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PREDICTABILITY AND CONTROL SYNTHESIS

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Processes modeled by a timed event graph may be represented by a linear model in dioïd algebra. The aim of this paper is to make temporal control synthesis when state vector is unknown. This information loss is compensated by the use of a simple model, the "ARMA" equations, which enables to introduce the concept of predictability. The comparison of the predictable output trajectory with the desired output determines the reachability of the objective.

1. INTRODUCTION

Discrete Event Dynamic Systems (DEDS) represent a great number of systems such as flexible manufacturing systems, multiprocessor systems, and transportation networks that are characterized as being concurrent, asynchronous, distributed and parallel. Among formalisms used to represent DEDS, Timed Petri Nets explicitly integrate time. Timed Event Graphs are a subclass that plays an important role because of its deterministic behavior. Its evolution is described by linear systems defined on a dioïd. The interpretation of each variable is, for example, of "dater" type for (max,+) algebra: each function $x_i(k)$ represents the date of the kth firing of transition x_i; $\oplus$ stands for the max operation while the usual addition plays the role of the multiplication, denoted $\otimes$.

An important objective is to make temporal control synthesis of systems. Historically, the PERT graph and potentials-tasks are the first well-known classical approaches enabling the definition of the execution calendar of a given project [13]. The results can be given by two algorithms that give respectively the earliest times and the latest times of the tasks. Using the dioïd algebra, [1, 7] generalize for processes with repetitive tasks. They solve the following classical problem: given a production system, how can we compute the latest dates of the part inputs in such a way that the parts be produced *at the latest before* the desired dates? It can be proved that, for the system which dater equations gives the lowest solution (the earliest times), the greatest solution (the latest times) is explicitly given by the backward recursive equations where the co-state vector plays the role of the state vector. This control theory is similar to the adjoint-state equations of optimal control theory. The difference between the co-state and the state represent the "spare time" or the "margin" which is available for the firing of the transitions. A negative difference prevents the future deadlines from being met.

Thus, this approach requires the vector state values. The knowledge of the model and of the initial conditions enable us to characterize the state vector with a state equation iteration. Unfortunately, this solution disregards unavoidable model errors and must start from a known state. To overtake these difficulties, we propose the use of a different model composed of equations called "ARMA" by analogy with ARMA equations used in classical control system theory. We show the possibility of using "ARMA" model to make a temporal control synthesis without knowing the state vector [8]. By example, this situation occurs when the process undergoes a failure and must be recovered. The model presents a description rupture, generating a misappreciation of the state vector. In this case, the problem is to compute, *after this past evolution of the system*, the latest firing dates of the input transitions in such a way that the output events occur at the latest before the desired date [9]. The model is supposed to be exact in the horizon of application of the control synthesis [10].

This paper is organized as follows. We first give notations and background concerning dioïds. We then, present the problem and study the "predictability" concept for the "d-cyclic" systems. Finally, we propose a multi-step control synthesis based on the "ARMA" model. The approach is applied to a short example in the annex.

2. PRELIMINARY

One of the tools used in this paper is (max,+) algebra, a particular example of the algebraic structure generally called dioïd. In this introduction, we shall limit ourselves to present notations and main concepts. A complete description may be found in [1][11].

A semi-ring S is a triplet $(S, \oplus, \otimes)$ where $(S, \oplus)$ and $(S, \otimes)$ are monoïds, $\oplus$ is commutative, $\otimes$ is distributive with respect to $\oplus$ and the zero element of $\oplus$ is the absorbing element of $\otimes$. A dioïd D is an idempotent semi-ring. The set $\Re \cup \{-\infty\}$ provided with max denoted $\oplus$ and with addition denoted $\otimes$ is usually called (max,+) algebra and is an example of dioïd.

We have: $\Re_{\max} = (\Re \cup \{-\infty\}, \oplus, \otimes\}$ with
$a \oplus b = \max(a, b)$; $\varepsilon = -\infty$ is the zero element of $\oplus$
$a \otimes b = a + b; e = 0$ is the identity element of $\otimes$
$a \oplus a = a$ (idempotency of $\oplus$)
$a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon$ (absorbing element ε).

The sign $\otimes$ will be omitted as usual when this causes no risk of confusion ($a \otimes b = ab$).

Cyclicity and residuation notions will be used again. Let λ be the maximum mean value of a circuit's weight of a graph associated with a general matrix A. λ is also the maximum eigenvalue of this matrix. A matrix A is cyclic if there exist d and m such that: $(\forall\, i \geq m)\ A^{i+d} = \lambda^d A^i$ with $\lambda^d = d \times \lambda$ in the usual notations.

d is called cyclicity and we say that A is d-cyclic. In this paper, we take the hypothesis that A is d-cyclic.

