost is zero. This is clearly a contradiction. □

Theorem 1. The optimal state and costate trajectories referred to Problem 1 are

$$x(t) = X(t)\,X^{-1}(0)\,x_0 \quad \text{and} \quad \lambda(t) = \Lambda(t)\,X^{-1}(0)\,x_0 \quad \forall t \in [0, t_f] \tag{11}$$

where, for all $t \in [0, t_f]$, $X(t)$ is defined by (10) and $\Lambda(t)$ is the $n \times n$ matrix defined by

$$\Lambda(t) := [\,P_1\,e^{(A-B\,K_1)\,(t-t_f)} + P_2\,e^{(A-B\,K_2)\,(t-t_f)}\,]\,e^{(A-B\,K_1)\,t_f} \tag{12}$$

Proof. Note that $x(t_f) = 0$ and $x(0) = x_0$; this means that the constraint on the terminal state is satisfied by (11) and the initial state is the one given in (1).
Note that the state and costate trajectories are optimal since they satisfy the Hamiltonian system (4): this can be shown through the same argument as in the proof of Lemma 1, by replacing $\bar{x}$ with $X^{-1}(0)\,x_0$ in the expressions of $e_1(t)$ and $e_2(t)$. □

The following lemma provides a first trivial expression for the optimal control law.

Lemma 2. The optimal control law is

$$u(t) = \begin{cases} -R^{-1}\left(B^T \lambda(t) + S^T x(t)\right) & t \in [0, t_f) \\ 0 & t = t_f \end{cases}$$

The following theorem provides an expression for the time-varying matrix that links the state and costate optimal trajectories.

Theorem 2. For each $t \in [0, t_f)$, a $n \times n$ time-varying matrix $P(t)$ exists such that $\lambda(t) = P(t)\, x(t)$ for all $t \in [0, t_f)$, and can be computed as

$$P(t) = \Lambda(t)\, X^{-1}(t) \tag{13}$$

where $X(t)$ and $\Lambda(t)$ are defined by (10) and (12) respectively, and satisfies, for each $t \in [0, t_f)$, the following Riccati differential equation:

$$\dot{P}(t) + P(t)\, A + A^T P(t) - \left(S + B\, P(t)\right) R^{-1} \left(S + B\, P(t)\right)^T + Q = 0 \tag{14}$$

Proof. First note that $X(t)$ is non-singular, hence invertible, for all $t \in [0, t_f)$, by virtue of Lemma 1. By direct substitution, it is possible to verify that $\lambda(t) = P(t)\, x(t)$ for all $t \in [0, t_f)$. Matrix $P(t)$ satisfies the Riccati differential equation (14), as both the state and costate functions satisfy the Hamiltonian system. □

Corollary 1. The optimal input is

$$u(t) = \begin{cases} -K(t)\, x(t) & t \in [0, t_f) \\ 0 & t = t_f \end{cases} \tag{15}$$

where, for all $t \in [0, t_f)$, $K(t)$ the $m \times n$ matrix defined by

$$K(t) = R^{-1}\left(S^T + B^T P(t)\right) \tag{16}$$

The following theorem provides the value of the performance index corresponding to the optimal solution.

Theorem 3. For the optimal solution we have

$$J_0 = x_0^T\, P(0)\, x_0 \tag{17}$$

Proof. Consider the input function given by (15); the state differential equation (1) and the performance index can consequently be expressed as

$$\dot{x}(t) = \bar{A}\, x(t) \qquad x(0) = x_0$$

where $\bar{A} := A - B\,K(t)$ and

$$J_0 = \lim_{s \to t_f^-} \int_0^s x^T(t)\,[\,Q - 2\,S\,K(t) + K^T(t)\,R\,K(t)\,]\,x(t)\,\mathrm{dt}$$

while (14) can be written as follows:

$$\dot{P}(t) + P(t)\,A + A^T\,P(t) - K^T(t)\,R\,K(t) + Q = 0$$

By virtue of the former,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dt}}[x^T(t)P(t)x(t)] &= 2\,x^T(t)\,P(t)\,\dot{x}(t) + x^T(t)\dot{P}(t)\,x(t)\\ &= x^T(t)\,[\,2P(t)\bar{A} - P(t)A - A^TP(t) + K^T(t)RK(t) - Q\,]\,x(t)\\ &= x^T(t)\,[\,2\,S\,K(t) - K^T(t)\,R\,K(t) - Q\,]\,x(t)\end{aligned}$$

so that

$$\begin{aligned}J_0 &= \lim_{s \to t_f^-} [-x^T(t)\,P(t)\,x(t)]_0^s = x_0^T\,P(0)\,x_0 - \lim_{s \to t_f^-} x^T(s)\,X^T(s)\,\Lambda(s)\,x(s) =\\ &= x_0^T\,P(0)\,x_0\end{aligned}$$

□

4. EXTENSION TO DISCRETE–TIME SYSTEMS

The extension of the results expounded in the previous section is straightforward: consider a linear time-invariant discrete-time system described by

$$x(k+1) = A\,x(k) + B\,u(k) \qquad x(0) = x_0 \tag{18}$$

where $A \in I\!R^{n \times n}$ and $B \in I\!R^{n \times m}$. Assume that A is non-singular. Consider the performance index

$$J\{x(k), u(k)\} = \sum_{k=0}^{k_f - 1} [\,x^T(k)\,Q\,x(k) + 2\,x^T(k)\,S\,u(k) + u^T(k)\,R\,u(k)\,]$$

where $k_f > 0$, R and $Q - S\,R^{-1}\,S^T$ are symmetric and positive semidefinite.

