KYBERNETIKA — VOLUME 37 (2001), NUMBER 2, PAGES 165 – 170

ON SOMEWHAT FUZZY SEMICONTINUOUS FUNCTIONS

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In this paper the concept of somewhat fuzzy semicontinuous functions, somewhat fuzzy semiopen functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. More examples are given to illustrate the concepts introduced in this paper.

1. INTRODUCTION

Ever since the introduction of fuzzy sets by L. A. Zadeh [14], the fuzzy concept has invaded almost all branches of Mathematics. The concept of fuzzy topological space was introduced and developed by C. L. Chang [3] and since then many fuzzy topologists [1, 6 – 13] have extended various notions in classical topology to fuzzy topological spaces. The concept of somewhat continuous functions was introduced in [5] and this concept was studied in connection with the idea of feebly continuous functions and feebly open functions introduced in [4]. The purpose of this paper is to extend this concept to fuzzy topological spaces. In this connection we have introduced the concept of somewhat fuzzy semicontinuous functions and somewhat fuzzy semiopen functions and studied their properties. Also further we have introduced the concept of fuzzy semi irresolvable and fuzzy semi resolvable spaces and we have given a characterization of fuzzy semi irresolvable spaces. Several examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

By a fuzzy topological space we shall mean a non-empty set X together with a fuzzy topology T (in the sense of Chang) and denote it by (X,T). A fuzzy set λ in X is called proper if $\lambda \neq 0$ and $\lambda \neq 1$. If λ and μ are any two fuzzy sets in X and Y respectively, we define [1] $\lambda \times \mu : X \times Y \to I$ as follows: $(\lambda \times \mu)(x,y) = \mathrm{Min}(\lambda(x), \mu(y))$. A fuzzy topological space X is product related [2] to a fuzzy topological space Y if for any fuzzy set υ in X and ξ in Y whenever $\lambda' (= 1 - \lambda) \not\geq \upsilon$ and $\mu' (= 1 - \mu) \not\geq \xi$ imply $\lambda' \times 1 \vee 1 \times \mu' \geq \upsilon \times \xi$ where λ is a fuzzy open set in X and μ is a fuzzy open set in Y, there exists a fuzzy open set λ_1 in X and a fuzzy open set μ_1 in Y such that $\lambda_1' \geq \upsilon$ or $\mu_1' \geq \xi$ and $\lambda_1' \times 1 \vee 1 \times \mu_1' = \lambda' \times 1 \vee 1 \times \mu_1'$. If (X, T) and (Y, S) are any two fuzzy topological spaces, we define the product fuzzy topology [1] $T \times S$ on $X \times Y$ to be that fuzzy topology for which $\mathcal{B} = \{\lambda \times \mu | \lambda \in \mathcal{T}, \mu \in \mathcal{S}\}$ forms a base.

A fuzzy set λ in a fuzzy topological space X is called fuzzy semiopen [2] if for some fuzzy open set υ we have $\upsilon \leq \lambda \leq cl\, \upsilon$ and the complement of a fuzzy semiopen set is called a fuzzy semiclosed set in X. A function f from a fuzzy topological space (X, T) to a fuzzy topological space (Y, S) is said to be fuzzy continuous if for each fuzzy open set λ in S the inverse image $f^{-1}(\lambda)$ is a fuzzy open set in T. f is called fuzzy open if the image of each fuzzy open λ in (X, T) is a fuzzy open set in (Y, S).

Let λ be any fuzzy set in the fuzzy topological space. Then we define the fuzzy semi interior of $\lambda = s\text{-}int(\lambda) = \vee\{\mu | \mu$ is fuzzy semiopen and $\mu \leq \lambda\}$ and the fuzzy semi closure of $\lambda = s\text{-}cl(\lambda) = \wedge\{\mu | \mu$ is fuzzy semiclosed and $\mu \geq \lambda\}$. For any fuzzy set δ in a fuzzy topological space, it is easy to see that $1 - s\text{-}cl(\delta) = s\text{-}int(1 - \delta)$. For a mapping $f : X \to Y$, the graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$ for each $x \in X$.

