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RTC-METHOD FOR THE CONTROL OF NUCLEAR REACTOR POWER

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In this paper, a new concept of the Reactivity Trace Curve (RTC) for reactor power control is presented. The concept is demonstrated for a reactor model with one group of delayed neutrons, where the reactivity trace curve is simply a closed form exponential solution of the RTC-differential equation identifier. An extended reactor model of multigroup (six groups) of delayed neutrons is discussed for power control using the RTC-method which is based on numerical solution of the governing equation for the RTC-differential equation identifier. In this numerical solution, an impeded analytical solution for the RTC-identifier in every sampling time step is used. Finally, the concept is applied to a more rigorous reactor model, namely; a model of multigroup of delayed neutrons with temperature feedback. The simulation studies for all of the above mentioned cases demonstrate the validity of the concept for reactor power control with absolute elimination of power shootings.

INTRODUCTION

Nuclear fission is a process that generates energy which is about a million greater in magnitude compared to the corresponding chemical processes. The basic initiator of this process is a neutron. One can identify two types of neutrons, namely prompt which are directly produced from the fissioning of the fissile nuclei, and the other one is reffered to as the delayed neutrons which come later in the process after the decay of the so called fission products or delayed neutron precursors. If there were no delyed neutrons, then the time constant of the fission process will be solely governed by the prompt neutrons, such time will be of the order of $10^{-4} - 10^{-6}$ sec. In such case it would be impossible to control the process, because there exists no control system with response of this order. Luckily, delayed neutrons come into picture and extend the overall process time constant to the second range, and hence one is capable of designing control systems that fulfill this time span. In most of the existing power and research reactors, PID-control is used. Such control causes overshoot in reactor power, and for power reactors, during power manoeuvring, such overshoot is avoided by simply raising the power to certain percent of the power required, and then operators start to manipulate the system reactivity untill the power settles at the desired power with no power shooting. Bernard et al [1-4] discussed this overshooting phenomena and related it to the effect of delayed neutrons on the reactor dynamics. It occurs because the production of precursors, being an integral part of fission process, is in equilibrium with the transient reactor power while, given that precursors have a finite life time, such is not the case with their decay. Bernard et al [1] showed from the manipulation of the equations of a reactor model with multigroup delayed neutrons that such mismatch between equilibrium and actual concentration during a power change is given by:

$$0 = \rho n - \Lambda \sum_i \left(\lambda_i C_i^0 - \lambda_i C_i\right), \tag{1}$$

where ρ – system reactivity, n – reactor power, Λ – neutron generation time, C_i^0 – equilibrium concentration of ith precursor, C_i – actual concentration, λ_i – decay constant of ith precursor.

The only way to keep the power constant during this mismatch is by time dependent adjustment of the system reactivity. Bernard et al [2] derived what is so called the dynamic period equation which is given by:

$$\tau(t) = \frac{\beta - \rho(t)}{\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} + \lambda_e(t)\,\rho(t) + \left(\frac{\beta-\rho(t)}{\lambda_e(t)}\right)\frac{\mathrm{d}\lambda_e(t)}{\mathrm{d}t}}, \tag{2}$$

where $\lambda_e(t)$ – the dynamic decay constant, β – the delayed neutron fraction.

The dynamic decay constant is given as an average weighted sum by the following equation [3]:

$$\lambda_e(t) = \frac{\sum_i \lambda_i C_i(t)}{\sum_i C_i(t)}, \tag{3}$$

Bernard et al [4] suggested a method of control which makes the reactor period infinite. Such method is called absolute reactivity constraint, and sufficient reactivity constraint; they are given by the following equations, respectively:

$$-\left|\frac{\mathrm{d}\rho}{\mathrm{d}t}\right| \leq \lambda_e \rho \leq \left|\frac{\mathrm{d}\rho}{\mathrm{d}t}\right|, \tag{4}$$

$$-\left(\frac{|\dot{\rho}|}{\lambda_e} + |\dot{\rho}|\,\tau \ln\left(\frac{p_f}{p}\right)\right) \leq \rho \leq \left(\frac{|\dot{\rho}|}{\lambda_e} + |\dot{\rho}|\,\tau \ln\left(\frac{p_f}{p}\right)\right), \tag{5}$$

where $\frac{\mathrm{d}\rho}{\mathrm{d}t} = \dot{\rho}$ – maximum possibly obtained rate of reactivity change, p_f – final power, p – actual power.

