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OPTIMAL DECENTRALIZED CONTROL DESIGN WITH DISTURBANCE DECOUPLING[1]

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In this paper we present an input-output point of view for the problem of closed loop norm minimization of stable plants when a decentralized structure and a disturbance decoupling property are imposed on the controller. We show that this problem is convex and present approaches to its solution in the optimal ℓ_1 sense in the nontrivial case which is when the block off-diagonal terms of the plant have more columns than rows.

1. INTRODUCTION

Decentralized control has been studied extensively over the last thirty five years or so and a rich literature has been generated on various aspects of the problem. Yet, it is fair to say that the problem of optimal performance still remains a challenge to the control community due to each complexity, notably the lack of a convex characterization of the problem (e. g., [6, 12] and references therein). The decoupling and noninteracting control problem has also been investigated intensively during the same time frame and several results on the problem of optimal design have been obtained (e. g., [10, 15, 16] and references therein).

In this paper we consider a combination of the the above problems for the special case of stable plants. In particular, we are investigating the problem of closed loop norm minimization with a decentralized controller which also provides decoupling of the effect of disturbances (or reference commands for that matter) occurring at the output of the plant. The interesting feature of this problem is that, using the Youla–Kučera (YK) parametrization of all stabilizing controllers [9, 17], the constraints on the controller become *convex* constraints on the YK parameter Q. Thus, the resulting model matching problem is convex irrespective of the norm used. This basic observation allows for investigation of various approaches to solve the underlying closed loop minimization problem. In the paper we provide a more detailed exposition in the case of ℓ_1 optimal control and furnish a solution procedure to achieve performance within any *prespecified* accuracy from optimal.

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The notation used in the paper is standard and we refer to textbooks (e. g., [5]) for further details: c^0 is the space of matrices the elements of which are 1-sided sequences converging to zero, ℓ_1 is the subspace of c^0 which contains the absolutely summable sequences, $\mathcal{L}_2$ is the the space of complex valued transfer function matrices that are square integrable on the unit circle, $\mathcal{H}_2$ is the subspace of $\mathcal{L}_2$ that contains the functions that are analytic in the unit disc, and, $\mathcal{H}_\infty$ is the subspace of $\mathcal{H}_2$ which contains the essentially bounded functions on the unit circle.

2. PROBLEM DEFINITION

Consider the feedback configuration of a stable, finite dimensional, discrete-time plant P with the controller C as in Figure 1, where the control input u and plant output y are partitioned into two (possibly vector) components u_1, u_2 and y_1, y_2 respectively, i. e., $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. Let P and C be partitioned accordingly as

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix} \quad C = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix}.$$

Let $S := (I - PC)^{-1} = \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix}$ denote the sensitivity function where its partitioning is inherited from P and C in the obvious manner. Moreover, let Φ represent a closed loop map that captures the overall input-output characteristics of interest to the design between exogenous disturbances and regulated variables. In relation to Figure 1, Φ could be as

$$\Phi = \begin{pmatrix} W_{11}(I-PC)^{-1} & W_{12}P(I-CP)^{-1} \\ W_{21}C(I-PC)^{-1} & W_{22}(I-CP)^{-1}CP \end{pmatrix}$$

where W_{ij}'s are weights selected by the designer. Let $\|\Phi\|$ refer to any norm, e. g., $\mathcal{H}_2$, $\mathcal{H}_\infty$ or ℓ_1. Since its particular type is not important at this stage we will use the norm symbol generically. The problem of interest is as follows:

Problem (P). Find, if possible, a decentralized C, i. e., $C_{12} = C_{21} = 0$, such that S is decoupled, i. e., $S_{12} = S_{21} = 0$, and the norm $\|\Phi\|$ is minimized subject to internal stability.

In the problem formulation above, there are two constraints on the allowable stabilizing controllers: (i) the decentralized structure and (ii) the (output) decoupling property that the resulting S is (block) diagonal and hence the effects of disturbances d_o to the output channels y of the plant decouple. Note that the effects of these disturbances to the control inputs u also decouple since C is constrained to be (block) diagonal. Also note that since the plant P is assumed stable, the trivial case $C = 0$ is always a feasible point in the optimization posed in (P). Finally, we mention that the input decoupling case, i. e., requiring $(I - CP)^{-1}$ be (block)-diagonal, can be treated analogously and will not be presented here.

