KYBERNETIKA — VOLUME 35 (1999), NUMBER 2, PAGES 215–223

FUZZY β-OPEN SETS AND FUZZY β-SEPARATION AXIOMS

GANESAN BALASUBRAMANIAN

In this paper fuzzy separation axioms have been introduced and investigated with the help of fuzzy β-open sets.

1. INTRODUCTION

The fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept by Zadeh [10]. Fuzzy sets have applications in many fields such as information [9] and control [8]. The theory of fuzzy topological spaces was introduced and developed by Chang [6] and since then various notions in classical topology have been extended to fuzzy topological spaces. The concept of β-open sets was introduced in [1] and studied also by Allam and El Hakeim [2]. In [3] this concept has been generalized to fuzzy setting. Our motivation in this paper is to define fuzzy β-interiors and fuzzy β-closures and investigate their properties. Also more attention is given to the extension of separation axioms to fuzzy topological spaces by using fuzzy β-open sets. A new approach is followed in defining these separation axioms. In this approach separation axioms are defined in terms of fuzzy sets where as in the usual ones, they are given in terms of fuzzy points.

2. PRELIMINARIES

Definition 1. Let (X, T) be a fuzzy topological space and let λ be any fuzzy set in X.

λ is called fuzzy α-open [4] if $\lambda \leq \operatorname{Int} \operatorname{cl} \operatorname{Int} \lambda$

λ is called fuzzy preopen [4] if $\lambda \leq \operatorname{Int} \operatorname{cl} \lambda$

λ is called fuzzy semiopen [4] if $\lambda \leq \operatorname{cl} \operatorname{Int} \lambda$.

Definition 2. For $t \in (0,1)$, x_t stands for the function from $(X, T) \to [0,1]$ defined by

$$\begin{aligned} x_t(y) &= 0 \quad \text{if} \quad y \neq x \\ &= 1 \quad \text{if} \quad y = x \end{aligned}$$

and is referred to as the fuzzy point with support x and value t.

Definition 3. Let λ and μ be any two fuzzy sets in (X,T). Then we define $\lambda \vee \mu : X \to [0,1]$ as follows:

$$(\lambda \vee \mu)(x) = \text{Max}\{\lambda(x), \mu(x)\}.$$

Also we define $\lambda \wedge \mu : X \to [0,1]$ as follows:

$$(\lambda \wedge \mu)(x) = \text{Min}\{\lambda(x), \mu(x)\}.$$

Definition 4. A fuzzy topological space (X,T) is called fuzzy T_1 or FT_1 [5] iff for every pair of non-zero fuzzy sets α, β with $\alpha \not\leq \beta$ there exists an open fuzzy set λ such that $\beta \leq \lambda$ and $\alpha \not\leq \lambda$.

Definition 5. A fuzzy topological space (X,T) is called fuzzy T_4 or FT_4 [5] iff for every pair of non-zero fuzzy closed sets α, β with $\alpha \wedge \beta = 0$, there exist fuzzy open sets λ and μ such that $\alpha \leq \lambda$, $\beta \leq \mu$ and $\lambda \wedge \mu = 0$.

Definition 6. Let X and Y be any two non-empty sets. Let $f : X \to Y$ be any function from X to Y. Let λ be a fuzzy set in X. Then the inverse image of λ under f written as $f^{-1}(\lambda)$ is the fuzzy set in X defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$. Also the image of λ under f written as $f(\lambda)$ is the fuzzy set defined by

$$\begin{aligned} f(\lambda)(y) &= \sup_{z \in f^{-1}(y)} \lambda(z) \quad \text{if } f^{-1}(y) \text{ is not empty} \\ &= 0, \qquad\qquad \text{otherwise.} \end{aligned}$$

3. FUZZY β-OPEN SETS AND FUZZY β-CLOSED SETS

Definition 7. Let λ be any fuzzy set in the fuzzy topological space (X,T). λ is called fuzzy β-open [3] if $\lambda \leq \text{cl}\,\text{Int}\,\text{cl}\,\lambda$. The complement of a fuzzy β-open set is called fuzzy β-closed.