Theorem 1. Every irreducible matrix is d-cyclic.

The following definition expresses the output trajectory characteristic. It may also be applied to the control or to the desired output after a past evolution of the process.

Definition 2. The output y follows a *d-cyclic* trajectory starting from $k = k_s$ to k_f if $y(k) \geq \lambda^d y(k-d)$ with $k_s \leq k \leq k_f$.

We denote $a \backslash b = \max\{x \mid ax \leq b\}$ the left residuated of b by a (also called the subsolution of equality $ax = b$).

We denote $A \backslash B = \max\{x \mid Ax \leq B\}$ with $A \in \Re_{\max}^{m.n}, x \in \Re_{\max}^{n}, B \in \Re_{\max}^{m}$ and $A \backslash B = A^t \odot B$, where $\odot$ is a matrix product where operation $\oplus$ and $\otimes$ of are replaced respectively by $\wedge$ (minimum) and $\backslash$ of $\Re_{\max}$. The matrix product $\odot$ enables us to calculate easily $A \backslash B$.

3. MODELS

3.1. State equation

In the dioïd (max,+), the model has the following expression

$$\begin{aligned} x(k+1) &= Ax(k) \oplus Bu(k+1) \\ y(k) &= Cx(k) \\ x(0) &= x_0 \end{aligned} \tag{1}$$

where the control u, the output y and the state x are defined on $\Re \cup \{-\infty\}$. In this paper, we consider the Single-Input/Single-Output case. $x(k)$ is a $n.1$ vector of completion times for the k^{th} event.

We note

$$Y_{k_2}^{k_1} = \begin{pmatrix} y(k_1) \\ y(k_1+1) \\ \vdots \\ y(k_2) \end{pmatrix}, \quad U_{k_2}^{k_1} = \begin{pmatrix} u(k_1) \\ u(k_1+1) \\ \vdots \\ u(k_2) \end{pmatrix}$$

and $g_i = CA^iB$. To simplify the notations, we write equally Y for $Y_{k_2}^{k_1}$ and U for $U_{k_2}^{k_1}$, if the context specifies the vectors without ambiguity. From the state equation, we deduce

$$y(k+h) = CA^h x(k) \oplus \sum_{j=0}^{h-1} g_i u(k+h-j)$$

and

$$Y_{k+h}^{k+1} = Hx(k) \oplus GU_{k+h}^{k+1} \quad \text{with} \quad H = \begin{pmatrix} CA \\ CA^2 \\ \vdots \\ CA^{h-2} \\ CA^{h-1} \end{pmatrix}$$

and

$$G = \begin{pmatrix} g_0 & \varepsilon & \dots & \varepsilon & \varepsilon \\ g_1 & g_0 & \dots & \varepsilon & \varepsilon \\ \vdots & \vdots & \dots & \vdots & \vdots \\ g_{h-2} & g_{h-3} & \dots & g_0 & \varepsilon \\ g_{h-1} & g_{h-2} & \dots & g_1 & g_0 \end{pmatrix}.$$

3.2. "ARMA" model

Let us recall the principle of the generation of the "ARMA" equations.

From the state equation, we deduce the two following equations:

$$\begin{aligned} \lambda^d y(k-d) &= \lambda^d C A^m x(k-m-d) \oplus \lambda^d \sum_{j=0}^{m-1} g_j u(k-d-j) \\ y(k) &= C A^{m+d} x(k-m-d) \oplus \sum_{j=0}^{m+d-1} g_j u(k-j). \end{aligned}$$

We note

$$a_1 = \sum_{j=0}^{m-1} g_j u(k-d-j); \quad a_2 = \sum_{j=0}^{m+d-1} g_j u(k-j)$$

and we respectively add a_2 and $\lambda^d a_1$ to the previous equations.

$\lambda^d y(k-d) \oplus a_2 = \lambda^d C A^m x(k-m-d) \oplus \lambda^d a_1 \oplus a_2$

$y(k) \oplus \lambda^d a_1 = C A^{m+d} x(k-m-d) \oplus a2 \oplus \lambda^d a_1.$

As the matrix A is cyclic, we can eliminate the state. We deduce

$$y(k) \oplus \lambda^d a_1 = \lambda^d y(k-d) \oplus a_2$$

or

$$y(k) \oplus \lambda^d \sum_{j=0}^{m+d-1} g_{j-d} u(k-j) = \lambda^d y(k-d) \oplus \sum_{j=0}^{m+d-1} g_j u(k-j). \tag{2}$$

Each term contains a single output and a function of the control. However, as the addition does not have the property of symmetry, we cannot express the output $y(k)$ from the other terms. One of the objectives will be to reduce and to exploit this structure.