3. SOMEWHAT FUZZY SEMICONTINUOUS FUNCTIONS

Definition 1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called somewhat fuzzy semicontinuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0 \Rightarrow$ there exists a fuzzy semiopen set μ of X such that $\mu \leq f^{-1}(\lambda)$.

It is clear from the definition that every fuzzy continuous function is somewhat fuzzy semicontinuous. The following example shows that the reverse implication need not be true.

Example 1. Let μ_1, μ_2 be fuzzy sets of I where $I = [0, 1]$, defined as

$$\mu_1(x) = \begin{cases} 0 & 0 \leq x \leq \frac{1}{2}; \\ 2x - 1 & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & 0 \leq x \leq \frac{1}{4}; \\ -4x + 2 & \frac{1}{4} \leq x \leq \frac{1}{2}; \\ 0 & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Clearly $T = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ is a fuzzy topology on I. Let $f : (I, T) \to (I, T)$ be defined as $f(x) = \frac{x}{2}$. Then f *is somewhat fuzzy semicontinuous* and f *is not fuzzy continuous.*

PROPERTIES

The following properties in connection with somewhat fuzzy semicontinuous function can be established easily.

Proposition 1. Let (X,T), (Y,S) and (Z,Q) be fuzzy topological spaces. Suppose that $f : (X,T) \to (Y,S)$ is a somewhat fuzzy semicontinuous function and $g : (Y,S) \to (Z,Q)$ is a somewhat fuzzy continuous. Then $g \circ f : (X,T) \to (Z,Q)$ is a somewhat fuzzy semicontinuous function.

Definition 2. A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy semidense if there exists no fuzzy semiclosed set μ such that $\lambda < \mu < 1$.

Proposition 2. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let $f : (X,T) \to (Y,S)$ be a function. Then the following are equivalent.

1. f is somewhat fuzzy semicontinuous.
2. If λ is a fuzzy closed set of Y such that $f^{-1}(\lambda) \neq 1$, then there exists a fuzzy semiclosed set μ of X such that $\mu > f^{-1}(\lambda)$.
3. If λ is a fuzzy semidense set in X, then $f(\lambda)$ is a fuzzy semidense set in Y.

Proposition 3. Let (X_1,T_1), (X_2,T_2), (Y_1,S_1) and (Y_2,S_2) be fuzzy topological spaces such that X_1 is product related to X_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of somewhat fuzzy semicontinuous functions $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$, is somewhat fuzzy semicontinuous.

Proposition 4. Let $f : (X,T) \to (Y,S)$ be a function from a fuzzy topological space (X,T) to another fuzzy topological space (Y,S). Then if the graph $g : X \to X \times Y$ of f is somewhat fuzzy semicontinuous then f is also somewhat fuzzy semicontinuous.

The following example shows that the somewhat fuzzy semicontinuity of f need not imply the somewhat fuzzy semicontinuity of the graph of f.

Example 2. Let $I = [0,1]$. Let μ_1, μ_2 be fuzzy sets of I defined as

$$\mu_1(x) = \begin{cases} 0 & 0 \le x \le \frac{1}{2}; \\ 2x-1 & \frac{1}{2} \le x \le 1. \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & 0 \le x \le \frac{1}{4}; \\ -4x+2 & \frac{1}{4} \le x \le \frac{1}{2}; \\ 0 & \frac{1}{2} \le x \le 1. \end{cases}$$

Clearly $T = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ and $T_1 = \{0, \mu_1, 1\}$ are fuzzy topologies on I. Let $f : (I,T) \to (I,T_1)$ be defined as $f(x) = \min(2x,1)$ for each $x \in I$. Then *f is somewhat fuzzy semicontinuous.* But the *graph of f is not somewhat fuzzy semicontinuous.*

Proposition 5. Let X, X_1 and X_2 be fuzzy topological spaces and $p_i : X_1 \times X_2 \to X_i (i = 1, 2)$ be the projection mappings. If $f : X \to X_1 \times X_2$ is a somewhat fuzzy semicontinuous, then $p_i \circ f$ is also a somewhat fuzzy semicontinuous function.