1. THE REACTIVITY TRACE CURVE (RTC) CONCEPT

Ratemi [6] proposed the concept of Reactivity Trace Curve (RTC) based on the work of Bernard et al, which was just briefly described above. The concept is simply based on the idea of trying to solve a differential equation which forces the dynamic period to be infinity, and hence terminating any transient power and levelling it at a steady state value without any power shooting. This implies that such differential equation is actually resulted from equating the denominator of (2) to zero, that is;

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} + \lambda_e(t)\,\rho(t) + \left(\frac{\beta - \rho(t)}{\lambda_e(t)}\right)\frac{\mathrm{d}\lambda_e(t)}{\mathrm{d}t} = 0. \tag{6}$$

Such differential equation represents the RTC-identifier, where the RTC is to be the solution of this differential equation. In the next section, we select a simplified reactor model to be controlled by the RTC-method, namely; a model based on one group of delayed neutrons so to easily define the RTC as an exponential form and hence to demonstrate the applicability of the concept.

2. RTC–METHOD FOR A REACTOR MODEL WITH ONE GROUP OF DELAYED NEUTRONS

A point kinetic reactor model with either one or multi-group of delayed neutrons are frequently used in literature [5] for describing the dynamic behavior of nuclear reactors. In this section, a one group of delayed neutrons model is chosen to demonstrate the RTC-method of control. The equations governing such reactor model are given by:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\rho - \beta}{\Lambda} n + \lambda C, \tag{7}$$

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{\beta}{\Lambda} n - \lambda C. \tag{8}$$

For such model the decay constant λ is a static decay constant since it is a physical parameter which is not subjected to a change, this is quite different from the case of the multigroup delayed neutrons case, since the effective decay constant $\lambda_e(t)$ for that case is time dependent and is given by (3). Therefore, one can easily determine the dynamic period equation from the more general one given by (2), that is

$$\tau(t) = \frac{\beta - \rho(t)}{\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} + \lambda\rho(t)}. \tag{9}$$

To make this period infinite, one can state that:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} + \lambda\rho(t) = 0. \tag{10}$$

This has a direct analytical solution in an exponential form which is given by:

$$\rho_{\mathrm{RTC}} = \rho_0\, e^{-\lambda t}, \tag{11}$$

where ρ_0 is the system reactivity at the start of control, and t is the time elapsed from initiating the control. Equation (11) is what we define as the Reactivity Trace Curve which is capable of terminating the power transient and levelling it at the desired value, whereas, (10) defines the RTC-identifier. Simulation studies for this case is shown in Figure 1 for power increase case, and in Figure 2 for power decrease case.

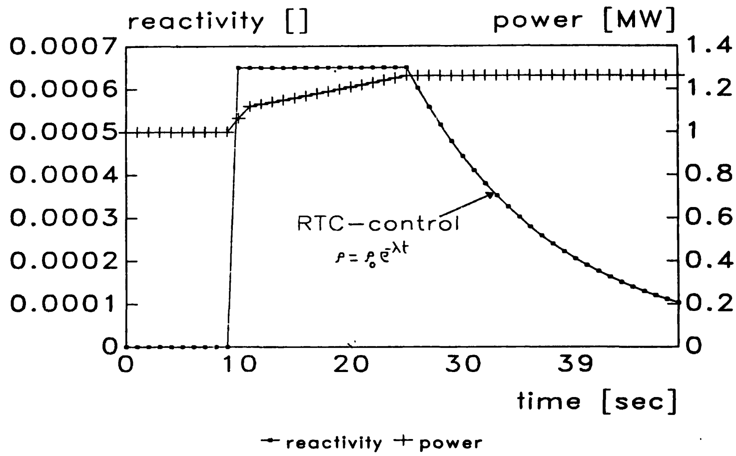

Fig. 1. RTC-control for power increase case.

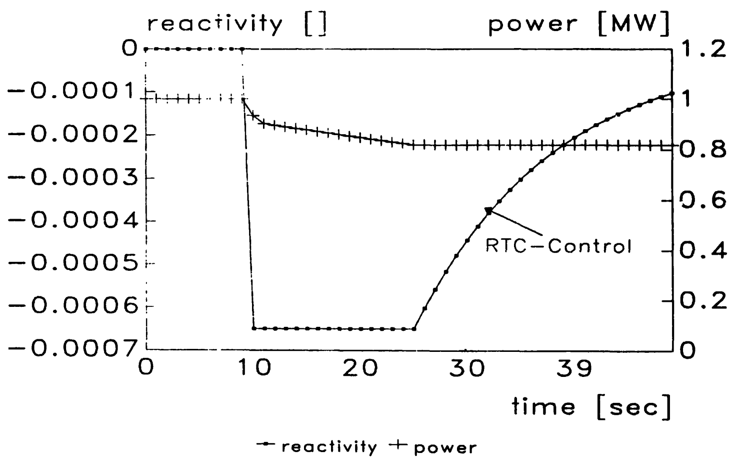

Fig. 2. RTC-control for power decrease case.