Fuzzy β-closure and fuzzy β-interior of a fuzzy set are defined as follows:

Definition 8. Let λ be any fuzzy set in the fuzzy topological space (X,T). Then we define

$$\begin{aligned} \beta - \text{cl}(\lambda) &= \wedge\{\mu \mid \mu \text{ is fuzzy } \beta\text{-closed and } \mu \geq \lambda\} \\ \beta - \text{Int}(\lambda) &= \vee\{\mu \mid \mu \text{ is fuzzy } \beta\text{-open and } \mu \leq \lambda\}. \end{aligned}$$

Proposition 1. Let λ be any fuzzy set in (X,T). Then $\beta - \text{cl}(1-\lambda) = 1 - \beta - \text{Int}(\lambda)$ and $\beta - \text{Int}(1-\lambda) = 1 - \beta - \text{cl}(\lambda)$.

Proposition 2. In a fuzzy topological space (X, T), the following hold for fuzzy β-closure.

(a) $\beta - \mathrm{cl}\,(0) = 0$

(b) $\beta - \mathrm{cl}\,(\lambda)$ is fuzzy β-closed in (X, T)

(c) $\beta - \mathrm{cl}\,(\lambda) \leq \beta - \mathrm{cl}\,(\mu)$ if $\lambda \leq \mu$

(d) $\beta - \mathrm{cl}\,(\beta - \mathrm{cl}\,(\lambda)) = \beta - \mathrm{cl}\,(\lambda)$

(e) $\beta - \mathrm{cl}\,(\lambda \vee \mu) \geq \beta - \mathrm{cl}\,(\lambda) \vee \beta - \mathrm{cl}\,(\mu)$

(f) $\beta - \mathrm{cl}\,(\lambda \vee \mu) \leq \beta - \mathrm{cl}\,(\lambda) \wedge \beta - \mathrm{cl}\,(\mu)$.

Similar results hold for fuzzy β-interiors.

Definition 9. Let (X, T) be fuzzy topological space and Y be an ordinary subset of X. Then $T_Y = (\lambda / Y \mid \lambda \in T)$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology. The pair (Y, T_Y) is called a fuzzy subspace of $(X, T) : (Y, T_Y)$ is called an fuzzy open / fuzzy closed / fuzzy β-open fuzzy subspace if the characteristic function of Y viz χ_Y is fuzzy open / fuzzy closed / fuzzy β-open respectively.

Proposition 3. Let (X, T) be a fuzzy topological space. Suppose $Z \subseteq Y \subseteq X$ and (Y, T_Y) is a fuzzy β-open fuzzy subspace of (X, T). Then Z is fuzzy β-open fuzzy subspace in $X \Leftrightarrow Z$ is fuzzy β-open fuzzy subspace in Y.

Proof. Suppose Z is fuzzy β-open fuzzy subspace in X. Then $\chi_Z \leq \mathrm{cl}\,\mathrm{Int}\,\mathrm{cl}\,(\chi_Z)$. But $Z \subseteq Y$ implies $\chi_Z \wedge \chi_Y = \chi_Z$ so that

$$\chi_Z \wedge \chi_Y = \chi_Z \leq \mathrm{cl}\,\mathrm{Int}\,\mathrm{cl}\,\chi_Z = \chi_Z \wedge \chi_Y .$$

This implies χ_Z is fuzzy β-open in Y. That is Z is fuzzy β-open fuzzy subspace in Y. □

Notation. $F\beta O(X, T)$ denotes the set of all fuzzy β-open sets in (X, T).

Definition 10. Let $f : (X, T) \to (Y, S)$ be a function. f is called fuzzy β-continuous if the inverse image of each fuzzy open set in Y is fuzzy β-open in X. f is called M-fuzzy β-continuous if the inverse image of fuzzy β-open set in Y is fuzzy β-open in X, f is called fuzzy β-open if the image of each fuzzy β-open set in X is fuzzy β-open in Y.

Proposition 4. For a fuzzy topological space (X, T) the following are valid.

(a) $T \subseteq F\beta O(X, T)$.

(b) If μ is a fuzzy set in X and λ is a fuzzy preopen set in X such that $\lambda \leq \mu \leq \mathrm{cl}\,\mathrm{Int}\,\lambda$, then μ is a fuzzy β-open set.