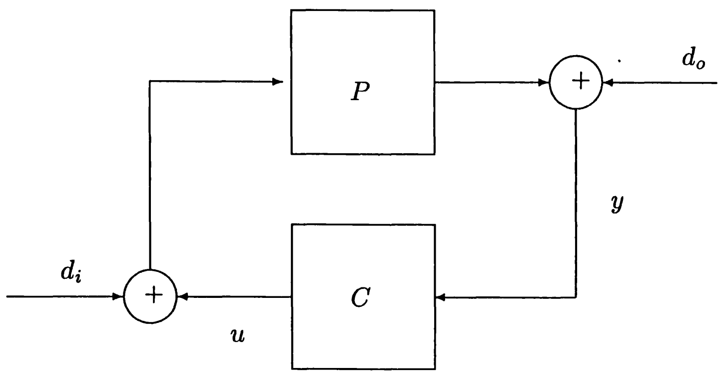

Fig. 1. Feedback configuration.

3. PROBLEM SOLUTION

3.1. Problem transformation

Considering the YK parametrization [17] in the case of stable plants, all stabilizing controllers, not necessarily decentralized or decoupling, are given as $C = -Q(I - PQ)^{-1}$ where Q is any stable map. With this parametrization the closed loop becomes $\Phi = H - UQV$ with H, U, V stable, finite dimensional and fixed depending only on the problem data. In addition, the sensitivity map becomes $S = I - PQ$. Thus, the decentralization and decoupling requirements on C transform to constraints on Q. In particular, as it can be seen from the previous relationships, S is block diagonal if and only if PQ is block diagonal. Hence, C and S are block diagonal if and only if Q and PQ block diagonal, or equivalently

$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} \quad \text{with} \quad Q_{12} = Q_{21} = 0$$

and

$$P_{12}Q_{22} = 0, \quad P_{21}Q_{11} = 0.$$

Thus, all the constraints on Q are convex and therefore the problem

$$\mu := \inf_C ||\Phi|| = \inf_Q ||H - UQV||$$

with C stabilizing, decentralized and decoupling, is an infinite dimensional (the pulse response coefficients of Q) convex problem. Clearly from the above description of the constraints on Q, the problem has a nontrivial solution i. e., $C \neq 0$, if and only if any of the block off-diagonal terms P_{12} and P_{21} have more columns than rows, i. e., they are "fat"; more precisely if and only if they have normal row rank which is strictly smaller than their column dimension. In this case not both Q_{11} and Q_{22} are forced to be zero which in turn implies that not both C_{11} and C_{22} are zero.

In the sequel we explore further the constraints on Q to recast the problem in a more suitable for our purposes form. We also assume that both P_{12} and P_{21} have full normal row rank which is strictly smaller than their column dimension. We start by reformatting the constraint $P_{12}Q_{22} = 0$. To this end, let P_{12} be written in the Smith form (e. g., [5]) as

$$P_{12} = U_1 \Sigma_1 (I\ 0)\, U_2$$

where U_1 and U_2 are stable and stably invertible, square transfer function matrices, and Σ_1 is diagonal. Then the constraint $P_{12}Q_{22} = 0$ becomes

$$(I\ 0)\, U_2 Q_{22} = 0.$$

Let

$$\overline{Q}_2 = U_2 Q_{22} := \begin{pmatrix} \overline{Q}_{21} \\ \overline{Q}_{22} \end{pmatrix}$$

then the resulting constraint is that $\overline{Q}_{21} = 0$ while $\overline{Q}_{22}$ is a free stable parameter. Hence in terms of the original parameter Q_{22} we have that

$$Q_{22} = U_2^{-1} \begin{pmatrix} 0 \\ \overline{Q}_{22} \end{pmatrix} = U_2^{-1} \begin{pmatrix} 0 \\ I \end{pmatrix} \overline{Q}_{22}.$$

