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DETECTION AND ACCOMMODATION OF SECOND ORDER DISTRIBUTED PARAMETER SYSTEMS WITH ABRUPT CHANGES IN INPUT TERM: STABILITY AND ADAPTATION

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In this note, we employ nonlinear on-line parameter estimation methods based on adaptive neural network approximators for detecting changes due to actuator faults in a class of second order distributed parameter systems. The motivating example is a cantilevered beam actuated via a pair of piezoceramic patches. We examine changes in the control input term, which provide a simple and practical model of actuator failures. Using Lyapunov redesign methods, a stable learning scheme for fault diagnosis is proposed. The resulting fault diagnosis scheme is utilized for control reconfiguration in order to accommodate the system's actuator failure. A numerical algorithm is provided for the implementation of the detection and accommodation scheme and simulation studies are used to illustrate the applicability of the theoretical results.

1. INTRODUCTION

Demanding operating conditions for many modern engineering systems result in higher possibility of system failures. In general, feedback control algorithms, which are designed to handle small system perturbations that may arise under "normal" operating conditions (typically, in the linear regime), cannot accommodate abnormal behavior due to faults. Such system failures can potentially result not only in the loss of productivity but also in the loss of expensive equipment and, ultimately, of human lives. Moreover, difficult and often dangerous environments limit the ability of humans to perform any supervisory and/or corrective tasks. For example, automated maintenance for early detection of worn equipment is becoming a crucial problem in many practical applications. In general, the development of automated health monitoring architectures is becoming a more crucial component in the design of truly intelligent systems.

The design and analysis of fault diagnosis architectures for finite dimensional systems using the model-based analytical redundancy approach has received considerable attention during the last two decades. According to this approach, quantitative nominal models (for example, state space models) of the physical system, together with sensory measurements, are used to provide estimates of measured and/or unmeasured variables. The deviations between the estimated and measured signals provide a *residual* vector which is utilized to detect and isolate system failures. Survey papers by Frank [8], Gertler [9], Isermann [11] and Willsky [18], among others, present excellent overviews of various model-based fault diagnosis algorithms.

Most fault diagnosis studies so far have been based on a linear system formulation. The learning based approach to fault diagnosis is a new methodology for nonlinear systems subject to faults that are nonlinear functions of the input and state variables. The main idea behind this approach is to monitor the plant for any off-nominal behavior due to faults utilizing a neural network or other types of on-line approximators. In the presence of a failure, the neural network can be used as an estimate of the nonlinear fault function for fault identification and accommodation purposes. Furthermore, during the initial stage of monitoring, the learning capabilities of the neural network can be used to learn the modeling errors, thereby enhancing the robustness properties of the fault diagnosis scheme.

The learning based approach to fault diagnosis was developed for the finite dimensional case over a number of studies. The initial formulation was proposed in [14], where the stability properties of the learning scheme for the restricted case of abrupt faults and no modeling uncertainties, were established. In [16] the assumption of no modeling uncertainties was relaxed and in addition to stability, some robustness and sensitivity properties of the fault diagnosis scheme were obtained. The input-output formulation for the learning based fault diagnosis algorithm was developed in [17]. In [6] the assumption of abrupt faults was relaxed by allowing the time profile of the fault to be a drift-type, decaying exponential function (representing incipient faults), and in [15] the class of detectable faults was characterized and an upper bound on the detection time was derived analytically. The case of parametric faults, which describe faults of known structure but with unknown parameter vectors, were considered in [13].

In this paper we discuss a theoretical investigation and present a numerical scheme for a *model-based* fault diagnosis and accommodation algorithm applied to a class of distributed parameter systems with failures occurring in the input term. An estimated model (adaptive diagnostic observer) of the plant is used to monitor the plant for any changes due to these faults. The estimated model incorporates an on-line approximator [14], which estimates and monitors the parameters on-line via a learning algorithm. The output of the on-line approximator is used as an indicator of the occurrence of a fault and also as a method for identifying the location (fault isolation) and shape (fault identification) of system failures. A reconfiguration of the standard control is presented in order to accommodate the system failure.

The structure considered in the ensuing numerical example is taken to be a cantilevered beam with two piezoceramic patches attached on the opposite sides of the beam. As was already mentioned in many works, see for example [2] and the references therein, a general structural (and even structural-acoustic) control problem has dynamics described by the second order evolution equation

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t) \tag{1.1}$$

with (collocated velocity) output

$$y(t) = C\dot{x}(t) \tag{1.2}$$

where the state $x(t)$ belongs to a Hilbert space X and M, D, K and B, C are operators in the appropriate spaces.