Proof. (a) Straight forward. (b) Since λ is a fuzzy preopen set $\lambda \leq \operatorname{Int}\operatorname{cl}\lambda$. Now $\mu \leq \operatorname{cl}\operatorname{Int}\lambda \leq \operatorname{cl}\operatorname{Int}(\operatorname{Int}\operatorname{cl}\lambda) = \operatorname{cl}\operatorname{Int}\operatorname{cl}\lambda \leq \operatorname{cl}\operatorname{Int}\operatorname{cl}\mu$.
This shows that μ is fuzzy β-open. □

Proposition 5. Let $f : (X,T) \to (X,T')$ be a fuzzy continuous and fuzzy open. Then f is fuzzy β-open.

Proof. Let λ be any fuzzy β-open set. Then $\lambda \leq \operatorname{cl}\operatorname{Int}\operatorname{cl}\lambda$. Therefore $f(\lambda) \leq f[\operatorname{cl}\operatorname{Int}\operatorname{cl}\lambda] \leq \operatorname{cl}\operatorname{Int}\operatorname{cl} f(\lambda) \Rightarrow f(\lambda)$ is fuzzy β-open. That is f is fuzzy β-open. □

Proposition 6. If λ is fuzzy closed and μ is fuzzy β-open then $\lambda \vee \mu$ is fuzzy β-open.

Proof. By hypothesis $\mu \leq \operatorname{cl}\operatorname{Int}\operatorname{cl}\mu$. Now

$$\lambda \vee \mu \leq \lambda \vee \operatorname{cl}\operatorname{Int}\operatorname{cl}\mu = \operatorname{cl}\operatorname{Int}\operatorname{cl}\lambda \vee \operatorname{cl}\operatorname{Int}\operatorname{cl}\mu \leq \operatorname{cl}\operatorname{Int}\operatorname{cl}(\lambda \vee \mu).$$

This shows that $\lambda \vee \mu$ is fuzzy β-open. □

Proposition 7. Let $f : (X,T) \to (Y,T')$ be fuzzy continuous and fuzzy open. Then f is M-fuzzy β-continuous.

Proof. Let λ be any fuzzy β-open set in Y. Then $\lambda \leq \operatorname{cl}\operatorname{Int}\operatorname{cl}\lambda$. Since f is fuzzy continuous and fuzzy open it follows that

$$f^{-1}(\lambda) \leq f^{-1}(\operatorname{cl}\operatorname{Int}\operatorname{cl}\lambda) = \operatorname{cl} f^{-1}(\operatorname{Int}\operatorname{cl}\lambda) \leq \operatorname{cl}\operatorname{Int} f^{-1}(\operatorname{cl}\lambda) = \operatorname{cl}\operatorname{Int}\operatorname{cl} f^{-1}(\lambda).$$

This shows that $f^{-1}(\lambda)$ is fuzzy β-open. That is f is M-fuzzy β-continuous. □

Proposition 8. A function $f : X \to Y$ is M-fuzzy β-continuous $\Leftrightarrow$ For every fuzzy β-closed set λ of Y, $f^{-1}(\lambda)$ is fuzzy β-closed.

Proposition 9. In a fuzzy topological space (X,T) the following are valid:

(a) λ is fuzzy β-open $\Leftrightarrow \beta - \operatorname{Int}(\lambda) = \lambda$.

(b) λ is fuzzy β-closed $\Leftrightarrow \beta - \operatorname{cl}(\lambda) = \lambda$.

Proposition 10. $f : (X,T) \to (Y,T')$ is M-fuzzy β-continuous $\Leftrightarrow$ For every fuzzy set λ of X, $f(\beta - \operatorname{cl}\lambda) \leq \beta - \operatorname{cl} f(\lambda)$.

Proof. Suppose f is M-fuzzy β-continuous. Now $\beta - \operatorname{cl} f(\lambda)$ is fuzzy β-closed. By hypothesis $f^{-1}[\beta - \operatorname{cl} f(\lambda)]$ is fuzzy β-closed. And $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(\beta - \operatorname{cl} f(\lambda))$. Hence by the definition of β-fuzzy closure, $\beta - \operatorname{cl}(\lambda) \leq f^{-1}(\beta - \operatorname{cl}(\lambda))$. That is $f(\beta - \operatorname{cl}(\lambda)) \leq \beta - \operatorname{cl}(f(\lambda))$.