Similarly for the other constraint $P_{21}Q_{11} = 0$ we can obtain that

$$Q_{11} = V_2^{-1} \begin{pmatrix} 0 \\ \overline{Q}_{11} \end{pmatrix} = V_2^{-1} \begin{pmatrix} 0 \\ I \end{pmatrix} \overline{Q}_{11}$$

where $\overline{Q}_{11}$ is a free stable parameter and V_2^{-1} is a stable unimodular map associated with the Smith form $P_{21} = V_1 \Sigma_2 (I\ 0) V_2$. Summarizing the previous discussions we have the following

Proposition 3.1. Problem (P) is equivalent to

$$\mu = \inf_{\overline{Q}} \|H - \bar{U}\overline{Q}V\|$$

where

$$\bar{U} = U \begin{pmatrix} V_2^{-1} \begin{pmatrix} 0 \\ I \end{pmatrix} & 0 \\ 0 & U_2^{-1} \begin{pmatrix} 0 \\ I \end{pmatrix} \end{pmatrix}$$

and

$$\overline{Q} = \begin{pmatrix} \overline{Q}_{11} & 0 \\ 0 & \overline{Q}_{22} \end{pmatrix}$$

with $\overline{Q}_{11}$ and $\overline{Q}_{22}$ arbitrary stable maps.

In general, the resulting model matching problem of Proposition 3.1 is, although convex, nonstandard since the off-diagonal blocks of $\overline{Q}$ have to be zero. Certain problems however that involve only the effect of the output disturbance d_o in Φ readily transform to decoupled standard problems. For example, if $\Phi = S$, based on the previous analysis $\Phi = \begin{pmatrix} I - \bar{P}_{11}\overline{Q}_{11} & 0 \\ 0 & I - \bar{P}_{22}\overline{Q}_{22} \end{pmatrix}$ where $\bar{P}_{11} = P_{11}V_2^{-1}\begin{pmatrix} 0 \\ I \end{pmatrix}$ and $\bar{P}_{22} = P_{22}U_2^{-1}\begin{pmatrix} 0 \\ I \end{pmatrix}$ which is clearly decoupled. In the paper we are interested in a more general Φ that may include couplings from input disturbances d_i and the weight selections. Such Φ provides better robustness and flexibility in the overall design.

3.2. Approaches for solving the equivalent problem

In principle, one can solve the problem by considering truncations of the $\overline{Q}$ parameter [1] and thus approximating the problem with a finite dimensional (the pulse response coefficients of the truncated $\overline{Q}$) convex programming problem

$$\mu_N := \inf_{\overline{Q}} \left\| H - \bar{U}\overline{Q}_N V \right\|$$

where $\overline{Q}_N$ is a Finite Impulse Response (FIR) of length N (block-diagonal) system. It can be easily checked that $\mu_N \to \mu$ monotonically from above as $N \to \infty$. The main shortcoming of this method is that it cannot indicate how close to the optimal solution is the converging upper bound μ_N. To do so, one needs converging lower bounds as well. In the sequel we specialize the discussion to ℓ_1 related problems where we provide solutions within any *predefined* accuracy.

3.2.1. ℓ_1-norm minimization

In this case one can use an extension of the scaled-Q method in [8] to provide converging upper and lower bounds to μ. In particular, for the problem at hand let P_N denote the Nth truncation operator and define the two, finite dimensional, linear programs (LP)s:

$$\nu_N(\alpha) := \min \max\{\|H - R\|, \alpha \left\|\overline{Q}\right\|\}$$

subject to

$$P_N(R) = P_N(\bar{U}\overline{Q}V), \quad P_N(\overline{Q}) \text{ is block diagonal}$$

and

$$\mu_N(\alpha) := \min \max\{\|H - R\|, \alpha \left\|\overline{Q}\right\|\}$$

subject to

$$R = \bar{U}P_N(\overline{Q})V, \quad P_N(\overline{Q}) \text{ is block diagonal}$$

where α is a scalar positive parameter. Note that the linear programs above will be finite dimensional if the λ-transforms $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ are polynomials. This however can always be done, for example, by absorbing the least common multiple of the denominator polynomials of $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ into $\hat{\overline{Q}}(\lambda)$. The variables in the first LP for computing $\nu_N(\alpha)$ are $R(0), \ldots, R(N), \overline{Q}(0), \cdots, \overline{Q}(N)$ related through