Assumptions. It is assumed that

1. the pair $(A,\,B)$ is controllable
2. the extended symplectic pencil

$$z \begin{bmatrix} I_n & O & O \\ O & -A^T & O \\ O & -B^T & O \end{bmatrix} - \begin{bmatrix} A & O & B \\ Q & -I_n & S \\ S^T & O & R \end{bmatrix}$$

is regular and has no generalized eigenvalues on the unit circle.

Problem 2. Find the feasible control law $u|_{\{0,\dots,k_f\}}$ that minimizes the performance index $J\{x(k),u(k)\}$ with the constraint on the terminal state

$$x(k_f) = 0$$

The dynamics of the reverse-time system associated to (18) can be expressed as a backward recursion

$$x(k) = A_b\, x(k+1) + B_b\, u(k) \tag{19}$$

where $A_b = A^{-1}$ and $B_b = -A^{-1} B$, and the weighting matrices are modified as follows:

$$\begin{aligned} Q_b &= A^{-T} Q\, A^{-1} \\ R_b &= R - S^T A^{-1} B - B^T A^{-T} S + B^T A^{-T} Q\, A^{-1} B \\ S_b &= A^{-T} S - A^{-T} Q\, A^{-1} B \end{aligned}$$

(see for example [5], page 179). Denote by P_1 and P_2 the symmetric positive semidefinite solutions of the following DAREs (discrete-time algebraic Riccati equations) referred to (18) and (19) respectively:

$$P_1 + (A^T P_1 B + S)\,(R + B^T P_1 B)^{-1} (B^T P_1 A + S^T) - A^T P_1 A - Q = 0 \tag{20}$$
$$P_2 + (A_b^T P_2 B_b + S_b)(R_b + B_b^T P_2 B_b)^{-1} (B_b^T P_2 A_b + S_b^T) - A_b^T P_2 A_b - Q_b = 0 \tag{21}$$

and by K_1 and K_2 the infinite-horizon optimal gain matrices referred to (18) and (19):

$$K_1 = (R + B^T P_1 B)^{-1} (B^T P_1 A + S^T) \tag{22}$$
$$K_2 = (R_b + B_b^T P_2 B_b)^{-1} (B_b^T P_2 A_b + S_b^T) \tag{23}$$

The expressions of the optimal state and costate trajectories as functions of time and of the initial state x_0 are provided by the following theorem, which is the extension of Theorem 1 for discrete-time systems.

Theorem 4. The optimal state and costate trajectories referred to Problem 2 are, for each $k \in \{0,\dots,k_f\}$, given by

$$x(k) = X(k)\, X^{-1}(0)\, x_0 \quad \text{and} \quad \lambda(k) = \Lambda(k)\, X^{-1}(0)\, x_0 \tag{24}$$

where, for each $k \in \{0,\dots,k_f\}$, $X(k)$ and $\Lambda(k)$ are $n \times n$ matrices defined by

$$X(k) := (A - B K_1)^k - (A_b - B_b K_2)^{(k-k_f)} (A - B K_1)^{k_f} \tag{25}$$
$$\Lambda(k) := [P_1 (A - B K_1)^{(k-k_f)} + P_2 (A_b - B_b K_2)^{(k-k_f)}]\, (A - B K_1)^{k_f} \tag{26}$$

and the optimal control law is

$$u(k) = \begin{cases} -K(k)\, x(k) & k \in \{0,\dots,k_f - 1\} \\ 0 & k = k_f \end{cases} \tag{27}$$

where, for all $k \in \{0, \ldots, k_f - 1\}$, $K(k)$ is the $m \times n$ matrix defined by

$$K(k) = \Big(R + B^T P(k+1)\, B \Big)^{-1} \Big(B^T P(k+1)\, A + S^T \Big) \tag{28}$$

where $P(k)$ is the $n \times n$ matrix that links the state and costate optimal trajectories, and can be computed as

$$P(k) = \Lambda(k)\, X^{-1}(k)$$

Furthermore, matrix $P(k)$ satisfies, for all $k \in \{0, \ldots, k_f - 1\}$, the Riccati difference equation

$$P(k) = A^T P(k+1) A - \Big(A^T P(k+1) B + S\Big) \Big(R + B^T P(k+1) B\Big)^{-1} \Big(B^T P(k+1) A + S^T\Big) + Q$$