Conversely suppose that λ is fuzzy β-closed in Y. Now by hypothesis

$$f[\beta - \operatorname{cl} f^{-1}(\lambda)] \leq \beta - \operatorname{cl} f[f^{-1}(\lambda)] = \lambda.$$

This implies $\beta - \operatorname{cl}(f^{-1}(\lambda)) \leq f^{-1}(\lambda)$ so that $f^{-1}(\lambda) = \beta - \operatorname{cl}(f^{-1}(\lambda))$.

That is $f^{-1}(\lambda)$ is fuzzy β-closed and so f is M-fuzzy β-continuous. □

Proposition 11. $f : X \to Y$ is M-fuzzy β-continuous $\Leftrightarrow$ For all fuzzy sets λ of Y, $\beta - \mathrm{cl}\, f^{-1}(\lambda) \leq f^{-1}(\beta - \mathrm{cl}\, \lambda)$.

Proof. Suppose f is M-fuzzy β-continuous. Now $\beta - \mathrm{cl}\,(\lambda)$ is fuzzy β-closed so that $f^{-1}(\beta - \mathrm{cl}\, \lambda)$ is fuzzy β-closed. Since $f^{-1}(\lambda) \leq f^{-1}(\beta - \mathrm{cl}\, \lambda)$, it follows from the definition of β-fuzzy closure that $\beta - \mathrm{cl}\,(f^{-1}(\lambda)) \leq f^{-1}(\beta - \mathrm{cl}\, \lambda)$.

Conversely suppose λ is fuzzy β-closed in Y. Then $\beta - \mathrm{cl}\, \lambda = \lambda$. Now by hypothesis

$$\beta - \mathrm{cl}\, f^{-1}(\lambda) \leq f^{-1}(\beta - \mathrm{cl}\,(\lambda)) = f^{-1}(\lambda).$$

Therefore

$$\beta - \mathrm{cl}\,[f^{-1}(\lambda)] = f^{-1}(\beta - \mathrm{cl}\, \lambda) = f^{-1}(\lambda).$$

Thus $f^{-1}(\lambda)$ is fuzzy β-closed and so f is M-fuzzy β-continuous. □

The following results are easy to establish.

Proposition 12. Suppose $X \xrightarrow{f} Y \xrightarrow{g} Z$ are both M-fuzzy β-continuous. Then $g \circ f : X \to Z$ is M-fuzzy β-continuous.

Proposition 13. Let $f : X \to Y$ be fuzzy continuous and fuzzy open. Then

(a) f is M-fuzzy β-continuous.

(b) $f^{-1}(\beta - \mathrm{cl}\, \lambda) = \beta - \mathrm{cl}\, f^{-1}(\lambda)$, λ being a fuzzy set in Y.

Definition 11. Let X and Y be fuzzy topological spaces. X and Y are said to be M-fuzzy β-homeomorphic $\Leftrightarrow$ There exists $f : X \to Y$ such that f is $1-1$, onto, M-fuzzy β-continuous and fuzzy β-open. Such an f is called fuzzy β-homeomorphism.

Proposition 14. If $f : X \to Y$ is fuzzy β-homeomorphism, then $\beta - \mathrm{cl}\, f^{-1}(\lambda) = f^{-1}(\beta - \mathrm{cl}\,(\lambda))$ where λ is a fuzzy set in Y.

Corollary. If $f : X \to Y$ is a fuzzy β-homeomorphism, then

(i) $\beta - \mathrm{cl}\,(f(\lambda)) = f(\beta - \mathrm{cl}\, \lambda)$

(ii) $f(\beta - \mathrm{Int}\, \lambda) = \beta - \mathrm{Int}\,[f(\lambda)]$

(iii) $f^{-1}(\beta - \mathrm{Int}\, \lambda) = \beta - \mathrm{Int}\,[f^{-1}(\lambda)]$.