$P_N(R) = P_N(\bar{U}\overline{Q}V)$ that generates finitely many constraints on $R(i)$s and $\overline{Q}(i)$s with $\overline{Q}(i)$ being block diagonal. The second LP for computing $\mu_N(\alpha)$ has variables $\overline{Q}(0), \ldots, \overline{Q}(N)$ with $\overline{Q}(i)$ being block diagonal that generate finitely many $R(i)$s via $R = \bar{U}P_N(\overline{Q})V$ since $\bar{U}P_N(\overline{Q})V$ corresponds to a polynomial transfer function matrix.

In the sequel we assume that $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ do not loose rank for any λ on the unit circle, i. e., $\lambda = \exp^{j\theta}$, and, they have full normal column and row rank respectively. Moreover, we assume $\mu > 0$. The following can now be shown.

Theorem 3.1. An optimal solution $\overline{Q}$ to the problem in Proposition 3.1 exists. Moreover, there exists $\alpha_0 > 0$ such that for all α with $0 < \alpha \leq \alpha_0$ it holds that $\mu_N(\alpha) \to \mu$ monotonically from above and $\nu_N(\alpha) \to \mu$ monotonically from below as $N \to \infty$.

Proof. Existence follows from the fact that $\overline{Q}$ can be constrained to be bounded in order to achieve optimal (or near-optimal) performance. Indeed, it is enough to search for $R = \bar{U}\overline{Q}V$ with $\|R\| \leq 2\,\|H\|$ since otherwise

$$\|H - R\| \geq \|R\| - \|H\| > \|H\| \geq \mu$$

where we have used that a legitimate $\overline{Q}$ is $\overline{Q} = 0$ which obeys obviously the constraint of being block diagonal and hence $\|H - \bar{U}0V\| = \|H\| \geq \mu$. If now $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ do not loose rank for any λ on the unit circle and in addition they have full normal column and row rank respectively we have that $\overline{Q} = \bar{U}^{-l}RV^{-r}$ where $\bar{U}^{-l}$ and V^{-r} are left and right inverses of $\bar{U}$ and V respectively. Thus as in [8]

$$\|\overline{Q}\| \leq 2\,\|\bar{U}^{-l}\|\,\|H\|\,\|V^{-l}\|$$

where the norms $\|\bar{U}^{-l}\|$, $\|V^{-l}\|$ are taken to be ℓ_1 norms of 2-sided sequences (which will be bounded since $\bar{U}^{-l}$, and V^{-l} are rational without poles at on the unit disk). Based on Alaoglu's Theorem (e. g., [2], p. 130) we can guarantee the existence of a sequence $\{\overline{Q}_n\}$ of block diagonal $\overline{Q}_n$ converging weak $*$ to a $\overline{Q}^o \in \ell_1$ with $\liminf_n \|H - \bar{U}\overline{Q}_nV\| = \mu$; it follows easily that $\overline{Q}^o$ is block triangular and by standard semicontinuity properties of weak $*$ convergence

$$\left\|H - \bar{U}\overline{Q}^oV\right\| \leq \liminf_n \|H - \bar{U}\overline{Q}_nV\| = \mu$$

which establishes existence. The rest of the theorem is a condensation of results in [8] which can be reproduced with exactly the same arguments as in [8]. The block diagonal requirement on $\overline{Q}$ introduces additional functionals in c^0 that need to be added in the characterization of $M^\perp$ in Lemma 2 of [8]. These are of the form $(0, W_a^{ijk})$ $k = 0, 1, 2, \ldots$ where (i, j) correspond to the indices of the elements in $\overline{Q}$ that are constrained to be zero and $W_a^{ijk}(m) = A$ when $m = k$ and $W_a^{ijk}(m) = 0$ when $m \neq k$ where A is a constant matrix of the same dimension as $\overline{Q}$ having zeros (0) in all its entries except the ijth which is set to one (1). The scalar $\alpha_0 > 0$ in the theorem can be taken as $\alpha_0 = \frac{\mu}{\|\overline{Q}^o\|}$. □