4. CONTROL SYNTHESIS

4.1. Presentation of the problem

Suppose that some events be designated as controllable, meaning that their input transitions may be delayed from firing until some arbitrary time. The delayed enabling times $u(k)$ for the controllable events are to be provided by a supervisor. Let us suppose that we wish to slow the system down as much as possible without causing any event to occur later than some sequence of execution times Z. We are equally, interested by a regular behavior of the output trajectory and a constraint will be the d-cyclicity: one application is high-frequency transportation systems, for instance [2]. Moreover, we consider a past evolution of the process: it enables changing the desired output, therefore a modification of the production rate. So, let us consider the following problem.

Knowing the dates' values of the control and the output, the number of events being inferior or equal to k_0 and a sequence of the desired output $z(k)$, k ranging from $k_s = k_0 + 1$ to $k_f = k_0 + h$, the problem is to determine the greatest control sequence $u(k)$ such that the output trajectory under the control effect satisfy the following points:

a) each output $y(k)$ occurs at the latest before $z(k)$

b) the output trajectory is d-cyclic.

4.2. Input trajectory

First, we consider the classical problem presented in the introduction. We introduce the following theorem.

Theorem 3. The non-decreasing greatest control such that the output $y(k)$ occur at the latest before the desired output $z(k)$ is given by: for $j = 1$ to h, $u(k_0 + j) = H \backslash Z_{k_0+h}^{k_0+1} = \bigwedge_{i=0}^{h-j} gi \backslash z(k_0 + j + i)$ under the initial constraints $Z_{k_0+h}^{k_0+1} \geq Hx(k_0)$ and $u(k_0 + j) \geq \sum_{i=1}^{n} x_i(k_0); u(k_0 + 1) \geq u(k_0)$.

Proof. We want to calculate the greatest control such that $Y_{k_0+h}^{k_0+1} \leq Z_{k_0+h}^{k_0+1}$. The model is

$Y_{k_0+h}^{k_0+1} = Hx(k_0) \oplus GU_{k_0+h}^{k_0+1}$.

If $Hx(k_0) \nleq Z_{k_0+h}^{k_0+1}$, the classical problem has no solution.

If $Hx(k_0) \leq Z_{k_0+h}^{k_0+1}$, the greatest control is $H \backslash Z_{k_0+h}^{k_0+1}$. In this case,

$Y_{k_0+h}^{k_0+1} = Hx(k_0) \oplus G(G \backslash Z_{k_0+h}^{k_0+1})$ is maximum and $Y_{k_0+h}^{k_0+1} \leq Z_{k_0+h}^{k_0+1}$. In the single-input single output case, we have $G \backslash Z_{k_0+h}^{k_0+1} = \bigwedge_{i=0}^{h-j} gi \backslash z(k_0 + j + i)$. □

Actually, we can easily prove that this result is another formulation of the Backward equations [1] in a more general case. In the following property, we give another expression of the optimal control for the Backward equations which realizes a connection between the control and the production rate. The calculus is divided into a transient part of length m and a periodic part using the d-cyclicity concept. The following definition expresses the production rate characteristic and can be equally applied to the control or to the desired output after a past evolution of the process.

Proposition 4. Let a desired output trajectory be $(z(k_0+1),\dots,z(k_0+h))^t$. $y(k) = z(k) \wedge \lambda^d \backslash y(k+d)$ with $y(k)=+\infty$ for $k>k_s$. For $j=1$ to h, $u(k_0+j) = \bigwedge_{i=0}^{m-1} g_i\backslash z(k_0+j+i) \wedge \bigwedge_{i=m}^{m+d-1} g_i\backslash y(k_0+j+i)$ with y d-cyclic.