The paper is organized as follows. In Section 2 we set up the abstract equations that govern the dynamics of the plant (which is assumed to be infinite dimensional) and in Section 3 we propose a model for the fault, which in this case is simply taken to be a change in actuator gain that is a function of the measurable output signal. The state observer along with the failure on-line approximator used for failure monitoring and detection is presented in Section 4. A standard controller for the nominal plant is proposed in Section 5 and a modification to the standard control is presented to account for the actuator changes due to failures; thus failure accommodation. Simulations studies with discussion follow in Section 6 and conclusions with future directions are summarized in Section 7.

2. PLANT DYNAMICS

Let X be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and corresponding induced norm $|\cdot|$, and let V and W be real reflexive Banach spaces with norms denoted by $\|\cdot\|_V$ and $\|\cdot\|_W$, respectively. We assume that V is embedded densely and continuously in W and that W is embedded densely and continuously in X. It then follows that (see, for example, [2])

$$V \hookrightarrow W \hookrightarrow X \cong X^* \hookrightarrow W^* \hookrightarrow V^*, \tag{2.1}$$

where X^*, W^*, and V^* denote the conjugate duals of X, W, and V, respectively. We denote the usual operator norms on W^* and V^* by $\|\cdot\|_{W^*}$ and $\|\cdot\|_{V^*}$, respectively. The duality pairing, denoted by $\langle \cdot, \cdot \rangle_{V^*,V}$, is the extension by continuity of the inner product $\langle \cdot, \cdot \rangle$ from $H \times V$ to $V^* \times V$; hence, elements $\varphi^* \in V^*$ have the representation $\varphi^*(\varphi) = \langle \varphi^*, \varphi \rangle_{V^*,V}$, see [2]. As it was pointed out in [2, 7], W can either be V, X or some intermediate space, depending on the damping form chosen.

Assume that we have the following second order system

$$\begin{aligned} &M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t) \quad \text{in } V^*, \\ &y(t) = C\dot{x}(t), \\ &x(0) \in V, \;\; \dot{x}(0) \in X, \end{aligned} \tag{2.2}$$

where $K : V \to V^*$ is the *stiffness* operator which is assumed to be a symmetric, $V - V^*$-bounded (i.e. there exists a $\alpha_1 > 0$ such that $|\langle K\phi, \psi \rangle| \leq \alpha_1 \|\phi\|_V \|\psi\|_V$, for $\phi, \psi \in V$) and V-coercive operator (i.e. there exists $k > 0$ such that $\langle K\phi, \phi \rangle \geq k\|\phi\|_V^2$ for $\phi \in V$), $D : W \to W^*$ is the *damping* operator which is a symmetric , $W - W^*$-bounded (i.e. there exists a $\alpha_2 > 0$ such that $|\langle D\phi, \psi \rangle| \leq \alpha_2 \|\phi\|_W \|\psi\|_W$, for $\phi, \psi \in W$) and W-coercive operator, (i.e. there exists $d > 0$ such that $\langle D\phi, \phi \rangle \geq$

$d\|\phi\|_W^2$ for $\phi \in W$), $M : X \to X$ is the *mass* operator which is a symmetric, X-bounded and X-coercive operator, $B : I\!R \to V^*$ is the *input* operator, $C : W \to I\!R$ is the *output* operator, $u(\cdot) \in I\!R$ is the input signal to the system, and $y(\cdot) \in I\!R$ is the measured output signal.

3. MODELING OF FAILURE

The above system given by (2.2) is termed as the nominal or "*healthy*" plant. It is assumed that *only* the input u and output y signals can be used to monitor the above plant for possible failures. The unanticipated actuator failure (i. e. failure in the input term) takes the form of

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t) + \beta(t-T)\,Bf(y)\,u(t) \tag{3.1}$$

where $\beta(t-T)$ is the *time profile* [14] of the actuator failure and is given by

$$\beta(\tau) = \begin{cases} 0 & \text{if} \quad \tau < 0 \\ 1 - e^{-\lambda\tau} & \text{if} \quad \tau \geq 0 \end{cases}$$

with $\lambda > 0$ an unknown constant whose upper bound is assumed to be known, $f : I\!R \to I\!R$ is a smooth vector field and $T > 0$ is the time at which failure occurs and is desired to be detected. If $\lambda < \infty$ we have *incipient* failures and if $\lambda = \infty$ (i. e. $\beta(\tau) = H(\tau)$, H is the Heaviside function) we have *abrupt* failures.

We will now make the assumption that for the class of systems under study, we have *admissible plants.*

Assumption 3.1. (Admissible plant) We assume that the (perturbed) system

$$\begin{aligned} &M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t) + \beta\,Bf(y)\,u \\ &y = C\dot{x}(t) \end{aligned}$$

for $t \geq T$, is well posed in the sense that a weak solution $x \in L^2(0,\infty;V)$ with $\dot{x} \in L^2(0,\infty;X)$, $\ddot{x} \in L^2(0,\infty;V^*)$ exists that satisfies (3.1) (see [2, 19]) and that has $y \in L^\infty(0,\infty;I\!R)$.