The bound $\alpha_0 > 0$ can be taken as any lower bound on $\frac{\mu}{\|\overline{Q}^o\|}$ where $\overline{Q}^o$ is any optimal solution. A lower bound of μ is the unconstrained optimal ℓ_1 cost denoted as μ_u. An upper bound on $\overline{Q}^o$ has been established in the proof of Theorem 3.1. Hence, α_0 can be computed apriori assuming $\mu_u > 0$; even if no nonzero lower bounds on μ can be computed (as a matter of fact, even if $\mu = 0$) still converging lower and upper bounds to μ can be obtained as indicated [8]. The above theorem guarantees that performance arbitrarily close to optimal within any prespecified tolerance can be delivered by selecting $\overline{Q}$ as the FIR solution that achieves $\mu_N(\alpha)$ for appropriate N.

We would like to mention that if $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ have full normal row and/or column rank respectively, i. e., fat and/or tall matrices respectively, the existence of an optimal $\overline{Q}^o$ is not guaranteed even if $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ do not loose rank for any λ on the unit circle. This is in contrast to the unconstrained case. The following is an example to illustrate the point in such a situation.

Consider a simple constrained case where

$$(\phi_1 \ \phi_2) = (h_1 \ h_2) - (\bar{u}_1 \ \bar{u}_2)\begin{pmatrix} \bar{q}_1 & 0 \\ 0 & \bar{q}_2 \end{pmatrix}$$

with all the systems appearing above being stable and scalar. A simple manipulation leads to

$$(\phi_1 \ \phi_2) = (h_1 - \bar{u}_1\bar{q}_1 \ \ h_2 - \bar{u}_2\bar{q}_2).$$

Hence minimizing $\|(\phi_1 \ \phi_2)\|$ amounts to minimizing $\|h_1 - \bar{u}_1\bar{q}_1\|$ over $\bar{q}_1$ and $\|h_2 - \bar{u}_2\bar{q}_2\|$ over $\bar{q}_2$. Let

$$\hat{\bar{u}}_2(\lambda) = (0.5 - \lambda)(1 - \lambda)$$

and

$$\hat{h}_2(\lambda) = 2(1 - \lambda)$$

and let $\hat{\bar{u}}_1(\lambda)$ be anything as long it does not have a zero at $\lambda = 1$. Then $(\hat{\bar{u}}_1 \ \hat{\bar{u}}_2)(\lambda)$ does not loose rank on the unit circle. Yet, $\|h_2 - \bar{u}_2\bar{q}_2\|$ does not have a minimizer $\bar{q}_2^o$ as it is indicated in Example 10.6.1 in [5].

Despite the absence of an existence guarantee in the case when $\hat{\bar{U}}(\lambda)$ and $\hat{V}(\lambda)$ have full normal row and/or column rank respectively, the same techniques as in [8] can be used to provide arbitrarily close to optimal solutions [14].

3.2.2. An example

Consider

$$P = g\left(\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 \end{array}\right)$$

where

$$g = \frac{0.1(\lambda + 0.1)}{\lambda + 0.2}.$$

We are interested in minimizing the ℓ_1 norm $||\Phi||$ where

$$\Phi = \left((I-PC)^{-1} \quad W(I-CP)^{-1}\right)$$

with $W = wI$,

$$w = \frac{-0.3333\lambda}{\lambda - 0.3333}$$

while ensuring the structural constraints of decoupling

$$(I-PC)^{-1} = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix}$$

and decentralization

$$C = \left(\begin{array}{c|c} * & 0 \\ * & 0 \\ * & 0 \\ \hline 0 & * \\ 0 & * \end{array}\right).$$

