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TIME SERIES MODELS FOR EARTH'S CRUST KINEMATICS[1]

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Deterministic and stochastic approach to modeling common trends has been applied to time series of horizontal coordinates of the permanent GPS station Modra-Piesky (recorded weekly during the period of 4 years).

1. ONE DIMENSIONAL ANALYSIS OF COORDINATE DATA

The U.S. Global Positioning System (GPS) is the satellite navigation system applied in geodesy for precise determination of the position on the Earth's surface. GPS is used for the regular monitoring of recent kinematics of the Earth's crust. The daily geocentric Cartesian coordinates of about 80 permanent GPS stations are determined within the framework of the European GPS network EUREF (see [4]). One site in Slovakia is participating in this project – the Modra-Piesky (MOPI) permanent observation station. The MOPI station has been operating regularly since June, 1996 (see [6]). In this paper we analyze a series transformed to a horizontal coordinate system covering the interval from November 1996 to October 2000. The two-dimensional series consist of N–S and E–W components representing the variations in the north-south and east-west directions. Each element of the series represents the value obtained during a 1-week interval of observations by GPS satellites (in meters). The values are reduced so that zero corresponds to January 2000. The linear trend observed in the N–S and E–W series is the consequence of the long-term drift of the Eurasian tectonic plate. Its approximate value is about 2.5 cm per year towards the northeast. The residual variations in the N–S and E–W series are due to the random and systematic disturbing effects shown by the GPS observations (see [1]).

The descriptive statistic and regression analysis output for Nord-South coordinates (see Figure 1) are: Mean $= -0.01454\,\mathrm{m}$; Variance $= 0.00027\,\mathrm{m}^2$; Standard Deviation $= 0.01632\,\mathrm{m}$; Residual Variance $= 0.0000016\,\mathrm{m}^2$; Adjusted R Squared $= 0.994$, $a_2 = -0.04286$, $b_2 = 0.000271$.

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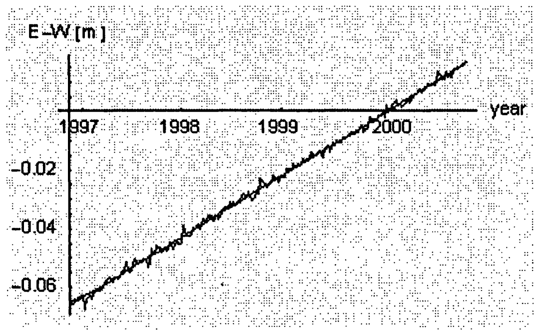

Fig. 1. N–S coordinates with linear trend function $x_t = a_2 + b_2 t$.

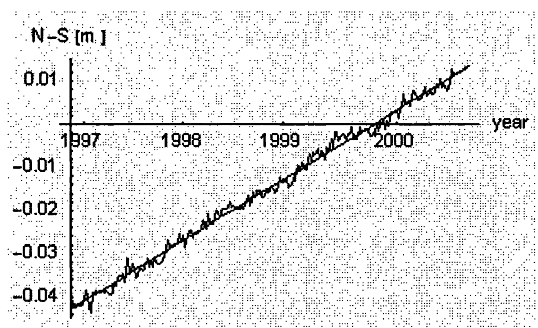

Fig. 2. E–W coordinates with linear trend function $y_t = a_1 + b_1 t$.

The descriptive statistic and regression analysis output for East – West coordinates (see Figure 2) are: Mean $= -0.02467\,\mathrm{m}$; Variance $= 0.00057\,\mathrm{m}^2$; Standard Deviation $= 0.02396\,\mathrm{m}$; Residual Variance $= 0.0000013\,\mathrm{m}^2$; Adjusted R Squared $=$ 0.998, $a_1 = -0.06632$, $b_1 = 0.000399$.

2. COMMON DETERMINISTIC TREND

Simple graphical and linear regression analyses indicate that both time series follow linear trends with a very high level of fit. This fact naturally leads to the idea of investigating a common trend. Inspired by the methodology of common trends (see [5]), we look for linear combinations

$$\begin{aligned} u &= \gamma_1 y + \delta_1 x \\ v &= \gamma_2 y + \delta_2 x \end{aligned} \tag{1}$$

such that u represents a common trend direction and v is a stationary trend-free variable, orthogonal to u. It is an easy exercise to obtain the following suitable form for variables u and v:

$$\begin{aligned} u &= y\cos(\alpha) + x\sin(\alpha) \\ v &= -y\sin(\alpha) + x\cos(\alpha) \end{aligned} \tag{2}$$

where the angle α is determined by

$$\begin{aligned} \tan(\alpha) &= \frac{b_2}{b_1}, \qquad \alpha = 0.597(= 34.21^\circ), \\ \gamma_1 &= \delta_2 = \cos(\alpha) = 0.827, \qquad \delta_1 = -\gamma_2 = \sin(\alpha) = 0.562. \end{aligned}$$

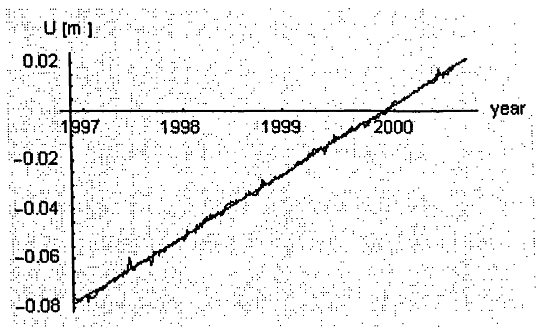

Fig. 3. Direction v.