The well posedness of the plant before and after the actuator failure is given in detail in the companion paper [1], where in addition to the justification of Assumption 3.1, the well posedness of the adaptive estimator (presented in the next section) along with a detailed treatment of the finite dimensional approximation is presented.

4. MODEL ESTIMATOR AND CONVERGENCE

We first propose the following state estimator (adaptive detection/diagnostic observer)

$$\begin{aligned} &M\ddot{\hat{x}}(t) + D\dot{\hat{x}}(t) + K\hat{x}(t) = Bu(t) + B\hat{f}(y;\hat{\theta}(t))\,u \\ &\hat{y}(t) = C\dot{\hat{x}}(t) \\ &\hat{x}(0) = x(0), \;\; \dot{\hat{x}}(0) = \dot{x}(0) \end{aligned} \tag{4.1}$$

where $\widehat{x}(t)$ is the estimated plant state, $\widehat{y}(t)$ is the estimator output and $\widehat{f}(y;\widehat{\theta}) : \mathbb{R} \times \mathbb{R}m \to \mathbb{R}$ is the $\widehat{\theta}$-parameterized estimate (*on-line approximator*) of the failure term $\beta(t-T)\, f(y)$.

The objective is to define adaptation laws (i.e. differential equations) that generate on-line the unknown parameter estimates $\widehat{\theta}(t)$, while at the same time guarantee that all the signals are bounded. To this end we use *Lyapunov redesign method* [10, 12], which essentially forces the derivative of an energy-type functional to be nonpositive.

As a starting point we make the assumption that the failure term $f(y)$ can be expressed as

$$f(y) = Z^T(y)\,\theta^* = \sum_{i=1}^{m} Z_i(y)\,\theta_i^* = \widehat{f}(y;\theta^*) \tag{4.2}$$

where $Z_i(y)$ are known bounded functions of the output y and θ_i^* are some unknown (optimal) parameters, i.e. we parameterize the unknown failure term in terms of the on-line approximator.

When we denote by $e(t) := x(t) - \widehat{x}(t)$ the *state error* and by $\varepsilon(t) := y(t) - \widehat{y}(t)$ the *output error*, we arrive at the following equations by combining (3.1), (4.1) and (4.2)

$$\begin{aligned} M\ddot{e}(t) + D\dot{e}(t) + Ke(t) &= B\left[\beta(t-T)\,f(y) - \widehat{f}(y;\widehat{\theta})\right]u(t) \\ &= BZ^T(y)\left[\beta(t-T)\theta^* - \widehat{\theta}(t)\right]u(t) \\ &= -BZ^T(y)\left[(1-\beta(t-T))\theta^* + (\widehat{\theta}(t) - \theta^*)\right]u(t) \\ &= -BZ^T(y)\left[\Phi(t)\theta^* + \tilde{\theta}(t)\right]u(t) \end{aligned} \tag{4.3}$$

where $\Phi(t) := 1 - \beta(t-T)$ and $\tilde{\theta}(t) := \widehat{\theta}(t) - \theta^*$. Note that in this case we have

$$\dot{\Phi}(t) = -\lambda\Phi(t), \quad t \geq T, \quad \Phi(T) = 1.$$

In order to study the stability properties of the learning scheme and derive an adaptation law for $\widehat{\theta}(t)$, we use the following Lyapunov-like functional that was also used in [5]

$$V(t) = \frac{1}{2}\langle M\dot{e}(t), \dot{e}(t)\rangle + \frac{1}{2}\langle Ke(t), e(t)\rangle + \frac{1}{2\gamma}|\tilde{\theta}(t)|^2_{\mathbb{R}m} + \frac{\mu}{2}|\Phi(t)|^2_{\mathbb{R}}, \tag{4.4}$$

where the positive constant μ will be defined below. Using (4.3), the time derivative of (4.4) is

$$\begin{aligned} \dot{V} &= -\langle D\dot{e}, \dot{e}\rangle - \langle \dot{e}, BZ^T(y)\,\theta^*\Phi u\rangle - \langle \dot{e}, BZ^T(y)\,\tilde{\theta}u\rangle + \frac{1}{\gamma}\,\langle\dot{\tilde{\theta}}, \tilde{\theta}\rangle_{\mathbb{R}m} - \mu\lambda|\Phi|^2_{\mathbb{R}} \\ &\leq -d\|\dot{e}\|^2_W + \|\dot{e}\|_W|\Phi|\cdot|BZ^T(y)\,\theta^*u| - \langle B^*\dot{e}, Z^T(y)\,\tilde{\theta}u\rangle_{\mathbb{R}m} + \frac{1}{\gamma}\langle\dot{\tilde{\theta}}, \tilde{\theta}\rangle_{\mathbb{R}m} - \mu\lambda|\Phi|^2_{\mathbb{R}} \\ &= -d\|\dot{e}\|^2_W + \|\dot{e}\|_W|\Phi|_{\mathbb{R}}|BZ^T(y)\,\theta^*u| - \varepsilon(t)Z^T(y)\,\tilde{\theta}u + \frac{1}{\gamma}\dot{\tilde{\theta}}^T\tilde{\theta} - \mu\lambda|\Phi|^2_{\mathbb{R}}, \end{aligned}$$

