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STATISTICAL-LEARNING CONTROL OF MULTIPLE-DELAY SYSTEMS WITH APPLICATIONS TO ATM NETWORKS

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Congestion control in the ABR class of ATM network presents interesting challenges due to the presence of multiple uncertain delays. Recently, probabilistic methods and statistical learning theory have been shown to provide approximate solutions to challenging control problems. In this paper, using some recent results by the authors, an efficient statistical algorithm is used to design a robust, fixed-structure, controller for a high-speed communication network with multiple uncertain propagation delays.

1. INTRODUCTION

This paper illustrates the application of statistical-learning control results for an Available Bit Rate (ABR) congestion control algorithm in an Asynchronous Transfer Mode (ATM) communications network. The ABR service category is a best-effort class used in ATM networks to handle highly bursty and varying data applications. ATM was selected by the International Telecommunication Union (ITU) for Broadband Integrated Service Digital Network (B-ISDN), and is detailed in [2]. ATM requires the transmission of fixed size cells (each containing 53 bytes) and is a connection-based network combining the advantages of packet and circuit switching [8].

ABR traffic sources receive explicit feedback from the ATM switches and adjust their transmission rates in order to match their share of the network resources. ATM networks and specifically their ABR service control has provided a fertile area of applications for control designers as witnessed by the recent flury of papers [3, 4, 6]. Most of these papers however have made simplifying assumptions regarding the model of the network or its connectivity. In particular, few of these papers have considered the fact that multiple delays exist in such networks and that such delays are usually uncertain.

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The study of dynamical systems with delays has recently seen a fleury of activities as witnessed by the references [1, 5, 10, 11]. We now understand how to design various controllers for linear and nonlinear systems to achieve various performance objectives. We can also account for various uncertainties in the modeling of the open-loop systems. The presence of delays in dynamical systems may have destabilizing effects [11], cause chaotic behavior [11], and in general the design of stabilizing controllers for such systems may be NP-hard [13]. In [4], the authors used tools from robust control to account for the fragility of the standard controllers in the presence of uncertain time delays in an ATM network.

The presence of multiple delays in physical systems has been illustrated in commodity markets, active displacement control, and other systems. In communications networks, it is also understood that delays are not only present, but that they have an important impact on the performance and even on the stability of such networks. Moreover, and if one also accounts for the fact that the delays are uncertain, one is faced with the challenging problem of trying to control a large, interconnected network, with many uncertain delays and to do so in the face of modeling errors and disturbances. In this paper, we shall focus on one type of networks (ATM), under one type of operational characteristics (ABR) and only consider the uncertainties in the time delays.

The current paper will first address the problem of designing fixed-structure controllers which will meet various performance objectives for an open-loop system with multiple uncertain delays. The results of this design will be illustrated on the aforementioned ATM networks.

In what follows, we will follow the ATM network model used in [9] which addressed the ABR control in the presence of delays using a Smith-predictor control architecture [12].

2. THE NETWORK MODEL AND THE CONTROL PROBLEM

The ABR class is designed as a best-effort class for applications such as file transfer and e-mail. Thus, no service guarantees are required, but the source of data packets controls its data rate, using a feedback signal provided by switches downstream that measure the congestion of the network. Due to the presence of this feedback, many classical and advanced control theory concepts have been suggested to deal with the congestion control problem in the ATM/ABR case.

We will focus our modeling on one particular queue in the network which is associated with a link shared by many virtual circuits. A virtual circuit is established for any two stations wishing to communicate, by informing all switches bewteen them of their requirements. Assuming that the flow of packets is conserved, the queue level model for each buffer in the ATM network (as proposed in [9]) is

$$\dot{x}(t) = \sum_{i=1}^{n} u_i(t - T_i) - d(t)$$

or stated otherwise

$$x(t) = \int_0^t \left(-d(\tau) + \sum_{i=1}^{n} u_i(\tau - T_i) \right) d\tau \tag{1}$$

where

- $x(t)$ is the queue level associated with the considered link;
- n is the number of *virtual circuits* sharing the queue associated with the considered link;
- $u_i(t)$ is the inflow cell rate caused by the ith *virtual circuit*;
- T_i is the propagation delay from the ith source to the queue, and is usually uncertain;
- $d(t)$ is the rate of packets leaving the queue.

More details on this model and its physical interpretation is available from [9]. Equation (1) appears to be linear since the saturation effect due to the limited buffer capacity has been neglected. Our control design however, must ensure that this condition is actually satisfied, so that the controller is not saturated.