Proof. If

$$\begin{aligned} Hx(k_0) &\le Z_{k_0+h}^{k_0+1}, u(k_0+j) = \bigwedge_{i=0}^{h-j} g_i\backslash z(k_0+j+i) \quad \text{(Theorem 2)} \\ u(k_0+j) &= \bigwedge_{i=0}^{m-1} g_i\backslash z(k_0+j+i) \wedge \bigwedge_{i=m}^{h-j} g_i\backslash z(k_0+j+i). \end{aligned}$$

We note

$$u_1(k_0+j) = \bigwedge_{i=0}^{m-1} g_i\backslash z(k_0+j+i)$$

and

$$\begin{aligned} u_2(k_0+j) &= \bigwedge_{i=m}^{h-j} g_i\backslash z(k_0+j+i) \\ u_2(k_0+j) &= \bigwedge_{i=m}^{h-j} g_i\backslash z(k_0+j+i) = \bigwedge_{l=0}^{d-1}\bigwedge_{p=0}^{+\infty} g_{m+l+pd}\backslash z(k_0+j+m+l+pd) \end{aligned}$$

with $z(k_0+j+m+l+pd)=+\infty$ for $j+m+l+pd>h$.

However, $g_{m+l+pd} = (\lambda^d)^p g_{m+l}$ because $g_i = \lambda^d g_{i-d}$ for $i \ge m+d$.

(For example, $g_{m+l+pd} = g_{m+d+l+(p-1)d} = \lambda^d g_{m+l+(p-1)d}$).

So,

$$\begin{aligned} u_2(k_0+j) &= \bigwedge_{l=0}^{d-1}\bigwedge_{p=0}^{+\infty} [(\lambda^d)^p g_{m+l}]\backslash z(k_0+j+m+l+pd) \\ &= \bigwedge_{l=0}^{d-1} g_{m+l}\backslash \left[\bigwedge_{p=0}^{+\infty} (\lambda^d)^p\backslash z(k_0+j+m+l+pd)\right]. \end{aligned}$$

As

$$\begin{aligned} y(k) &= z(k)\bigwedge \lambda^d\backslash y(k+d) = z(k)\bigwedge \lambda^d\backslash[z(k+d)\bigwedge \lambda^d\backslash y(k+2d)] \\ &= z(k)\bigwedge \lambda^d\backslash z(k+d)\bigwedge \lambda^{2d}\backslash y(k+2d) = \dots = \bigwedge_{p=0}^{+\infty} (\lambda^d)^p\backslash z(k+pd), \end{aligned}$$

we finally obtain

$$u_2(k_0+j) = \bigwedge_{l=0}^{d-1} g_{m+l}\backslash y(k_0+j+m+l) = \bigwedge_{i=m}^{m+d-1} g_i\backslash y(k_0+j+i). \qquad \square$$

The following result is immediate.

Proposition 5. Let a d-cyclic desired output trajectory be $(z(k_0+1),\ldots,z(k_0+h))^t$. For $j=1$ to h, $u(k_0+j) = \bigwedge_{i=0}^{m+d-1} g_i \backslash z(k_0+j+i)$.

Consequently, if the desired trajectory is d-cyclic, the optimal control can be calculated without knowing the values over a horizon of length $d+m$.

Proposition 6. A control sequence deduced from a d-cyclic desired output trajectory z by $u(k_0+j) = \bigwedge_{i=0}^{m+d-1} g_i \backslash z(k_0+j+i)$ is also d-cyclic.

Proof.

$$u(k_0+j) = \bigwedge_{i=0}^{m+d-1} g_i \backslash z(k_0+j+i), \quad u(k_0+j+d) = \bigwedge_{i=0}^{m+d-1} g_i \backslash z(k_0+j+i+d).$$

As $\lambda^d z(k_0+j+i) \le z(k_0+j+i+d)$, we have

$$g_i \backslash z(k_0+j+i+d) \ge g_i \backslash (\lambda^d z(k_0+j+i)) = \lambda^d [g_i \backslash z(k_0+j+i)].$$

So,

$$\begin{aligned} u(k_0+d+j) &\ge \bigwedge_{i=0}^{m+d-1} \lambda^d [g_i \backslash z(k_0+j+i)] \\ &= \lambda^d \bigwedge_{i=0}^{m+d-1} g_i \backslash z(k_0+j+i) = \lambda^d u(k_0+j). \end{aligned}$$ □

4.3. Output trajectory

In this part, we assume that the control is known. As we have taken the hypothesis that the state is unknown, the problem is to anticipate the effects on the output and to predict it. We shall exploit the "ARMA" structure 2 which is a relation between the input and the output on a finite horizon $m+d$.

In the following theorem, we show that an output trajectory deduced from a d-cyclic input trajectory is also d-cyclic after a transient period and is given by a simple relation.