Following Section 3, we have

$$\begin{aligned} P_{12} &= g(1\ 0) \\ P_{21} &= g(1\ 0\ 0) \end{aligned}$$

$$Q_{11} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \bar{Q}_{11}$$

$$Q_{22} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bar{Q}_{22}$$

with

$$\bar{Q}_{11} = \begin{pmatrix} \bar{q}_{11} \\ \bar{q}_{21} \end{pmatrix}, \quad \bar{Q}_{22} = \bar{q}_{22}.$$

Moreover,

$$\bar{U} = \begin{pmatrix} g & g & g \\ 0 & 0 & g \end{pmatrix}$$

$$V = w\begin{pmatrix} 1 & 0 & g & g & g & g & 0 \\ 0 & 1 & g & 0 & 0 & g & g \end{pmatrix}$$

$$H = V$$

$$\bar{Q} = \left(\begin{array}{c|c} \bar{Q}_{11} & 0 \\ \hline 0 & \bar{Q}_{22} \end{array}\right)$$

$$\Phi = H - \bar{U}\bar{Q}V.$$

With this structure it is easy to check that the structural constraints are met:

$$I - PQ = (I - PC)^{-1} = \begin{pmatrix} \frac{1}{1-g(\bar{q}_{11}+\bar{q}_{21})} & 0 \\ 0 & \frac{1}{1-g\bar{q}_{22}} \end{pmatrix}$$

and

$$C = -Q(I - PQ)^{-1} = \left(\begin{array}{c|c} 0 & 0 \\ \frac{-\bar{q}_{11}}{1-q(\bar{q}_{11}+\bar{q}_{21})} & 0 \\ \frac{\bar{q}_{21}}{1-g(\bar{q}_{11}+\bar{q}_{21})} & 0 \\ \hline 0 & 0 \\ 0 & \frac{\bar{q}_{22}}{1-q\bar{q}_{22}} \end{array}\right).$$

Solving the constrained problem with the methods of [8] leads to

$$\begin{aligned} \bar{q}_{11} &= 5.0004 + 0.4999\lambda - 0.0498\lambda^2 + 0.0051\lambda^3 \\ &\quad +0.0034\lambda^4 + 0.0003\lambda^6 + 0.0001\lambda^7 + 0.0002\lambda^9 \end{aligned}$$

$$\begin{aligned} \bar{q}_{21} &= 4.9996 + 0.5001\lambda - 0.0502\lambda^2 + 0.0049\lambda^3 + 0.0036\lambda^4 \\ &\quad -0.0004\lambda^5 - 0.0001\lambda^6 + 0.0001\lambda^7 - 0.0002\lambda^8 \end{aligned}$$

$$\bar{q}_{22} = 4.9217 + 0.5726\lambda - 0.0368\lambda^2 - 0.0015\lambda^3 + 0.0025\lambda^4 - 0.0001\lambda^6$$

and the optimal value for the norm is

$$||\Phi|| = 0.3195.$$

Solving the model matching problem without the constraint on $\bar{Q}$ being blocked diagonal leads to

$$||\Phi|| = 0.0005.$$

Both values were computed with 10^{-14} accuracy from optimal. The resulting (constrained) linear program involved 66 variables and 33 constraints and it was set-up in Matlab software as described in [13, 14].

3.2.3. Pareto optimal ℓ_1 design

An alternative ℓ_1-based design criterion that we investigate is a weighted sum of the ℓ_1 norms of the rows of Φ. In particular, we are interested in

$$\sum_{i=1}^{n} c_i \, ||\phi_i||$$

where ϕ_i is the ith row of Φ and $\sum_{i=1}^{n} c_i = 1$, $c_i \geq 0$. It can be shown [3] that minimizing the above criterion for various c_i's captures all Pareto optimal solutions Φ^p. A Pareto optimal solution Φ^p in this case means that there is no other feasible Φ such that

$$\|\phi_i\| \leq \|\phi_i^p\|, \forall i = 1, \ldots, n \quad \text{and} \quad \|\phi_i\| < \|\phi_i^p\|, \text{ for some } i = 1, \ldots, n$$

where ϕ_i^p is the ith row of Φ^p. It should be clear then that, assuming existence, the set of ℓ_1 optimal solutions of the previous section (since they may not be unique in general) should include a Pareto optimal one. Thus, by solving the proposed weighted sum criterion for varying c_i's there will be a combination that will generate an ℓ_1 optimal solution. This is obviously not the best way to solve ℓ_1 problems but it could provide insightful trade-offs. Next we proceed in minimizing the proposed cost and show that this amounts to a standard ℓ_1 minimization problem.