2.1. The control problem

The author in [9] proposes the control scheme of Figure 1. The objectives of the control law proposed in [9] are to guarantee

- *Stability*:
$$x(t) \leq r^0 \quad t \geq 0$$
where r^0 is the queue capacity. This condition guarantees no cell loss and is not the usual stability requirement;
- *Full Link Utilization*:
$$x(t) \geq 0 \qquad t > T_{tr}. \tag{2}$$

The time T_{tr} in (2) mainly accounts for the transient time of the dynamics.

Assuming that $\hat{T}_i = T_i$ and disregarding for the moment the presence of d, the scheme in Figure 1 can be shown to be equivalent to the one in Figure 2 [9]. In this case, the designer knows exactly the value of all delays $T_i, i = 1, \cdots, n$, and the closed-loop system exhibits the following nice properties:

- Considering $r(t) = r^0 \cdot 1(t)$ where $1(t)$ is the unit step, and $d(t) = a \cdot 1(t)$, with $0 \leq a \leq 1$ the *stability property* is satisfied.
- Considering $d(t) = a \cdot 1(t)$, with $0 \leq a \leq 1$ the *full utilization property* is satisfied, provided that
$$r^0 > a \left(\frac{1}{k} + \frac{1}{n} \sum_{i=1}^{n} T_i \right).$$

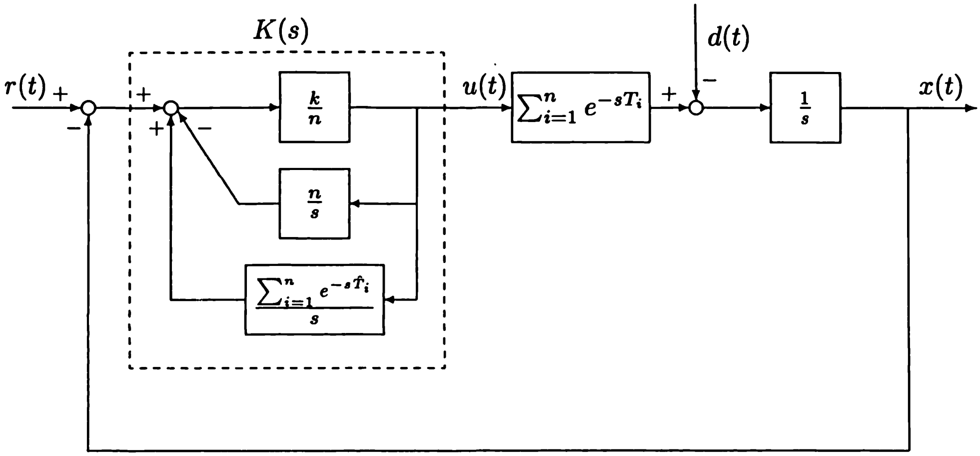

Fig. 1. The controlled system.

The technique which is used in order to obtain the desired closed-loop system is the well-known Smith's principle [12].

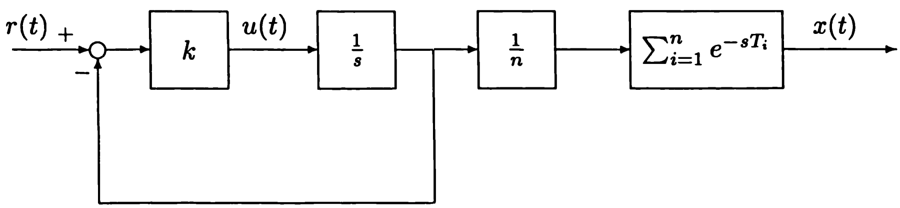

Fig. 2. The desired closed-loop system.

The main drawback of the approach of [9] is that it works well as long as the propagation delays T_i are *exactly* known. When this is not the case, even stability, *in the sense defined above*, can be lost. In order to illustrate this limitation and the potential of losing stability, we carried out the following simulation: Assuming that $n = 4$, $T_1 = 10$s, $T_2 = 30$s, $T_3 = 60$s, $T_4 = 120$s, $k = 0.1$, and $r^0 = 40$, the stability condition is satisfied (see Figure 3) if we choose $\hat{T}_i = T_i$, $i = 1, \ldots 4$.

Then we perturbed all the T_i, $i = 1, \cdots, n$ by 5 %, without modifying the assumed $\hat{T}_i$, $i = 1, \cdots, n$. As shown in Figure 4, stability is lost since $x(t)$ is greater than r^0 during some time intervals. However, we found that stability is regained if the controller gain k is changed to $k = 0.01$ (see Figure 5) at the expense of a slower response. When the T_is however are not known, the designer cannot a-priori design a Smith-predictor-type controller in order to guarantee the desired degree of stability and performance.