Theorem 7. If

$$\lambda^d u(k) \le u(k+d) \quad \text{for} \quad k \ge k_0+1$$

then

$$y(k) = \lambda^d y(k) \oplus \bigwedge_{j=0}^{m+d-1} g_j u(k-j) \quad \text{for} \quad k \ge k_0+m+d.$$

Proof. If u is d-cyclic, then we have $\lambda^d g_{j-d} u(k_0+i-j) \le g_{j-d} u(k_0+i-j+d)$ for $i \ge j+1$.

If j belongs to $[d, m+d-1]$, then the minimal value of i is $m+d$.

If $i \geq m+d$,

$$\bigwedge_{j=d}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j) \leq \bigwedge_{j=d}^{m+d-1} g_{j-d} u(k_0+i-j+d)$$
$$= \bigwedge_{j=0}^{m-1} g_j u(k_0+i-j) \leq g_j u(k_0+i-j).$$

As $y(k_0+i) = CA^i x(k_0) \oplus \bigwedge_{j=0}^{i-1} g_j u(k_0+i-j)$, $y(k_0+i) \geq \bigwedge_{j=d}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$.

Finally, we obtain $y(k_0+i) = \lambda^d y(k_0+i-d) \oplus \bigwedge_{j=0}^{m+d-1} g_j u(k_0+i-j)$ for $i \geq m+d$.

□

Theorem 8. If the following initial constraint is verified

$$y(k_0+i) \geq \bigwedge_{j=d\oplus i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j) \text{ for } 1 \leq i \leq m+d-1 \tag{3}$$

and if the input is d-cyclic $\lambda^d u(k) \leq u(k+d)$ for $k \geq k_0+1$ then

$$y(k_0+i) = \lambda^d y(k_0+i-d) \oplus \bigwedge_{j=0}^{m+d-1} g_j u(k_0+i-j) \text{ for } i \geq 1. \tag{4}$$

Proof. There are three cases:

a) for $i \geq m+d$
We apply the Theorem 7.

b) for $d+1 \leq i \leq m+d-1$
As u is d-cyclic,

$$\sum_{j=d}^{i-1} \lambda^d g_{j-d} u(k_0+i-j) \leq \sum_{j=d}^{i-1} g_{j-d} u(k_0+i-j+d)$$
$$= \sum_{j=0}^{i-d-1} g_j u(k_0+i-j) \leq \sum_{j=0}^{i-1} g_j u(k_0+i-j).$$

As

$$y(k_0+i) = CA^i x(k_0) \oplus \sum_{j=0}^{i-1} g_j u(k_0+i-j),$$

$y(k_0+i)$ is greater than equal $\sum_{j=d}^{i-1} \lambda^d g_{j-d} u(k_0+i-j)$.

As $\sum_{j=d}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j) = \sum_{j=d}^{i-1} \lambda^d g_{j-d} u(k_0+i-j) \oplus \sum_{j=i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$,

the condition $y(k_0+i) \geq \sum_{j=d}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$ is reduced to

$y(k_0+i) \geq \sum_{j=i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$ for $d+1 \leq i \leq m+d-1$.

c) For $1 \leq i \leq d$, the condition remains $y(k_0+i) \geq \sum_{j=d}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$.

We can shortly write $y(k_0+i) \geq \sum_{j=d\oplus i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$ for $1 \leq i \leq m+d-1$.

Consequently, if u is d-cyclic and the condition 3 holds, the condition $y(k_0+i) \geq \sum_{j=d}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$ is true and we can write the following equality:

$$y(k_0+i) = \lambda^d y(k_0+i-d) \oplus \bigwedge_{j=0}^{m+d-1} g_j u(k_0+i-j) \text{ for } i \geq 1. \qquad \square$$

Remark 1. Let us suppose that the input and output trajectory are known from $k_0 - m - d + 2$ to k_0. We can calculate the right hand term of the inequality 3 from the known values of the control. However, we cannot calculate $y(k_0+i)$ for $1 \leq i \leq m+d-1$ with the equation 4 to verify the condition 3 because this equality needs that condition.

Proposition 9. A sufficient condition of 3 is for $1 \leq i \leq m+d-1$,

$$\sum_{j=0}^{l+i-1} g_j u(k_0+i-j) \geq \sum_{j=d\oplus i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j) \text{ with } l = m+d-1. \tag{5}$$

P r o o f. The state equation gives $y(k_0+i) = CA^{l+i} x(k_0-l) \oplus \sum_{j=0}^{l+i-1} g_j u(k_0+i-j)$.

As $m+d$ is the minimal horizon necessary to exploit the "ARMA" structure, we take $l = m+d-1$ to obtain the maximal information.