4. FUZZY β-SEPARATION AXIOMS

In this section we consider separation axioms using fuzzy β-open sets. Let $\mathcal{I}(X)$ denote the set of all topologies on X and $\omega(X)$, the set of all fuzzy topologies on X. On the real line R we consider the topology $\mathcal{I}_r = \{(\alpha,\infty)\,|\,\alpha \in R\}\,U\{\phi\}$. The topological space obtained by giving I, the induced topology is denoted by I_r. Define the two mappings [7]

$$I : \omega(X) \to \mathcal{I}(X);\ \ \delta \to I(\delta)$$

where $I(\delta)$ is the initial topology on X for the family of functions δ and the topological space I_r. $\omega : \mathcal{I}(X) \to \omega(X) : \mathcal{I} \to \omega(\mathcal{I})$ where $\omega(\mathcal{I}) = C(X, I_r) =$ The family of all continuous functions from $(X,\mathcal{I})$ to I_r. If $\delta = \omega(\tau)$ for some topology τ, then δ is said to be topologically generated.

Definition 12. A fuzzy topological space (X,δ) is called fuzzy β-open T_1 (in short $F\beta OT_1$) $\Leftrightarrow$ For every pair of non-zero fuzzy sets α, β with $\alpha \nleq \beta$ there exists a fuzzy β-open set λ such that $\beta \leq \lambda$ and $\alpha \nleq \lambda$.

Since every fuzzy open set is fuzzy β-open, it follows that every FT_1 space [5] is $F\beta OT_1$ space.

Notation. The notation $i_{F\beta O}(X,\delta)$ stands for the initial topology generated by $F\beta O(X,\delta)$.

Proposition 15. If (X,δ) is $F\beta OT_1$, then $(X,\, i_{F\beta O}(\delta))$ is T_1.

Proof. Let x, y be any two points of X such that $x \neq y$. Then for any $t \in (0,1)$ we have $x_t \nleq y_t$. As (X,δ) is $F\beta OT_1$, there exists a fuzzy β-open set λ such that $y_t \leq \lambda$ and $x_t \nleq \lambda$. Now $x_t \nleq \lambda \Rightarrow \lambda(x) < t$ and therefore we choose ε in $(0,1)$ such that $\lambda(x) < \varepsilon < t$. Then $x \notin \lambda^{-1}\{(\varepsilon,1]\}$. But since $y_t \leq \lambda$, $\lambda(y) \geq t$ and so $y \in \lambda^{-1}\{(\varepsilon,1]\}$. Now $\lambda \in F\beta O(X,\delta) \Rightarrow \lambda^{-1}(\varepsilon,1] \in i_{F\beta O}(X,\delta)$. As $y_1 \leq x_t$, by the above method one can again show that there is an open set U such that $x \in U$ and $y \notin U$. Thus we find $(X,\, i_{F\beta O}(X,\delta))$ is T_1. □

Proposition 16. If (X,τ) is T_1, then $(X,\, \omega(\tau))$ is $F\beta OT_1$.

Proof. By Theorem 1.2.2 in [3], $(X,\, \omega(\tau))$ is FT_1 and therefore $(X,\, \omega(\tau))$ is $F\beta OT_1$. □

Proposition 17. If (X,δ) is an $F\beta OT_1$ space and A, a subset of X, then $(A,\, \delta/A)$ is an $F\beta OT_1$ space.

Proposition 18. If (X,δ) is an $F\beta OT_1$ space then any fuzzy set λ in X is the intersection of fuzzy β-open sets which contain λ.

Proof. If $\lambda = 0$, there is nothing to prove and we assume $\lambda > 0$. Let

$$\mu = \wedge\{g\,|\,g \in F\beta O(X,\delta),\ \lambda \leq g\}$$

clearly

$$\lambda \leq \mu. \tag{1}$$

Suppose now that $\mu \not\leq \lambda$. Hence by hypothesis there exist a fuzzy β-open set

$$h \in F\beta O(X,\delta) \text{ such that } \lambda \leq h \text{ and } \mu \not\leq h. \tag{2}$$

But $h \in F\beta O(X,\delta)$ and $\lambda \leq h$ imply

$$\mu \leq h. \tag{3}$$

(2) and (3) are contradictions. Hence we must have

$$\mu \leq \lambda. \tag{4}$$

From (1) and (4) we have $\mu = \lambda$. □

Proposition 19. If (X,δ) is an $F\beta OT_1$ space, then every fuzzy set λ in X is the union of fuzzy β-closed sets contained in λ.