where we used the fact that $B^*\dot{e}(t) = C\dot{e}(t) = \varepsilon(t)$ (i.e. assumed a collocated system). When the adaptation law is chosen as

$$\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) = \gamma Z(y)\,\varepsilon(t)u(t) \tag{4.5}$$

we get

$$\dot{V} \leq -d\|\dot{e}\|_W^2 + \|\dot{e}\|_W|\Phi|c_2 - \lambda\mu|\Phi|_{\mathbb{R}}^2 \leq -\left(d - \frac{1}{2\alpha}c_2\right)\|\dot{e}\|_W^2 - \left(\lambda\mu - \frac{\alpha}{2}c_2\right)|\Phi|_{\mathbb{R}}^2$$

where we used

$$c_2 = \sup_{t\geq 0}(|BZ^T(y)\,\theta^*|).$$

We choose α such that $d - \frac{1}{2\alpha}c_2 > 0$ and then choose the parameter μ such that $\lambda\mu - \frac{\alpha}{2}c_2 > 0$. Then we end up with

$$\dot{V} \leq -\kappa\left(\|\dot{e}\|_W^2 + |\Phi|_{\mathbb{R}}^2\right) \leq 0.$$

We consider first the case of $t < T$, i.e. prior to the failure occurrence. Then we have

$$V(t) + \kappa\int_0^t \|\dot{e}(\tau)\|_W^2\,\mathrm{d}\tau \leq V(0).$$

Using the fact that $e(0) = 0 = \dot{e}(t)$ and by choosing $\hat{\theta}(0)$ be such that $\hat{f}(y;\hat{\theta}(0)) = 0$, we have that $V(t) \equiv 0$ for all $t < T$. After the failure we have that $\beta(t-T)\,f(y) \neq 0$. Thus, integrating the above from T to some $t > T$ we have that

$$V(t) + \kappa\int_T^t \|\dot{e}(\tau)\|_W^2\,\mathrm{d}\tau \leq V(T).$$

This implies that by monitoring either the state output $\varepsilon(t)$ or the output of the *on-line approximator* $\hat{f}(y(t);\hat{\theta}(t))$, we can detect the failure; when either of the above signals becomes nonzero, we have failure detection. Using previous results, it can be shown that the state error ($\|e\|_W$ and $|\dot{e}|$) will go to zero asymptotically, see [7] for a proof of asymptotic convergence of second order adaptive distributed parameter systems. Furthermore, we have that $\hat{\theta}$ is bounded, and if we have *persistence of excitation*, we can show $\hat{\theta} \to \theta^*$, see [3] for a proof of parameter convergence in the context of adaptive estimation.

The *diagnosis* of the failure comes after the failure occurrence, meaning that we try to identify the term $Z^T(y)\,\theta^*$. In the event that the actuator failure term $f(y)$ cannot be expressed by $f(y) = Z^T(y)\,\theta^*$ as in (4.2), we then use the following

$$M\ddot{e}(t) + D\dot{e}(t) + Ke(t) = B\nu(t)u(t) + B\left[\beta\hat{f}(y;\theta^*) - \hat{f}(y;\hat{\theta}(t))\right]u(t), \tag{4.6}$$

where $\nu(t)$ is the *approximation error* given by

$$\nu(t) = \beta(t-T)\left[f(y) - \hat{f}(y;\theta^*)\right]. \tag{4.7}$$

The "optimal" parameter θ^* is now chosen as the value of $\widehat{\theta}$ that minimizes the L_2-norm distance between $f(y)$ and $\widehat{f}(y;\widehat{\theta})$. Using once again Lyapunov redesign methods [10], the adaptation law for the adjustment of parameter estimates is now given by

$$\dot{\widehat{\theta}}(t) = \mathcal{P}\left\{Z(y)\,\varepsilon(t)u(t)\right\} \quad \widehat{\theta}(0) = 0, \tag{4.8}$$

where $Z \in I\!R\,m$ is $Z(y) = \frac{\partial \widehat{f}(y;\widehat{\theta})}{\partial \widehat{\theta}}$ and $\mathcal{P}$ is the projection operator that constrains the parameter $\widehat{\theta}$ to some selected compact, convex region of the parameter space, [10, 14]. As was mentioned in [14] for the finite dimensional treatment, in the case of the compact region being a hypersphere, the adaptive law can then be expressed as