The descriptive statistic of coordinates v (see Figure 3) is follow: Mean = 0.0018 m, Variance = 0.000018 m^2, Standard Deviation = 0.00134 m.

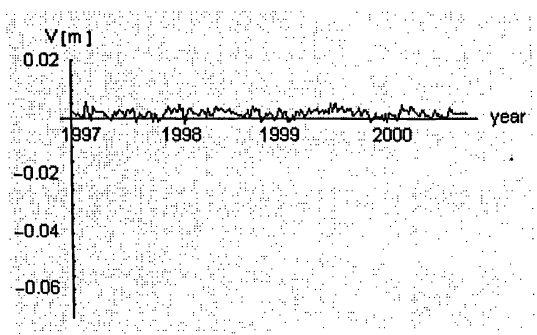

Fig. 4. Direction u with linear trend function $u_t = a + b\,t$.

The descriptive statistic and regression analysis output for coordinates u (see Figure 4) are: Mean = −0.0259 m; Variance = 0.00073 m^2, Standard Deviation = 0.02693 m; Residual Variance = 0.000001 m^2; Adjusted R Squared = 0.998, $a = -0.07273$, $b = 0.00045$.

From Figure 3 and Figure 4 we see that our intention to obtain a trend-free direction v and trend containing direction u has been fulfilled.

Applying standard tests we concluded that the series v_t has properties of white noise (shifted by an additive constant).

Next we analyze the series of residuals of the variable u (after removing the linear trend).

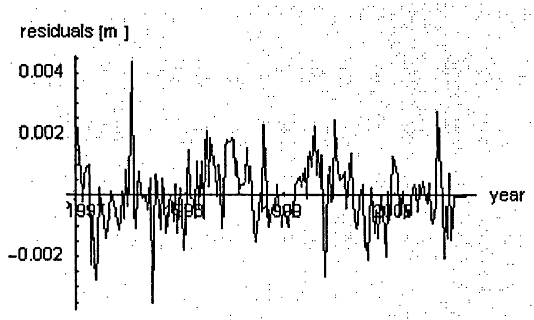

Fig. 5. Residuals of the variable u after removing the linear trend.

Applying again standard tests we concluded that the above series of residuals does not have independent components. Moreover, graphical analysis indicated possibility of periodic trends. Applying the periodogram and test for significant periods we concluded that the period 52 months = 1 year is significant. Therefore we calculated the cyclical component:

$$C_t = -0.0005 \cos\left(\frac{2\Pi}{52}t\right) - 0.00018 \sin\left(\frac{2\Pi}{52}t\right).$$

We repeatedly removed this component from the values of former residuals and received new residuals e_t that we also tested for independence. After rejecting independence we applied Box–Jenkins methodology and concluded that the investigated residuals are optimally fitted by the first order autoregression model:

$$e_t = 0.35209 e_{t-1} + w_t$$

where w_t is a white noise with variance $\sigma^2 = 0.00000094\,\mathrm{m}^2$.

3. COMMON STOCHASTIC TREND

By analysis of a common stochastic trend we use the linear regression

$$x_t = a_0 + b_0 y_t \quad (a_0 = 0.00218,\ b_0 = 0.67783).$$

We are again looking for new variables u and v given by linear transformations (1) and (2), where

$$\tan(\alpha) = b_0, \qquad \alpha = 0.596\ (= 34.13°).$$

New values of coefficients in (1) are:

$$\gamma_1 = \delta_2 = \cos(\alpha) = 0.828, \quad \delta_1 = -\gamma_2 = \sin(\alpha) = 0.561.$$

Because the new values of these coefficients only very slightly differ from their previous values, we will not change the variables u and v calculated above. However, we try to find a so-called stochastic trend model for the trend variable u. It has the form:

$$u_t - u_{t-1} = \beta + \epsilon_t, \quad \beta = 0.00047.$$

The value of stochastic trend for the common trend variable u is slightly bigger then its corresponding value for deterministic trend ($b = 0.00045$). Similar phenomenon has been observed for some econometric time series (see [5]).

4. CONCLUSION

Our analysis clearly proved that there exists a one dimensional direction that attract a substantial part of trend behavior of the Eurasia tectonic plate. Results of deterministic and stochastic trend analysis for this behavior are very close to each other.

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