To this end let q_{1i}^T and q_{2j}^T be the ith of the n_1 and the jth of the n_2 columns of $\overline{Q}_{11}$ and $\overline{Q}_{22}$ respectively and consider a row vector q defined by stacking all columns q_{1i}^T and q_{2j}^T in a row vector as

$$q := (q_{11} \cdots q_{1n_1} \; q_{21} \cdots q_{2n_2}).$$

Split $\bar{U} = (U_a \; U_b)$ and $V = \begin{pmatrix} V_a \\ V_b \end{pmatrix}$ so that $\bar{U}\overline{Q}V = (U_a\overline{Q}_{11}V_a \quad U_b\overline{Q}_{22}V_b)$ and denote u_{ai} and u_{bi} the ith row of U_a and U_b respectively.

Consider also $U_{ai} := \mathrm{diag}(u_{ai}^T, u_{ai}^T, \ldots, u_{ai}^T)$ with n_1 columns and similarly $U_{bi} := \mathrm{diag}(u_{bi}^T, u_{bi}^T, \ldots, u_{bi}^T)$ with n_2 columns. Define as

$$U_A := (U_{a1} \; \ldots \; U_{an_1}), \quad U_B := (U_{b1} \; \ldots \; U_{bn_2})$$

and

$$V_A := \mathrm{diag}(V_a, V_a, \ldots, V_a), \quad V_B := \mathrm{diag}(V_b, V_b, \ldots, V_b)$$

where V_A and V_B have respectively n_1 and n_2 diagonal blocks. Then, the rows of $U_a\overline{Q}_{11}V_a$ and $U_b\overline{Q}_{22}V_b$ stack together one after each other from first to last in a row vector, are respectively $q_1U_AV_A$ and $q_2U_BV_B$. Splitting $H = (H_a \; H_b)$ and denoting h_a and h_b the row vectors produced by stacking the rows of H_a and H_b respectively we have that minimizing the weighted sum amounts to minimizing over *unconstrained* $q \in \ell_1$ the norm

$$\|\phi\| := \left\|(h - q\bar{V})D\right\|$$

where $h := (h_a \; h_b)$, $\bar{V} = \mathrm{diag}(V_A, V_B)$, and, D is a constant weight

$$D = \begin{pmatrix} \mathrm{diag}(c_1I_a, \ldots, c_nI_a) & 0 \\ 0 & \mathrm{diag}(c_1I_b, \ldots, c_nI_b)) \end{pmatrix}$$

with I_a and I_b identity matrices of dimension equal to the column dimension of $U_a\overline{Q}_{11}V_a$ and $U_b\overline{Q}_{22}V_b$ respectively. The problem is thus a standard ℓ_1 optimization.

By looking at this equivalent problem associated to Pareto optimal solutions, certain properties of the optimal ℓ_1 solutions, assuming they exist, can also be established. For example, if an optimal closed loop map Φ^o for the ℓ_1 norm minimization corresponds to a combination of c_is reflected in D such that $\bar{V}D$ has full column normal rank and has no zeros on the circle, then the optimal ϕ^o is FIR and thus the same holds for Φ^o.