4. SOMEWHAT FUZZY SEMI OPEN FUNCTIONS

Definition 3. Let (X, T) and (Y, S) be fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called somewhat fuzzy semiopen function if and only if for all $\lambda \in T$, $\lambda \neq 0$ there exists a fuzzy semiopen set μ of Y such that $\mu \neq 0$ and $\mu \leq f(\lambda)$.

Clearly every fuzzy open function [1] is somewhat fuzzy semiopen. The following example shows that the reverse implication need not be true.

Example 3. Let μ_1, μ_2 be fuzzy sets of I defined as

$$\mu_1(x) = \begin{cases} 0 & 0 \leq x \leq \frac{1}{2}; \\ 2x - 1 & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & 0 \leq x \leq \frac{1}{4}; \\ -4x + 2 & \frac{1}{4} \leq x \leq \frac{1}{2}; \\ 0 & \frac{1}{2} \leq x \leq 1. \end{cases}$$

Clearly $T = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ is a fuzzy topology on I. Let $g : (I, T) \to (I, T)$ be defined by $g(x) = \min(2x, 1)$ for each $x \in X$. Then g *is somewhat fuzzy semiopen* whereas g *is not fuzzy open.*

PROPERTIES

The following properties in connection with somewhat fuzzy semiopen function can be proved easily.

Proposition 6. Let (X, T), (Y, S) and (Z, Q) be fuzzy topological spaces. Suppose that $f : (X, T) \to (Y, S)$ is fuzzy open and $g : (Y, S) \to (Z, Q)$ is somewhat fuzzy semiopen then $g \circ f : (X, T) \to (Z, Q)$ is somewhat fuzzy semiopen.

Proposition 7. Suppose (X, T) and (Y, S) be fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an onto function. Then the following conditions are equivalent.

1. f is somewhat fuzzy semiopen.

2. If λ is a fuzzy semidense set in Y, then $f^{-1}(\lambda)$ is a fuzzy semidense set in X.

Proposition 8. Suppose (X,T) and (Y,S) be fuzzy topological spaces. Let $f : (X,T) \to (Y,S)$ be a 1-1 and onto function. Then the following conditions are equivalent.

1. f is somewhat fuzzy semiopen.
2. If λ is a fuzzy closed set in X such that $f(\lambda) \neq 1$, then there exists a fuzzy semiclosed set μ in Y such that $\mu \neq 1$ and $\mu > f(\lambda)$.

5. CHARACTERIZATION OF FUZZY SEMI IRRESOLVABLE SPACES

Definition 4. Let (X,T) be a fuzzy topological space. (X,T) is called fuzzy semi resolvable if (X,T) has a pair of fuzzy semi dense sets λ_1 and λ_2 such that $\lambda_1 \leq 1 - \lambda_2$. Otherwise (X,T) is called fuzzy semi irresolvable.

Definition 5. A function $f : (X,T) \to (Y,S)$ is said to be weakly somewhat fuzzy semiopen if for each semidense fuzzy set λ in (Y,S) with $s\text{-}int\,\lambda \neq 0$, we have that $f^{-1}(\lambda)$ is also a fuzzy semidense set in (X,T).

The above definition leads to a characterization of fuzzy semi irresolvable spaces as follows:

Proposition 9. The following statements are equivalent for a fuzzy topological space (Y,S).

1. (Y,S) is fuzzy semi irresolvable.
2. For every fuzzy topological space (X,T), every weakly somewhat fuzzy semiopen function $f : (X,T) \to (Y,S)$ is somewhat fuzzy semiopen.

The following proposition shows the relationship between a weakly somewhat fuzzy semiopen function and its graph function.

Proposition 10. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Assume X and Y are product related. Let $f : (X,T) \to (Y,S)$ be a function. If the graph function of f, $g : (X,T) \to (X \times Y, T \times S)$ is weakly somewhat fuzzy semiopen, then f is weakly somewhat fuzzy semiopen.

(Received February 23, 2000.)

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