Figure 1 shows that the reactor power is increased due to external positive step of reactivity insertion corresponding to a withdrawals of a control rod. The power is increased till the desired power is achieved at 25 sec., then the RTC-control takes the role of power levelling with absolute elimination of power shooting. Figure 2, on the other hand, shows the reverse case of power decrease, namely; an insertion of control rod for negative reactivity step. Similar behavior is achieved. In the two cases, one notices that during the RTC-control, a reversal of the control rod motion is established for negating the existing system reactivity.

2.1. The ρ–z–t diagram

The required reactivity which will control the reactor power at the desired level is obtained in correspondence to the motion of the control rod. A ρ–z–t diagram which relates the RTC-reactivity manipulator (ρ), the position of the control rod (z), and the time required for control (t), is presented next. Such diagram demonstrates the required control rod position that corresponds to the required reactivity suggested by the RTC at every instant of time. Figure 3 presents the ρ–z–t diagram (Bird like diagram) for power increase case. One notices that as the time progresses, the control rod is inserted accordingly. Figure 4 (Concord like diagram) demonstrates the case for power decrease case, where in this case, one has to withdraw the control rods for power control to a prescribed rod position determined by the RTC.

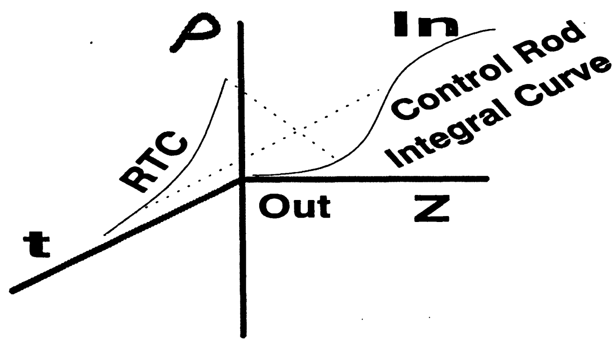

Fig. 3. ρ–z–t diagram for power increase case.

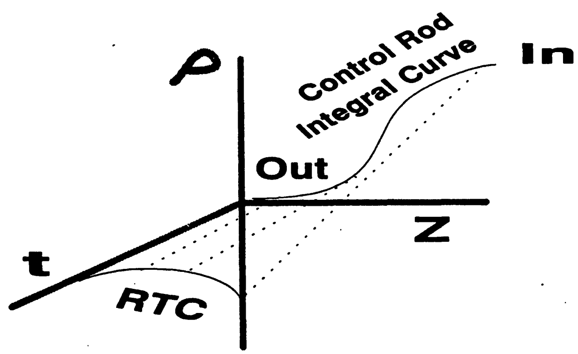

Fig. 4. ρ–z–t diagram for power decrease case.

3. RTC–METHOD FOR A REACTOR MODEL WITH MULTI–GROUPS OF DELAYED NEUTRONS

Nuclear reactors are better described with multigroups of delayed neutrons. Therefore, one has to extend the RTC-concept for controlling such reactors. This requires the solution of the related RTC-identifier which is given by (6), with the effective dynamic decay constant, $\lambda_e(t)$, is given by (3). One can, easily, derive an equation for the rate of change of the effective dynamic decay constant as:

$$\frac{\mathrm{d}\lambda_e(t)}{\mathrm{d}t} = \frac{\sum C_i \left(\sum \lambda_i \dot{C}_i\right) - \left(\sum \lambda_i C_i\right) \sum \dot{C}_i}{\left(\sum C_i\right)^2}. \tag{12}$$

Ratemi and Elbuni [7] solved, numerically, (6) with the auxiliary (3) and (12) for a reactor model with six groups of delayed neutrons. Their results showed complete power settling at the desired value with absolute elimination of power shootings. They were also able to correlate an empirical summed exponential terms that define two finger prints for the reactor. One of the exponential terms is governed by a one group decay constant λ, and the other is governed by an effective decay constant γ. Their empirical equation is given by:

$$\rho_{\mathrm{RTC}} = \rho_{01}\, e^{-\lambda t} + \rho_{02}\, e^{-\gamma t}. \tag{13}$$

Such empirical formula, although it is controlling the power, it does have some shooting, because the λ fitting they performed was first order fitting.