Proof. By defining

$$e_1(k) := (A - B\,K_1)^k\, X^{-1}(0)\, x_0\,, \quad e_2(k) := (A_b - B_b\, K_2)^{k-k_f}\, (A - B\,K_1)^{k_f}\, X^{-1}(0)\, x_0$$

so that

$$\begin{aligned} x(k+1) &= (A - B\,K_1)\, e_1(k) - (A_b - B_b\,K_2)\, e_2(k) \\ \lambda(k+1) &= P_1\,(A - B\,K_1)\, e_1(k) + P_2\,(A_b - B_b\,K_2)\, e_2(k) \end{aligned}$$

it can be shown by direct substitution that the state and costate trajectories satisfy

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) \\ \lambda(k) &= A^T\,\lambda(k+1) + Q\,x(k) + S\,u(k) \\ 0 &= R\,u(k) + S^T\,x(k) + B^T\,\lambda(k+1) \end{aligned}$$

obtained by applying the Lagrange-multiplier approach to Problem 2 owing to (20)–(23). □

Corollary 2. The optimal value of the cost is

$$J_0 = x_0^T\, P(0)\, x_0$$

The non-singularity of matrix A can be assumed with no loss of generality; in fact, since the pair $(A,\, B)$ is controllable, if A is singular a state feedback $u(k) = F\,x(k)$ can be performed to obtain a non-singular matrix $A + B\,F$ (see for example [5] and [10]). The weighting matrices in the performance index have to be modified as follows:

$$\hat{Q} = Q + F^T S^T + S\,F + F^T R\,F \tag{29}$$

$$\hat{S} = S + F^T R \tag{30}$$

$$\hat{R} = R \tag{31}$$

The solution of the original problem is $K(k) = \hat{K}(k) + F\,X(k)$, where $\hat{K}(k)$ is obtained by applying (28) to the modified Popov triplet $(A + B\,F; B; \hat{Q}, \hat{S}, \hat{R})$, and $X(k)$ is computed through (25).

5. A NUMERICAL EXAMPLE

Consider a system whose matrices and initial state are

$$A = \begin{bmatrix} -8.95 & -6.45 & 0 & 0 \\ 2.15 & -0.35 & 0 & 0 \\ -10.89 & -40.94 & -16.1 & -7.95 \\ 8.17 & 28.87 & 7.07 & -0.2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad x_0 = \begin{bmatrix} 4 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{32}$$

Using the theory explained in section (2.) we find the input control law that minimizes a performance index (2) with

$$Q = \begin{bmatrix} 5 & 4 & 13 & 16 \\ 4 & 5 & 11 & 14 \\ 13 & 11 & 34 & 42 \\ 16 & 14 & 42 & 52 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 5 \\ 4 & 6 \end{bmatrix} \quad t_f = 1 \tag{33}$$

In Figure 1 the state trajectory and the optimal control law are shown. The optimal cost corresponding to the constrained problem is $J_0 = 4.0544$, while that corresponding to the infinite-horizon non-constrained problem is zero, since the quadruple (A, B, C, D), with matrices C and D such that $Q = C^T C, R = D^T D$ and $S = C^T D$, is minimum-phase.

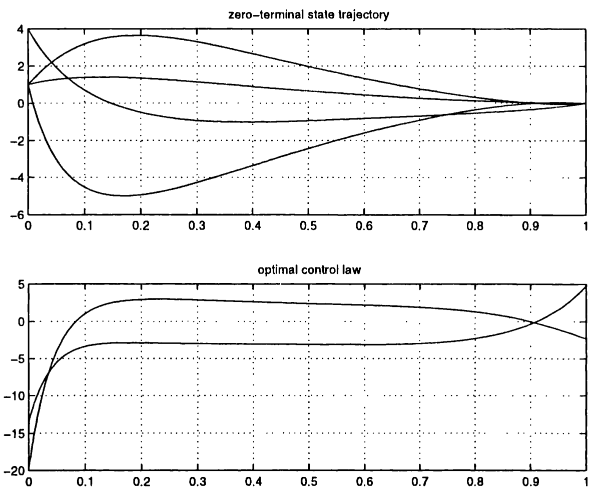

Fig. 1. Optimal state trajectory and control law.

6. CONCLUDING REMARKS

It has been shown how to derive simple expressions for the state and costate functions that solve the linear quadratic regulator problem with zero terminal state. It has

been proved that the extended trajectory thus obtained satisfies the Hamiltonian system: this result ensures optimality, and allows an expression to be derived for the corresponding optimal input. In this way, the time-varying matrix that solves the Riccati differential equation is easily determined; this matrix can be used to derive the optimal value of the performance index.
These considerations can be exploited to solve the reverse-time problem: starting from the origin, the control law that has to be applied to reach an assigned state in finite time can easily be computed in order to minimize a cost criterion of the type considered here.

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Lorenzo Ntogramatzidis, Dipartimento di Elettronica, Informatica e Sistemistica, Università di Bologna, viale Risorgimento 2, Bologna. Italy.
e-mail: lntogramatzidis@deis.unibo.it