Proof. By Proposition 18

$$1-\lambda = \wedge\{g \mid g \in F\beta O(X,\delta),\ 1-\lambda \leq g\}.$$

That is

$$\begin{aligned}\lambda &= \vee\{1-g \mid g \in F\beta O(X,\delta),\ 1-g \leq \lambda\} \\ &= \vee\{h \mid 1-h \in F\beta O(X,\delta),\ h \leq \lambda\}.\end{aligned}$$

Hence the result. □

Definition 13. A fuzzy topological space (X,δ) is $F\beta OT_2 \Leftrightarrow$ For every pair of non-zero fuzzy sets α, β with $\alpha \wedge \beta = 0$, there exists fuzzy β-open sets λ and μ such that $\alpha \wedge \lambda > 0$, $\beta \wedge \mu > 0$ and $\lambda \wedge \mu = 0$.

Proposition 20. If (X,δ) is $F\beta OT_2$, then $(X,\ i_{F\beta O}(X,\delta))$ is T_2

Proof. Let x, y be any two distinct points in X with $x \neq y$. Then for any $t \in (0,1)$, $x_t \wedge y_t = 0$. As (X,δ) is $F\beta OT_2$ there exists fuzzy β-open sets λ and μ such that

$$x_t \wedge \lambda > 0, \quad y_t \wedge \mu > 0 \quad \text{and} \quad \lambda \wedge \mu = 0.$$

Now

$$x_t \wedge \lambda > 0 \Rightarrow \lambda(x) > 0 \quad \text{and} \quad y_t \wedge \mu > 0 \Rightarrow \mu(y) > 0.$$

Put $\varepsilon' = \lambda(x)$ and $\eta' = \mu(y)$. Choose $\varepsilon < \varepsilon'$ and $\eta < \eta'$. Let $U = \lambda^{-1}\{(\varepsilon,1]\}$ and $V = \mu^{-1}\{(\eta,1]\}$. Then $x \in U$ and $y \in V$. Also $U \cap V = \phi$. (For $z \in U \cap V \Rightarrow \lambda(z) > 0$ and $\mu(z) > 0$, which contradicts the condition that $\lambda \wedge \mu = 0$). Moreover U, $V \in i_{F\beta O}(X,\delta)$. That is $(X,\ i_{F\beta O}(X,\delta))$ is T_2. □

Definition 14. Suppose (X, τ) is a topological space and $f : (X, \tau) \to I_r$ be a function. f is said to be β-continuous if $f^{-1}(\varepsilon, 1]$ is β-open [2] in X for all $\varepsilon > 0$. Define $\beta(\tau)$ to be the set of all β-continuous functions from (X, τ) to I_r.

Proposition 21. If the topological space (X, τ) is T_2 then the associated fuzzy topological space $(X, [\beta(\tau)])$ is $F\beta OT_2$ where $[\beta(\tau)]$ is the fuzzy topology generated by $\beta(\tau)$.

Proof. Let α and β be non-zero fuzzy sets with $\alpha \wedge \beta = 0$. Then there exist x, y in X such that $\alpha(x) > 0$, $\beta(y) > 0$. As $\alpha \wedge \beta = 0$, $x \neq y$ and so by hypothesis there exist open sets U and V (and therefore β-open sets) such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Let $\lambda = \chi_U$, $\mu = \chi_V$. Then $\lambda, \mu \in \beta(\tau)$. As $x \in U$ and $\alpha(x) > 0$, $\alpha \wedge \lambda > 0$. For similar reason $\beta \wedge \mu > 0$. Also $U \cap V = \phi \Rightarrow \lambda \wedge \mu = 0$. That is $(X, [\beta(\tau)])$ is $F\beta OT_2$. □

Proposition 22. Let (X, δ) be an $F\beta OT_2$ space and A, a subset of X. Then $(A, \delta/A)$ is an $F\beta OT_2$ space.