So, $y(k_0+i) \geq \sum_{j=0}^{l+i-1} g_j u(k_0+i-j)$ that is the minimal value of the output.

A sufficient condition is for $1 \leq i \leq m+d-1$,

$$\sum_{j=0}^{l+i-1} g_j u(k_0+i-j) \geq \sum_{j=d\oplus i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j). \qquad \square$$

4.4. The multi–step control synthesis in the single–input single–output case

The following algorithm gives the solution of the problem of the Section 4.1. when the state is unknown.

a) *d-cyclic desired output trajectory*

We deduce it from

$w(k) = \lambda^d w(k-d) \bigwedge \lambda^d \backslash w(k+d)$ for $k_s \leq k \leq k_f$ with $w(k) = +\infty$ for $k > k_f$.

b) *d-cyclic input trajectory*

The control sequence is deduced by $u(k) = \bigwedge_{i=0}^{m+d-1} g_i \backslash w(k+i)$ for $k_s \leq k \leq k_f$ with the condition $u(k) \geq u(k_0)$.

c) *Predictable output trajectory*

We predict a trajectory y_p with $y_p(k) = \lambda^d y_p(k-d) \oplus \bigwedge_{i=0}^{m+d-1} g_j u(k-i)$ for $k_s \leq k \leq k_f$ with the predictability condition $\bigwedge_{j=0}^{l+i-1} g_j u(k_0+i-j) \geq \bigwedge_{j=d\oplus i}^{m+d-1} \lambda^d g_{j-d} u(k_0+i-j)$ for $1 \leq i \leq m+d-1$ with $l = m+d-1$.

d) *Reachability Analysis*

We verify the following inequality $z(k) \geq y_p(k)$ for $k_s \leq k \leq k_f$.

Proof. The output defined by $w(k) = \lambda^d w(k-d) \bigwedge \lambda^d \backslash w(k+d)$ for $k_s \leq k \leq k_s$ gives the greatest d-cyclic output w such that $\forall\, k \in [k_s, k_s]$ $w(k) \leq z(k)$ and the Proposition 5 shows that the greatest output control is $u(k) = \bigwedge_{i=0}^{m+d-1} g_i \backslash w(k+i)$. As the desired output trajectory w is d-cyclic, the output control u is also d-cyclic after k_0 (Proposition 6). If the initial constraint 5 is verified and if the input is d-cyclic, then $y_p(k) = \lambda^d y_p(k-d) \oplus \bigwedge_{i=0}^{m+d-1} g_j u(k-i)$ for $i \geq 1$ (Theorem 8 and Proposition 9). Particularly, if the values of $y(k_0+i)$ are known for $1-d \leq i \leq 0$, we can deduce the output trajectory for $1 \leq i \leq k_s - k_0$ that allows us to test the just-in-time criteria. If it exists k such that $z(k) < y_p(k)$, there is not an optimal control such that $z(k) \geq y_p(k)$. □

Remark 2. As the control is applied after the calculus of the control synthesis, the calculated dates must be later than the initial data. Consequently, we have the causality condition $u(k_s) > y(k_0)$.

Remark 3. We can notice that the control is calculated on a finite horizon $d+m$. Precisely, if we consider the case of a d-cyclic desired output trajectory, we have $y(k) = z(k)$ in the first step and the calculus of the control $u(k)$ does not need the values of the desired output over $d+m$. In other words, if the desired output follows the internal rate of the system, the control calculus can consider the real values of the desired output trajectory on only a finite horizon $d+m$ without any optimality reduction: the knowledge of the trajectory under the d-cyclicity hypothesis can be introduced in a sequential and infinite manner. As a result, the desired output trajectory can be easily defined as the infinite repetition of a motif.

4.5. Related work

The reachability analysis verifies the existence of a control that satisfies the constraint a) of the problem and consequently uses only the output trajectory. It is analogous to the existence of non-negative difference between the co-state and the state for the backward approach [1]. In the spirit of the classical automatic control, [12] and [14] consider a strict definition of reachability where the state must exactly be reached. The reachability analysis corresponds partially to the concept of controllable desired output defined in [4, 5] and [6] with a different model. In this work, the state is known and the matrix C equals the identity matrix. The events of the transitions can be delayed or not. In the first case the events are designated as controllable and the matrix B equals I_c: I_c denote the matrix having the identity function on diagonal elements for which the events can be delayed and ε elsewhere. The transposition of the controllable output is $x = A^*(Bu \oplus v) \leq z$ where x, u and z are sequences of firing time vectors for events. v is a sequence of earliest allowable firing time vectors and generalizes the initial condition x_0. To compute the effect of uncontrollable events, the authors choose the equality between the control and the desired output which is a particular choice. The objective of our problem is precisely to determine this control.