$$\dot{\widehat{\theta}}(t) = Z(y)\,\varepsilon(t)\,u(t) - \chi^* \frac{\widehat{\theta}(t)\,\widehat{\theta}^T(t)}{|\widehat{\theta}(t)|^2_{I\!R\,m}} Z(y)\,\varepsilon(t)u(t), \tag{4.9}$$

where the *indicator function* χ^* is given by

$$\chi^* = \begin{cases} 0 & \text{if} \quad (|\widehat{\theta}| < L) \text{ or } (|\widehat{\theta}| = L \text{ and } \widehat{\theta}^T Z\varepsilon u \leq 0) \\ 1 & \text{if} \quad (|\widehat{\theta}| = L \text{ and } \widehat{\theta}^T Z\varepsilon u > 0) \end{cases}$$

and L is an upper bound for $|\theta^*|$. Using the smoothness assumption on $\widehat{f}$, it then follows from (4.6) that

$$\begin{aligned} M\ddot{e}(t) + D\dot{e}(t) + Ke(t) &= -\Phi Bu(t)\widehat{f}(y;\theta^*) - BZ^T(y)\,\widetilde{\theta}(t)\,u(t) \\ &\quad -B\Delta(y;\widehat{\theta})\,u(t) + B\nu(t)\,u(t) \end{aligned} \tag{4.10}$$

where $\Delta(y;\widehat{\theta})$ is given by

$$\Delta(y;\widehat{\theta}) = \widehat{f}(y;\theta^*) - \widehat{f}(y;\widehat{\theta}) - \frac{\partial \widehat{f}(y;\widehat{\theta})}{\partial \widehat{\theta}}.$$

If we let

$$\omega(t) = -\Delta(y(t);\widehat{\theta}(t)) + \nu(t),$$

we have

$$M\ddot{e}(t) + D\dot{e}(t) + Ke(t) = -BZ^T(y)\,\tilde{\theta}u(t) - \Phi(t)\,Bu(t)\widehat{f}(y;\theta^*) + B\omega(t)\,u(t). \tag{4.11}$$

When the derivative of $V(t)$ in (4.4) is evaluated along the trajectories of (4.9), (4.11), it yields

$$\dot{V}(t) = -2\langle \dot{e}, D\dot{e}\rangle - 2\langle \dot{e}, \Phi Bu\widehat{f}\rangle + 2\langle \dot{e}, B\omega u\rangle - 2\chi^* \widetilde{\theta}^T \frac{\widehat{\theta}\widehat{\theta}^T}{|\widehat{\theta}|^2} Z^T \varepsilon u - 2\lambda|\Phi|^2, \tag{4.12}$$

where we used the fact that

$$\dot{\Phi} = -\lambda\Phi.$$

Using established results in the theory of robust adaptive control [10] and in automated fault detection [14], we have that the projection term can only make the derivative of $V(t)$ more negative, i.e. the additional term in the right hand side of (4.9) pre-multiplied by $\widetilde{\theta}^T$ is negative semidefinite. Using the coercivity of the operator D, the smoothness of the input $u(t)$ and output $y(t)$, and the smoothness assumption on $\widehat{f}$, we have that

$$\dot{V}(t) \leq -c_1\|\dot{e}(t)\|_W^2 - c_2|\Phi(t)|^2 + c_3|\omega(t)|^2,$$

for some c_1, c_2, $c_3 > 0$. When $c_1\|\dot{e}(t)\|_W^2 + c_2|\Phi(t)|^2 \geq c_3|\omega(t)|^2$ we have that $\dot{V} \leq 0$, which yields the uniform boundedness of V and $\tilde{\theta}$. Thus by integrating over a finite interval $[T, T+\tau]$ we have that

$$V(T+\tau) + c_1 \int_T^{T+\tau} \|\dot{e}(t)\|_W^2 \, dt + c_2 \int_T^{T+\tau} |\Phi(t)|^2 \, dt \leq V(T) + c_3 \int_T^{T+\tau} |\omega(t)|^2 \, dt.$$

The above yields the uniform boundedness of $\widehat{\theta}$ and $V(t)$ for $t \geq T$. It is easily observed that $V(t) = 0$ for $t < T$. Using the observability condition, we have that both $\widehat{f}(y, \widehat{\theta}(t))$ and $\varepsilon(t)$ are zero prior to the failure time T and become nonzero for $t \geq T$. Hence, by monitoring either the output error $\varepsilon(t)$ or the on-line approximator output $\widehat{f}(y, \widehat{\theta}(t))$, we can detect the time of failure T. Furthermore, we have that the extended L^2 norm of the state estimation velocity error (and by observability, the output error) over any finite time interval is at most of the same order as the extended L^2 norm of $\omega(t)$.