3.1. Impeded RTC–analytical solution for reactor model with six groups of delayed neutrons

In this section, an impeded analytical solution for the RTC-identifier (6) at every sampling step is presented. The selected reactor model is based on six groups of delayed neutrons. With some mathematical manipulation of equations (6), (3), and (12), one can show that the identifier (6) can be rewritten as:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + a_k\, \rho(t) + c_k = 0, \tag{14}$$

where

$$a_k = \left(\frac{\sum \lambda_i C_i + \sum \dot{C}_i}{\sum C_i} - \frac{\sum \lambda_i \dot{C}_i}{\sum \lambda_i C_i}\right)_k, \tag{15}$$

$$c_k = -\beta \left(\frac{\sum \dot{C}_i}{\sum C_i} - \frac{\sum \lambda_i \dot{C}_i}{\sum \lambda_i C_i}\right)_k. \tag{16}$$

The a_k, and c_k are evaluated at each sampling time and therefore, for each sampling time, they are constants. The solution of the first order differential equation with constant coefficients a_k, and c_k (14) in each sampling interval k is given by:

$$\rho_{k+1} = \left(\rho_k + \frac{c_k}{a_k}\right) e^{-a_k t} - \frac{c_k}{a_k} \tag{17}$$

since (17) represents a recursive relation for ρ, then t represents the sampling time interval of RTC-control. A reactor model with six groups of delayed neutrons can be described by the following set of differential equations:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\rho-\beta}{\Lambda}n + \sum_i \lambda_i C_i \tag{18}$$

$$\frac{\mathrm{d}C_i}{\mathrm{d}t} = \frac{\beta_i}{\Lambda}n - \lambda_i C_i, \quad i = 1, 2, 3, \ldots, 6, \tag{19}$$

where λ_i is the decay constant of the ith precursor (physical parameter) β_i is the delayed neutron fraction of the ith group. Such reactor model is simulated and controlled with the impeded analytical RTC solution given by (17). Figure 5 presents the result for power increase case due to insertion of positive reactivity $\rho = .1\beta$, the figure demonstrates that the desired power (1.53 MW) is obtained at 30 sec., and is attained at that level with the RTC-control with absolute elimination of power shooting.

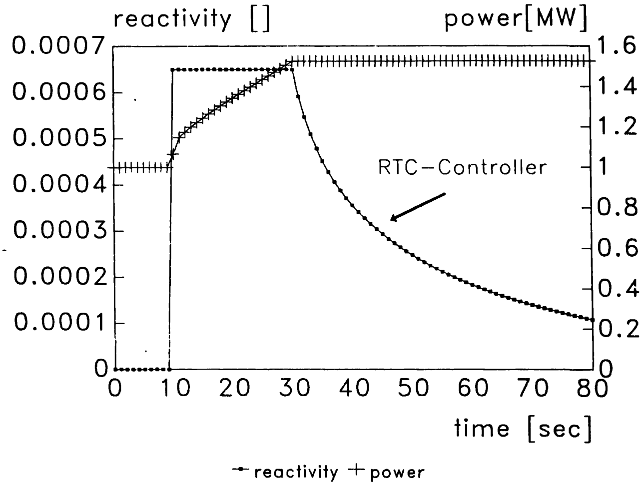

Fig. 5. RTC-control for reactor model with six groups of delayed neutrons.

4. RTC-CONTROL FOR REACTOR MODEL WITH SIX GROUPS OF DELAYED NEUTRONS AND WITH TEMPERATURE FEEDBACK EFFECTS

For reactors operating at low power, one can ignore temperature feedback effect. But for the case of power reactors operating at high power, such ignorance is not allowed. Therefore, in this section, we extend the reactor model to include temperature feedback effects. The reactor model with six groups of delayed neutrons and with temperature feedback effects can be described by the following set of eight coupled differential equations and two auxiliary algebraic equations:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\rho-\beta}{\Lambda}n + \sum_i \lambda_i C_i, \tag{20}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i C_i, \quad i = 1, 2, \ldots, 6, \tag{21}$$

$$\frac{dT}{dt} = Gn - \mu(T - T_{c0}), \tag{22}$$

$$\rho = \rho_{\text{ins}} + \rho_f, \tag{23}$$

$$\rho_f = \alpha(T - T_0) \tag{24}$$

where, Λ is the neutron generation time [10^{-4} sec.], n is the reactor power [MW], G is the reactor heat capacity [$0.15\, c^0/\text{MW} \cdot \text{sec}$], μ is the inverse of the time required to transfer heat to the coolant [$0.5 \,\text{sec}^{-1}$], T is the reactor temperature [c^0], T_{c0} is the coolant temperature [$200\, c^0$], T_0 is the reactor equilibrium temperature [$230\, c^0$], ρ_f is the feedback reactivity [ ], ρ_{ins} is the inserted reactivity [ ], and α is the temperature coefficient of reactivity [$-10^{-4}/c^0$].