In this paper, a basic assumption that allows us to model the system, is that places are First In First Out (FIFO) channels. A place is FIFO if the K^{th} token to enter this place is also the K^{th} that becomes available in this place. The interpretation is that tokens cannot overtake one another which is a necessary numbering condition of the events. We are in this case if the holding times are constant. However, if the holding times vary and if the event numbering is kept, the ARMA equation of the normal system can be used after a delay of $d+m$ occurrences if the system is restored in its usual behavior. This delay is a consequence of the state equation iteration on this horizon [10]. For example, if a place belongs to a cycle that contains one token, the overtaking is forbidden because the place contains at most one token by reason of the property of conservativeness of the event graphs. Consequently, the variation of the holding time does not disrupt the numbering of the state equation. Note that it corresponds to a classical situation where a machine can works on one piece at once. The generalization to the event-index varying case needs a more general Theorem 1 on d-cyclicity. In [4], [5] and [6], the Timed Event Graphs are modeled using a backshift operator which make it possible to consider this case but a difficult problem is to describe algorithms to calculate A star [3].

5. CONCLUSION

In this paper, we present a temporal control synthesis using "ARMA" model in Timed Event Graphs. This approach makes it possible the release of the knowledge of the state vector and enables having a non-stationarity of the model. It enables changes of the desired output and of the production rate in consequence of a modification in the desired output. Coherent with the spirit of the Backwards equations, the solution is modular and can easily be applied.

The control synthesis is based on the d-cyclicity of the desired trajectories relevant to the periodicity of the system and therefore to the production rate. Under this constraint, the desired output trajectory can be defined as the infinite repetition of a motif. In this paper, we also study the "predictability" concept through the "ARMA" model which depends on the system and its behavior. This notion brings up the problem of Observability and Commandability and for the time being, the study of these concepts is an open field for Timed Petri Nets.

APPENDIX

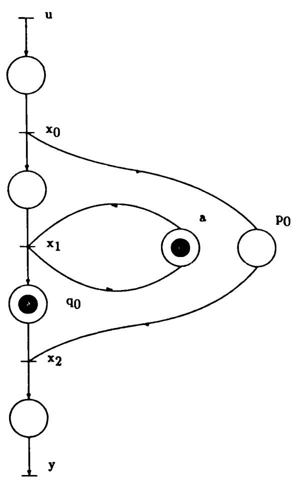

Fig. 1. Timed Event Graph.

A. State equation

We have $y(k) = x_2(k)$; $x_2(k) = q_0 x_1(k-1) \oplus p_0 u(k)$; $x_1(k) = a x_1(k-1) \oplus u(k)$

$$\begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} = \begin{pmatrix} a & \varepsilon \\ q_0 & \varepsilon \end{pmatrix} \begin{pmatrix} x_1(k-1) \\ x_2(k-1) \end{pmatrix} \oplus \begin{pmatrix} e \\ p_0 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} \varepsilon & e \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} \quad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} \varepsilon \\ \varepsilon \end{pmatrix}.$$

B. "ARMA" equation

$$(\forall i \geq 1)\ A^{i+1} = \lambda^d A^i, \quad m = 1, \quad d = 1$$
$$g_0 = CB = p_0, \quad g_1 = CAB = q_0$$
$$CA = (q_0, \varepsilon), \quad CA^2 = (q_0 a, \varepsilon)$$
$$\begin{pmatrix} y(k) \\ y(k+1) \end{pmatrix} = \begin{pmatrix} q_0 & \varepsilon \\ q_0 a & \varepsilon \end{pmatrix} x(k-1) \oplus \begin{pmatrix} p_0 & \varepsilon \\ q_0 & p_0 \end{pmatrix} \begin{pmatrix} u(k) \\ u(k+1) \end{pmatrix}$$
$$y(k+1) \oplus a g_0 u(k) = a y(k) \oplus g_1 u(k) \oplus g_0 u(k+1).$$

C. Control synthesis

a) $w(k) = z(k) \bigwedge a \backslash w(k+1)$

b) $u(k) = g_0 \backslash w(k) \bigwedge g_1 \backslash w(k+1)$

c) $y_p(k+1) = a y_p(k) \oplus g_1 u(k) \oplus g_0 u(k+1)$.