5. FAILURE ACCOMMODATION

The standard control law for the nominal plant (2.2) without failure terms can be chosen as $u(t) = u_0(t)$ with

$$u_0(t) = -G_1\widehat{x}(t) - G_2\dot{\widehat{x}}(t) + G_3 r(t), \tag{5.1}$$

where the gains G_1, G_2 are, for example, chosen as the LQR feedback gains obtained by solving an Algebraic Riccati equation for the nominal plant (2.2), and the signal r is a reference signal with G_3 a reference gain, that is used if the control objective is model reference.

In the presence of a failure, the nominal control law (5.1) needs to be modified to account for the additive failure term $Bu(t)\,f(y)$. This takes the form

$$u(t) = \frac{1}{1 + \widehat{f}(y; \widehat{\theta})} u_0(t). \tag{5.2}$$

The closed loop state estimator (4.1) is now given by

$$M\ddot{\widehat{x}} + [D + BG_2]\,\dot{\widehat{x}} + [K + BG_1]\,\widehat{x} = BG_3 r.$$

A modification to the control law (5.2) must be made in order to ensure that $1 + \widehat{f}(y; \widehat{\theta}) \neq 0$. Implicitly it was assumed that the fault term $f(y) \neq -1$ (hence no loss of controllability in (3.1)) and thus the output of the on-line approximator must not cancel out the control signal in (4.1). In practice, the control reconfiguration must ensure that $|1+\widehat{f}(y;\widehat{\theta})| > c$ with $0 < c \ll 1$ in order to avoid large inputs in the system. This leads to the reconfigured controller

$$u(t) = u_0(t) \begin{cases} (1+\widehat{f}(y;\widehat{\theta}))^{-1} & \text{if } |1+\widehat{f}(y;\widehat{\theta})| > c \\ 1/c & \text{if } |1+\widehat{f}(y;\widehat{\theta})| \le c. \end{cases}$$

Alternatively, one can switch the adaptation off for $\widehat{\theta}$ when $1+\widehat{f}(y;\widehat{\theta})$ is "near" 0 or impose additional constraints on the adaptation rule (4.7) such that $1+\widehat{f}(y;\widehat{\theta}) \neq 0$.

6. NUMERICAL RESULTS

It is assumed that the beam satisfies the Euler–Bernoulli displacement hypothesis with Kelvin–Voigt damping (damping proportional to strain rate) and air damping (damping proportional to velocity). Two piezoceramic patches are bonded to the beam at the location $\xi_l \le \xi \le \xi_r$ and are excited out-phase, which results in pure bending of the beam [2]. The moment due to patches is localized to the region covered by the patches. When the structure is subject to moments generated by the patches, it leads to the equation

$$\rho(\xi)\frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 \mathcal{M}}{\partial \xi^2} = -\frac{\partial^2 \mathcal{M}_p}{\partial \xi^2}, \quad 0 < \xi < l, \tag{6.1}$$

where $x = x(t,\xi)$ is the transverse displacement, $\mathcal{M}$ is the internal moment given by $\mathcal{M} = EI(\xi)\frac{\partial^2 x}{\partial \xi^2} + c_D I(\xi)\frac{\partial^3 x}{\partial \xi^2 \partial t}$, and $\mathcal{M}_p$ is the external moment due to the piezoceramic patches. This piezoceramic moment is given by $\mathcal{M}_p = -\mathcal{K}_A \chi_p(\xi)\, u(t)$, where $u(t)$ is the voltage applied to the patches, $\mathcal{K}_A$ is a constant that depends on the piezoceramic material properties [2] and $\chi_p(\xi)$ is the characteristic function, which is equal to 1 for $\xi_l \le \xi \le \xi_r$ and zero elsewhere. The above (spatially varying) parameters $\rho(\xi)$, $EI(\xi)$, and $c_D I(\xi)$ above are the *mass density*, *stiffness* coefficient and *damping* coefficient, respectively. Using the above equations for the moments, we arrive at the following partial differential equation (PDE) for the transverse displacement of the beam

$$\rho\ddot{x}'' + [EIx'' + c_D I\dot{x}'']'' = [\mathcal{K}_A \chi_p(\xi)\, u]'', \tag{6.2}$$

for $\xi \in \Omega = [0,1]$, with collocated output

$$y(t) = \int_0^l \mathcal{K}_S \chi_p(\xi)\dot{x}''(t,\xi)\, \mathrm{d}\xi,$$

where $\mathcal{K}_S$ is a sensor constant which is a piezoceramic material and geometry related quantity, [2]. Associated with the above beam equation are the appropriate boundary (cantilevered beam) and initial conditions given by $x(t,0) = x'(t,0) = 0 = x''(t,l) = x'''(t,l)$, and $x(0,\xi) = x_0(\xi)$, $\dot{x}(0,\xi) = x_1(\xi)$.