Proof. Let α, β be non-zero fuzzy sets in A such that $\alpha \wedge \beta = 0$. Define $\alpha_1 : X \to I$ by $\alpha_1(z) = \alpha(z)$ if $z \in A$ and $\alpha_1(z) = 0$ for $z \notin A$. Similarly define β_1 using β. then $\alpha_1 \wedge \beta_1 = 0$, $\alpha_1 > 0$, $\beta_1 > 0$. Hence by hypothesis there exists λ_1, μ_1 fuzzy β-open sets in X such that $\alpha_1 \wedge \lambda_1 > 0$, $\beta_1 \wedge \mu_1 > 0$ and $\lambda_1 \wedge \mu_1 > 0$. Put $\lambda = \lambda_1/A$, $\mu = \mu_1/A$. Then $\lambda, \mu \in \delta/A$, $\lambda \wedge \mu = 0$, $\alpha \wedge \lambda > 0$ and $\beta \wedge \mu > 0$. Hence $(A, \delta/A)$ is an $F\beta OT_2$. □

Definition 15. A fuzzy topological space (X, δ) is fuzzy β-open T_3 (or $F\beta OT_3$) $\Leftrightarrow$ For every pair of non-zero fuzzy sets α, β with β, fuzzy β-closed and $\alpha \wedge \beta = 0$, there exist fuzzy β-open fuzzy sets λ and μ such that $\alpha \wedge \lambda > 0$, $\beta \leq \mu$ and $\lambda \wedge \mu = 0$.

Proposition 23. If (X, δ) is $F\beta OT_3$ and δ is topologically generated then $(X, i_{F\beta O}(X, \delta))$ is T_3.

Proposition 24. If (X, δ) is $F\beta OT_3$ and $A \subseteq X$, then $(A, \delta/A)$ is $F\beta OT_3$.

Definition 16. A fuzzy topological space (X, δ) is $F\beta OT_4 \Leftrightarrow$ For every pair of non-zero fuzzy closed sets α, β with $\alpha \wedge \beta = 0$, there exists fuzzy β-open sets λ and μ such that $\alpha \leq \lambda$, $\beta \leq \mu$ and $\lambda \wedge \mu = 0$.

Clearly every FT_4 space [5] is $F\beta OT_4$ space.

Proposition 25. If (X, δ) is $F\beta OT_4$ space and δ is topologically generated then $(X, i_{F\beta O}(X, \delta))$ is T_4.

Proposition 26. If (X, δ) is an $F\beta OT_4$ space, $A \subseteq X$ with χ_A, δ-fuzzy closed, then $(A, \delta/A)$ is an $F\beta OT_4$ space.

(Received July 14, 1997.)

REFERENCES

[1] Abd. El. Monseb, S. N. El-Deeb and R. A. Mahmould: β-open sets and β-continuous mapping. Bull. Fac. Sci. Assiut Univ. (1982).
[2] A. A. Allam and Abd. El. Hakkim: On β-compact spaces. Bull. Calcuta Math. Soc. *81* (1989), 179–182.
[3] G. Balasubramanian: On fuzzy β-compact spaces and fuzzy β-extremally disconnected spaces. Kybernetika *33* (1997), 271–277.
[4] A. S. Bin Shahna: On fuzzy strong semi continuity and fuzzy precontinuity. Fuzzy Sets and Systems *44* (1991), 303–308.
[5] G. K. Chandrika: Fuzzy Topological Spaces. Ph.D. Thesis, Bharathiar University, 1993.
[6] C. L. Chang: Fuzzy topological spaces. J. Math. Anal. Appl. *24* (1968), 182–190.
[7] R. Lowen: Fuzzy topological spaces and fuzzy compactness. J. Math. Anal. Appl. *56* (1976), 621–633.
[8] M. Sugeno: An introductory survey of fuzzy control. Inform. Sci. *36* (1985), 59–83.
[9] P. Smets: The degree of belief in a fuzzy event. Inform. Sci. *25* (1981), 1–19.
[10] L. A. Zadeh: Fuzzy sets. Inform. and Control *8* (1965), 338–353.

Dr. Ganesan Balasubramanian, Department of Mathematics, Madras University, P. G. Centre, Salem – 636 011, Tamil Nadu. India.