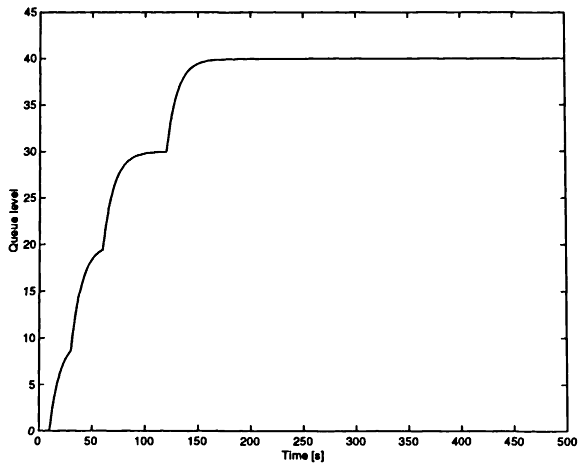

Fig. 3. Queue level when the propagation delays are assumed to be known.

Our choice of $k = 0.01$ came out of a trial-and-error procedure. How then should one choose the *best* controller gain k, assuming the controller structure proposed in [9]? In order to give an answer to this question we propose an algorithm based on statistical learning theory [7].

3. CONTROLLER DESIGN

In this section, using a randomized algorithm which is described in detail and proven elsewhere [7], we shall describe a way of choosing the controller gain k (see Section 2.1) in an *optimal* way. The basic idea of this control design approach is to convert a difficult control design problem into a sequence of efficient analysis problems. The number of these analysis problems will be determined by our algorithm, and each one of them must be efficiently resolved. This type of approach works well when we have a fixed-structure, multiobjective control design problem and the corresponding analysis problem is efficiently solvable.

The time-delays T_i are assumed to be uncertain but known to lie in a given interval. In fact, let them be uniformly distributed in the following intervals

$$T_1 \in [9, 11], \quad T_2 \in [27, 33], \quad T_3 \in [54, 66], \quad T_4 \in [106, 132]$$

i. e. there is a 10 % uncertainty in their value, and the amplitude a of the disturbance $d(t)$ is assumed to have the maximum value $a = 1$. Our objective is to find the controller which solves the following problem: given a certain, fixed value for r^0, find a controller gain k which guarantees the best possible robust performance, in terms of stability and full utilization as defined in Section 2.1 in the presence of the worst-case disturbance $d(t) = a \cdot 1(t)$, with $a = 1$. More formally our target is to find a controller gain k such that, given $r^0 = 95$.

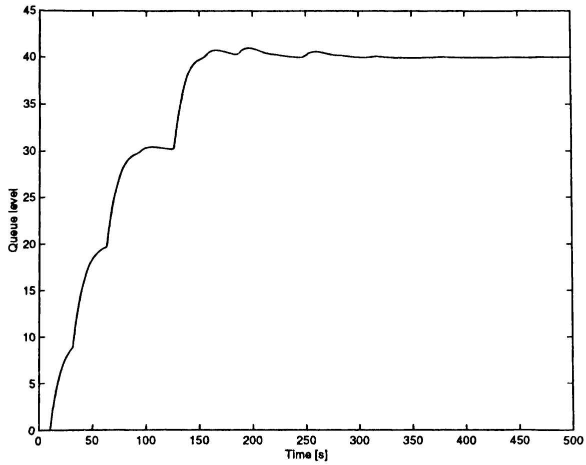

Fig. 4. Queue level when the propagation delays are perturbed by 5 %.

1. The closed-loop system with the nominal time delay is guaranteed to be both stable and to fully utilize the network's capabilities in the presence of the worst-case disturbance with a value of T_{tr} (see (2)) no greater than 20.

2. A certain cost function is minimized in the presence of uncertain time delays. This cost function accounts for the closed-loop stability and performance (in terms of full utilization) in the presence of variations of the time delays. The desired value for T_{tr} is $T_{tr} \leq 30$.

We assume no previous design experience in the value of the controller gain and so we choose k to be uniformly distributed in the interval $\mathcal{Y} = [0.01, 1]$.