Predictability condition: $g_0 u(k_0+1) \oplus g_1 u(k_0) \geq a g_0 u(k_0)$.
Causality condition: $u(k_s) > y(k_0)$.

Let $p_0 = 3$, $q_0 = 1$, $a = 2$.

If the holding times are constant, we can have for example the following evolution·

k	0	1	2	3
u		e	1	3
x_1	ε	e	2	4
x_2	ε	3	4	6
y	ε	3	4	4

But we assume that the holding time "a" of the recycled place has undergone a variation at $k = 2$.

We have $x_1(k) = a x_1(k-1) \oplus u(k)$ with $a = 4$ for $k = 2$ and $a = 2$ otherwise.

k	0	1	2	3
u		e	1	3
x_1	ε	e	4	6
x_2	ε	3	4	6
y	ε	3	4	6

Let a desired output trajectory be $(z(4),\ z(5),\ z(6))^t = (11, 14, 14)^t \cdot (k_0 = 3,\ k_s = 4,\ k_f = 6)$. We deduce a d-cyclic trajectory $(w(4),\ w(5),\ w(6))^t = (10, 12, 14)^t$, hence the control $(u(4),\ u(5),\ u(6))^t = (7, 9, 11)^t$.

The predictable output trajectory y_p is: $(y_p(4),\ y_p(5),\ y_p(6))^t = (10, 12, 14)^t$. The predictability condition $g_0 u(k_0+1) \oplus g_1 u(k_0) \geq a g_0 u(k_0)$ is verified ($u(k_0) = 3$, $u(k_0+1) = 7$) as the causality condition $u(k_s) = 7 > y\ (k_0) = 6$.

Using the state equations, the simulation confirms this result.

k	0	1	2	3	4	5	6
u		e	1	3	7	9	11
x_1	ε	e	4	6	8	10	12
x_2	ε	3	4	6	10	12	14
y	ε	3	4	6	10	12	14

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REFERENCES

[1] F. Baccelli, G. Cohen, G. J. Olsder and J. P. Quadrat: Synchronization and Linearity. An Algebra for Discrete Event Systems. Wiley, New York 1992.

[2] H. Bracker: Algorithms and Applications in Timed Discrete Event Systems. Ph.D thesis, Delft University of Technology, 1993.

[3] G. P. Brat and V. K. Garg: A max-plus algebra of signals for the supervisory control of real-time discrete event systems. In: 9th Symposium of the IFAC on Information Control in Manufacturing, Nancy-Metz 1998.

[4] D. D. Cofer and V. K. Garg: A timed model for the control of discrete event systems involving decisions in the max/plus algebra. In: Proc. 31st Conference on Decision and Control, Tucson 1992.

[5] D. D. Cofer: Control and Analysis of Real-Time Discrete Event Systems. Ph.D. Thesis, University of Texas, Austin 1995.

[6] D. D. Cofer and V. K. Garg: Supervisory control of real-time discrete-event systems using lattice theory. IEEE Trans. Automat. Control *41* (1996), 2, 199–209.

[7] G. Cohen, S. Gaubert and J.-P. Quadrat: From first to second-order theory of linear discrete event systems. In: 1st IFAC World Congress, Sydney 1993.

[8] Ph. Declerck: "ARMA" model and admissible trajectories in timed event graphs. In: CESA'96, IMACS, IEEE-SMC, Lille 1996.

[9] Ph. Declerck: Control synthesis using the state equations and the "ARMA" model in timed event graphs. In: 5th IEEE Mediterranean Conference on Control and Systems, Paphos 1997.

[10] Ph. Declerck and M. Mares: Temporal control synthesis and failure recovery. In: 9th Symposium of the IFAC on Information Control in Manufacturing, Nancy-Metz 1998.

[11] S. Gaubert: Théorie des systèmes linéaires dans les dioïdes. Ph.D Thesis, Ecole des Mines de Paris 1992.

[12] M. J. Gazarik and E. W. Kamen: Reachability and observability of linear systems over max-plus. In: 5th IEEE Mediterranean Conference on Control and Systems, Paphos 1997, revised version: Kybernetika *35* (1999), 2–12.

[13] M. Gondran and M. Minoux: Graphes et algorithmes. Edition Eyrolles 1995.

[14] J.-M. Prou and E. Wagneur: Controllability in the max-algebra. In: 5th IEEE Mediterranean Conference on Control and Systems, Paphos 1997, revised version: Kybernetika *35* (1999), 13–24.

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