When the beam equation (6.1) with collocated output is written as a second order evolution equation in the larger space $V^* = H^{-2}(0,l)$, it produces equation (1.1) with velocity output given by (1.2) see, for example, [2, 4] for details on how to write (6.1) in an abstract setting and the corresponding expressions of the operators M, D, K B and C in weak form. It is easily seen that the output operator C is a constant multiple of the adjoint of the input operator B, given by $C = \alpha_p B^*$ with $\alpha_p = \frac{\mathcal{K}_S}{\mathcal{K}_A}$.

We now summarize the numerical approximation scheme. Assume that the beam displacement is approximated by

$$x^n(t,\xi) = \sum_{i=1}^{n+1} \alpha_i(t)\,\phi_i^n(\xi), \qquad i = 1,2,\ldots,n+1$$

where $\phi_i^n(\xi)$, $i = 1,\ldots,n+1$ are modified cubic splines on $[0,l]$. Then using results in [2, 4] the beam equation can be written in a matrix form as

$$\begin{aligned} &M^n\ddot{\alpha}(t) + D^n\dot{\alpha}(t) + K^n\alpha(t) = B^n\left[1+\beta f(y)\right]u(t) \\ &y(t) = C^n\dot{\alpha}(t) \end{aligned}$$

where the above matrices are given explicitly in [2, 4]. In this simulation study, Radial basis function networks are used as the on-line approximator model given by

$$\widehat{f}(y,\widehat{\theta}) = \sum_{k=1}^{m} \widehat{\theta}_i(t)\exp\left(-\frac{(y-c_i)^2}{\sigma^2}\right) = Z^T(y)\,\widehat{\theta}(t).$$

The finite dimensional adaptive observer along with the on-line approximator are given by

$$\begin{aligned} &M^n\ddot{\widehat{\alpha}}(t) + D^n\dot{\widehat{\alpha}}(t) + K^n\widehat{\alpha}(t) = B^n\left[1+\widehat{f}(t,\widehat{\theta})\right]u(t) \\ &\dot{\widehat{\theta}}(t) = \gamma Z(t)\varepsilon(t)\,u^T(t), \quad \widehat{\theta}(0) = 0, \end{aligned}$$

where γ is the *adaptive gain*, [10].

For the specific set of simulations, we assumed that the beam length is $l = 0.4573\,\mathrm{m}$, with the patches placed at $x_l = 0.15\,\mathrm{m}$ and $x_r = 0.25\,\mathrm{m}$. The beam stiffness coefficient is $EI_b = 0.491\,\mathrm{Nm}^2$ and the beam damping coefficient is $c_D I_b = 0.48675 \times 10^{-3}\,\mathrm{s\,Nm}^2$. In the damping component, we assumed air damping with damping parameter $9.75 \times 10^{-3}\,\mathrm{s\,N/m}^2$. The corresponding values for the patch are $EI_p = 0.793\,\mathrm{Nm}^2$, $c_D I_p = 1.255 \times 10^{-3}\,\mathrm{s\,Nm}^2$ with patch linear mass density $\rho_p = 0.433\,\mathrm{kg/m}$ and thickness $h_p = 0.000254\,\mathrm{m}$. The beam had a mass density $\rho_b = 0.093\,\mathrm{kg/m}$, thickness $h_b = 0.0016\,\mathrm{m}$, and width $b = 0.0203\,\mathrm{m}$. The piezoceramic constant $\mathcal{K}_A = 1.746 \times 10^{-2}\,\mathrm{Nm/V}$ with the one used for sensing $\mathcal{K}_S = 1 \times 10^{-1}\mathcal{K}_A$.

The failure term is given by

$$\beta(t-0.01)\,f(y) = 10\left(1 - e^{-5.5(t-0.01)}\right)\sin(y)$$

which models an incipient fault commencing at $T = 0.01$ seconds. The adaptive gain is $\gamma = 3$. The feedback gains G_1, G_2 were found by solving the Riccati equation

$$\Pi\mathcal{A} + \mathcal{A}^T\Pi - \Pi\mathcal{B}\mathcal{B}\Pi + Q = 0$$

for the nominal system (2.2) written as a first order system with $\dot{x} = \mathcal{A}x + \mathcal{B}u$ and Q given by $Q = \mathrm{diag}(500K, 10^4 M)$. Finally, the reference term is $G_3 r(t) = 10\sin(0.155\pi t)$.

The evolution of the on-line approximator output $\widehat{f}(y;\widehat{\theta})$ (dashed) is presented in Figure 6.1. In the same figure we plot the actual failure term $\beta(t-0.01)\,f(y)$ (solid). It is observed that the on-line approximator (OLA) is able not only to detect but to diagnose the failure as well.