In order to use the randomized algorithm methodology, this problem has been reformulated in the following way (see also [7, 15]). We let $X = [T_1 \;\; T_2 \;\; T_3 \;\; T_4]$ and $Y = k$, and define a cost function

$$\Psi(Y) = \max\{\psi_1(Y), \psi_2(Y)\} \tag{3}$$

where

$$\psi_1(Y) = \begin{cases} 1 & \text{if the nominal plant is not} \\ & \text{stabilized or the full utilization} \\ & \text{property is not guaranteed with a } T_{tr} \leq 20 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and

$$\psi_2(Y) = E\left(\zeta(X, Y)\right) \tag{5}$$

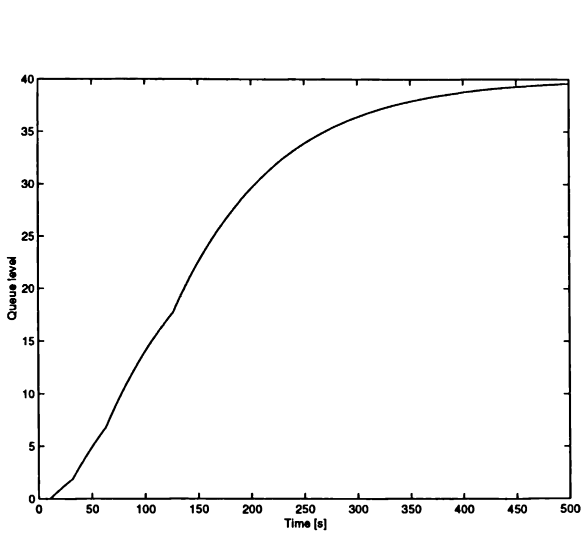

Fig. 5. Queue level when the propagation delays are perturbed by 5 % and $k = 0.01$.

where E indicates the *expected value* with respect to X, and

$$\zeta(X,Y) = \begin{cases} 1 & \text{if the randomly generated} \\ & \text{plant is not stabilized or the full utilization} \\ & \text{property is not guaranteed with a } T_{tr} \leq 30 \\ 0 & \text{otherwise.} \end{cases}$$

Our aim is to minimize the cost function (3) over $\mathcal{Y}$. The optimal controller is then characterized by the parameter Y^* for which

$$\Psi^* := \Psi(Y^*) = \inf_{Y \in \mathcal{Y}} \Psi(Y). \tag{6}$$

Finding the scalar Y^* which minimizes (6) would imply the evaluation of the expected value in (5) and then the minimization of (3) over the set $\mathcal{Y}$. What we shall find is a *suboptimal* solution, a probably approximate near minimum of $\Psi(Y)$ with confidence $1 - \delta$, level α and accuracy ϵ (see [7]).

Definition 1. Suppose $\Psi : \mathcal{Y} \to \mathbb{R}$, that P is a given probability measure on $\mathcal{Y}$, and that $\alpha \in (0,1)$, $\delta \in (0,1)$ and $\epsilon > 0$ are given. A number Ψ_0 is a a probably approximate near minimum of $\Psi(Y)$ with confidence $1 - \delta$, level α and accuracy ϵ, if

$$\text{Prob}\left\{ \inf_{Y \in \mathcal{Y}} \Psi(Y) - \epsilon \leq \Psi_0 \leq \inf_{Y \in \mathcal{Y} \setminus \mathcal{S}} \Psi(Y) + \epsilon \right\} \geq 1 - \delta \tag{7}$$

with some measurable set $\mathcal{S} \subseteq \mathcal{Y}$ such that $P(\mathcal{S}) \leq \alpha$. In (7), $\mathcal{Y} \setminus \mathcal{S}$ indicates the complement of the set $\mathcal{S}$ in $\mathcal{Y}$.

An interpretation of the definition is that we are not searching for the minimum over all of the set $\mathcal{Y}$ but only over its subset $\mathcal{Y} \setminus \mathcal{S}$, where $\mathcal{S}$ has a small measure (at most α). Unless the actual infinum Ψ^* is attained in the exceptional set $\mathcal{S}$, Ψ_0 is within ϵ from the actual infinum with confidence $1-\delta$. Although using Monte Carlo type minimization, it is unlikely to obtain a better estimate of Ψ^* than Ψ_0 (since the chances of getting into the set $\mathcal{S}$ are small), nothing can be said in practice about the size of the difference $\Psi_0 - \Psi^*$.

Based on the randomized algorithms discussed in [7], a probably approximate near minimum of $\Psi(Y)$ with confidence $1-\delta$, level α and accuracy ϵ, can be found with the following Procedure, which was derived in [7].

Procedure

Step 1. Let $j = 0$.

Step 2. Choose n controllers with random uniformly distributed coefficients $Y_1, \ldots, Y_n \in \mathcal{Y}$, where (we indicate by $\lfloor\cdot\rfloor$ the floor operator)

$$n = \left\lfloor \frac{\log(2/\delta)}{\log[1/(1-\alpha)]} \right\rfloor.$$

Evaluate for these controllers the function ψ_1 in (4) and discard those controllers for which $\psi_1 = 1$. Let $\hat{n}$ be the number of the remaining controllers.