For such test example reactor model, we solved the RTC-identifier (6) with ρ now is given by (23), where, both of the feedback reactivity and the inserted reactivity contribute to the total system reactivity. Simulation results for this case are listed in Figures 6 − 8. For such simulation, the reactor is kept operating at steady state for 10 seconds, then a positive insertion of reactivity of $0.1\,\beta$ is applied (by raising the control rod), and then at 30 seconds, the RTC -control is applied , where again we used the already described impeded analytical solution for every time step. Figure 6 shows both the power and reactor temperature response curves. One clearly sees that after 10 sec., the power increases promptly, then it slows down, because, of the temperature feedback effects, and at 30 sec. the RTC-control is applied which keeps the power at the prescribed level with no shooting. Figure 7 displays both the system reactivity, and the power curves. The figure shows that after 10 sec., the system reactivity (the inserted and the feedback drops down from the previous step value, and because it is not negative, the power continues to rise but at slower rate. At 30 sec., the control rod is moved according to the proposed required reactivity by the RTC. Note that the manipulation of the reactivity is done solely by the control rod, because feedback reactivity is an inherent process. Figure 8 displays, separately, the system reactivity, the inserted reactivity, and the feedback reactivity. The figure shows that after 30 sec., the RTC-controller is solely playing the role of power control at the prescribed level.

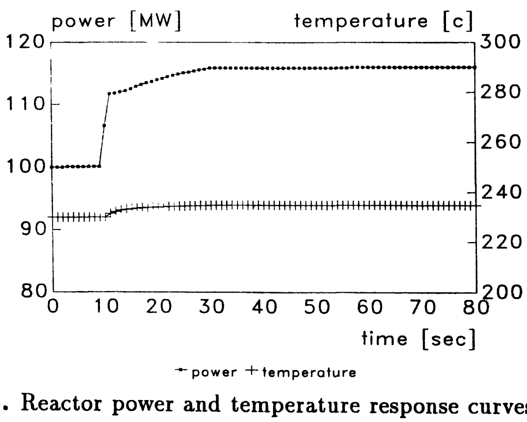

Fig. 6. Reactor power and temperature response curves.

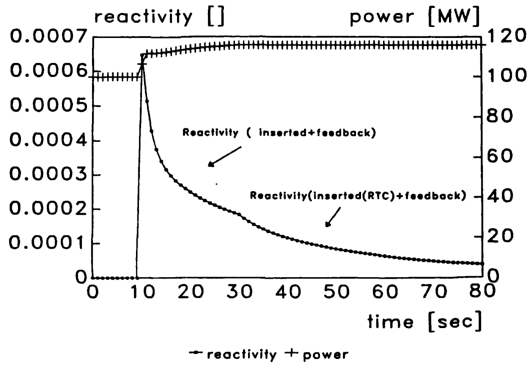

Fig. 7. The system reactivity and reactor power curves.

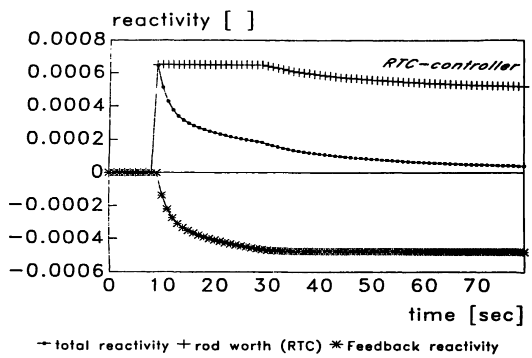

Fig. 8. System reactivity, inserted reactivity, and feedback reactivity.

5. CONCLUSION

In this paper, the concept of Reactivity Trace Curve (RTC) method for reactor power control was presented. The concept was illustrated with simulation results of a reactor model with one group of delayed neutrons. A closed form solution for such case was obtained. An extended model with six groups of delayed neutrons, and with a closed form solution of the RTC-identifier applied at every time step demonstrated further the validity of the concept. For making the concept close to real application for power plant control, a more complicated reactor model with temperature feedback effects was analyzed. The method seems to be working. Future study would be developing an RTC-based reference model controller for reactor power control of a power plant simulator. Such reference model will incorporate inverse kinetics which determines on-line reactivity from the monitored reactor power. Such on-line reactivity measurement is essential for the determination of the initial condition of the system reactivity for the solution of the RTC-identifier. Furthermore, the reference model will provide on line calculation for the precursors concentrations.

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