The Pareto optimal approach can also be used to solve $\mathcal{H}_2/\ell_1$ multiobjective type of problems as in [11] with constraints in Q. Finally, it should be clear that the pure $\mathcal{H}_2$ minimization problem corresponds to setting $c_i = 1$ all i; this amounts to the same approach given in [7] for converting $\mathcal{H}_2$ constrained problems to standard model matching. Performing a standard $\mathcal{H}_2$ minimization by projection (e.g., [5]) of $\|h - q\bar{V}\|$ leads to $q^{o,2} = (\Pi_{\mathcal{H}_2} h \bar{V}_i^*)\bar{V}_o^{-l}$ where $\bar{V} = \bar{V}_o\bar{V}_i$ is an outer-inner factorization [4] of $\bar{V}$ i.e., $\bar{V}_i\bar{V}_i^* = 1$ where $\bar{V}_o$ has a stable left inverse $\bar{V}_o^{-l}$ and $\Pi_{\mathcal{H}_2}$ is the standard projection of $\mathcal{L}_2$ onto $\mathcal{H}_2$.

4. CONCLUSIONS

In this paper we investigated the problem of closed loop norm minimization of stable plants subject to certain decentralization and decoupling constraints. It was shown that the problem is convex, however nonstandard, infinite dimensional optimization. Approaches to solve the optimal ℓ_1 control and variants were given. The case of unstable plants is a harder problem due to the fact that decentralization and decoupling may not always be possible. This a subject for future research.

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REFERENCES

[1] S. P. Boyd and C. H. Barratt: Linear Controller Design: Limits of Performance. Prentice Hall, Englewood Cliffs, N. J. 1991.

[2] J. B. Conway: A Course in Functional Analysis. Springer–Verlag, New York 1990.

[3] N. O. Da Cuhna and E. Polak: Constrained minimization under vector valued criteria in finite dimensional spaces. J. Math. Anal. Appl. *19* (1967), 103–124.

[4] B. A. Francis: A Course in H_∞ Control Theory. (Lecture Notes in Control and Information Sciences 88.) Springer–Verlag, New York 1987.

[5] M. A. Dahleh and I. J. Diaz-Bobillo: Control of Uncertain Systems: A Linear Programming Approach. Prentice Hall, Englewood Cliffs, N. J. 1995.

[6] A. N. Gundes and C. A. Desoer: Algebraic Theory of Linear Feedback Systems with Full and Decentralized Compensators. Springer–Verlag, Heidelberg 1990.

[7] G. C. Goodwin, M. M. Seron, and M. E. Salgado: $\mathcal{H}_2$ design of decentralized controllers. In: Proc. American Control Conference, San Diego 1999.

[8] M. Khammash: Solution of the ℓ_1 optimal control problem without zero interpolation. In: Proc. Control Decision Conference, Kobe 1996; IEEE Trans. Automat. Control, to appear.
[9] V. Kučera: Discrete Linear Control. Wiley, Chichester 1979.
[10] A. Linnemann and Q. Wang: Block decoupling with stability by unity output feedback-solution and performance limitations. Automatica *29* (1993), 735–744.
[11] M. V. Salapaka, M. Khammash, and M. Dahleh: Solution of MIMO $\mathcal{H}_2/\ell_1$ problem without zero interpolation. In: Proc. Control Decision Conference, San Diego 1997.
[12] K. A. Ünyelioglu and U. Özgüner: $\mathcal{H}_\infty$ sensitivity minimization using decentralized feedback: 2-input 2-output systems. Systems Control Lett. (1994), 99–109.
[13] X. Qi, M. H. Khammash, and M. V. Salapaka: Optimal controller synthesis with multiple objectives. In: Proc. 2001 American Control Conference, Arlington 2001, pp. 2730–2735.
[14] X. Qi, M. H. Khammash, and M. V. Salapaka: A Matlab package for multiobjective control synthesis. In: IEEE Conference on Decision and Control 2001, to appear.
[15] C. K. Quek and A. P. Loh: Robust decoupling of discrete systems using ℓ_1 optimization. IEEE Trans. Automat. Control *42* (1997), 549–553.
[16] J. W. van der Woude: Almost noninteracting control by measurement feedback. Systems Control Lett. *9* (1987), 7–16.
[17] D. C. Youla, H. A. Jabr, and J. J. Bongiorno: Modern Wiener-Hopf design of optimal controllers. Part 2: The multivariable case. IEEE Trans. Automat. Control *AC-21* (1976), 319–338.

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