Step 3. Choose m plants generating random parameters $X_1, \ldots, X_m \in \mathcal{X}$ with uniform distribution, where

$$m = 2^k \left\{ \left\lfloor \frac{100}{\epsilon^2} \log\left(\frac{8}{\delta}\right) \right\rfloor + 1 \right\}.$$

Step 4. Evaluate the stopping variable

$$\gamma = \max_{1 \le j \le \hat{n}} \left| \frac{1}{m} \sum_{i=1}^{m} r_i \zeta(X_i, Y_j) \right|$$

where r_i are *Rademacher* random variables, i.e. independent identically distributed random variables taking values $+1$ and -1 with probability $\frac{1}{2}$ each. If $\gamma > \frac{\epsilon}{5}$, let $j = j+1$ and go back to Step 3.

Step 5. Choose the controller which minimizes the function

$$\frac{1}{m} \sum_{i=1}^{m} \zeta(X_i, \cdot).$$

This is the *suboptimal* controller in the sense defined above.

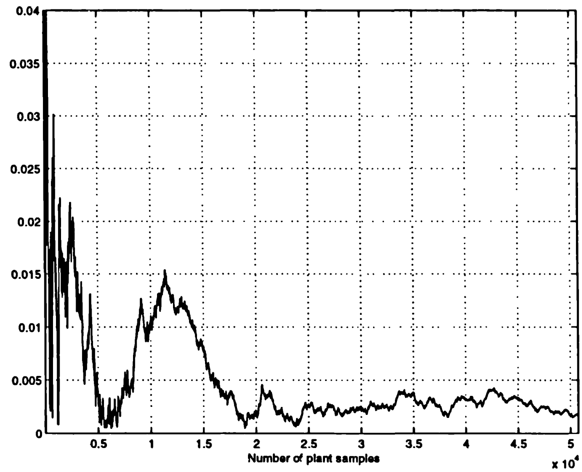

Fig. 6. The stopping variable.

Remark 1. The proposed algorithm consists of two distinct parts: the estimate of the expected value in (5), which is given with an accuracy ϵ and a confidence $1-\delta/2$, and the minimization procedure which is carried out with a confidence $1-\delta/2$ and introduces the level α. As it can be seen from the Procedure, the number m of samples in $\mathcal{X}$ which are needed to achieve the estimate of the expected value (5), known as the *sample complexity*, is not known *a priori* but is itself a random variable. The upper bounds for this random sample complexity however, are of the same order of those that can be found in [14].

In our case, the procedure needed just one iteration to converge, i.e. $j = 1$. Therefore, for $\delta = 0.05$, $\alpha = 0.005$ and $\epsilon = 0.1$, n evaluated to 736 controllers and m evaluated to 50,753 plants. In Figure 6, the stopping variable is shown. The *suboptimal* controller is $k = 0.9787$, and the corresponding value of the cost function is $\Psi_0 = 0.5020$. The performance of this controller is illustrated in Figure 7 for the noniman $T_i s$ and the case of no disturbance, although the non-nominal cases are also shown to meet all design specifications.

4. CONCLUSIONS

In this paper we have illustrated a new approach to the design of fixed-structure control design for multiple, uncertain time-delay systems. We illustrated our design approach for controlling the ABR case in an ATM communications network. We showed that by using statistical learning concepts, we were able to design a controller that will not only guarantee various performance objectives for the nominal ATM system, but to do so even when the multiple-time delays are not exactly known.

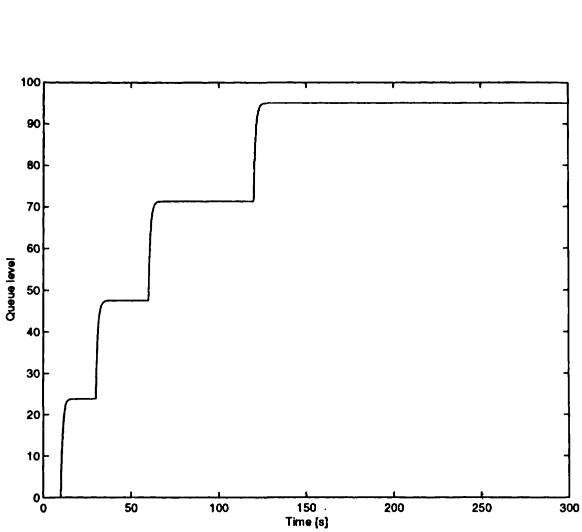

Fig. 7. The performance of the closed-loop system.

The same design approach may be used in other multi-delay control problems or to include other performance objectives.

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