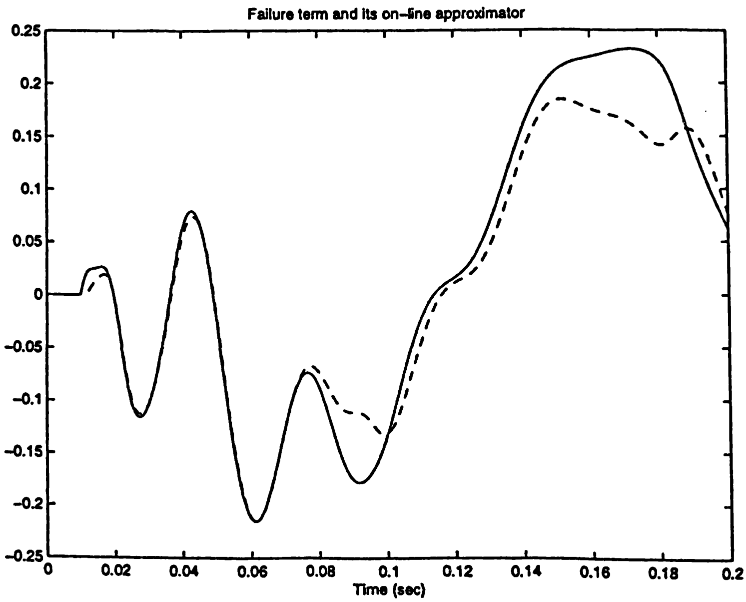

Fig. 6.1. Evolution of failure term $\beta(t - 0.01)\,f(y)$ (solid) and OLA $\widehat{f}(y;\widehat{\theta})$ (dashed).

When the difference between the output of the healthy plant with full state feedback (y_{ideal}) and that of the plant output with no-accommodated failure ($y_{\text{no accom}}$) is plotted against time in Figure 6.2a, we notice that it has a value of zero prior to $t = 0.01$ seconds and then attains a nonzero value. The difference between the output of the healthy plant with having state feedback (y_{ideal}) and the output of the plant with accommodated failure and output feedback ($y_{\text{with accom}}$) is presented in Figure 6.2b and is observed that it stays close to zero. Thus, the performance of the system with accommodated failure is near the performance of the system under ideal conditions and full state feedback. As a result we can conclude that both the on-line approximator output $\widehat{f}$ and the output error $\varepsilon(t)$ can be used for failure detection and furthermore the OLA output can be used for failure diagnosis as well.

Furthermore, the benefit of accommodating the actuator failure is evident on the control performance.

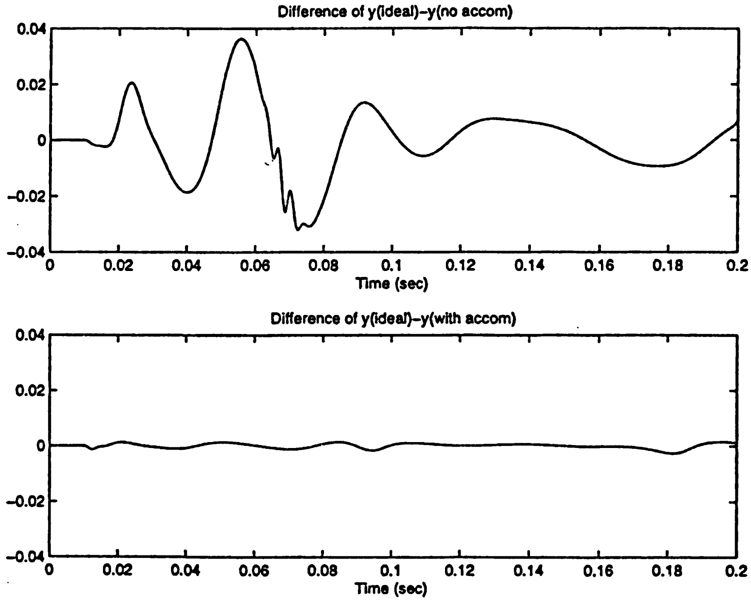

Fig. 6.2. Evolution of (a) $y_{\text{ideal}} - y_{\text{no accom}}$ and (b) $y_{\text{ideal}} - y_{\text{with accom}}$.

7. CONCLUSION

In this note an on-line approximation scheme was proposed for the detection, diagnosis and accommodation of actuator failures in a plant whose dynamics were governed by a second order partial differential equation. The plant describes the transverse vibration of a flexible cantilevered beam actuated with a pair of piezoceramic patches that are also used as sensors. The failure was modeled as a time varying additive perturbation of the actuator signal with a function of the output signal as the failure gain. The proposed scheme, through both theoretical and numerical results, was shown to actually detect, diagnose and accommodate the actuator failure with incipient time profiles.

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