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HEURISTIC SOLVING A BICRITERIA PARALLEL–MACHINE SCHEDULING PROBLEM

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In the paper we investigate the inject press optimization problem with parallel matrices on the machine. A production order must be realized in a whole assortment and it is requested to minimize the production time and the number of matrix exchanges. The problem can be formulated as a parallel–machine scheduling problem with two criteria: to minimize the makespan and to minimize the number of job preemption. A simple heuristic algorithm for solving this NP–hard problem is proposed.

1. INTRODUCTION

Real life scheduling problems require the consideration of a number of criteria for correct decision making. The trade-offs involved in considering several criteria provide useful information to the decision maker. Research in multicriteria scheduling problems has been scarce when compared to research in single criterion scheduling problems A detailed literature survey of multicriteria scheduling problems is presented in [6]. Most papers are devoted to bicriteria single machine scheduling problems. These problems are foundations for the more complex multiple machine problems.

Three solution approaches to the multicriteria scheduling models can be considered: a criteria aggregation, an interactive approach, a set of nondominated schedules (see [3]). In accordance with the first approach the criteria are aggregated by a multicriteria utility function. An implicit enumeration procedure is used for finding the preferred schedule (see [5]). By the interactive approach the scheduling model is transformed into a multicriteria integer programming problem. The solution is based on the idea of interactive methods for multicriteria programming problems (see [4]). For some simple cases the whole set of nondominated schedules can be obtained. This is particularly so in the case of a bicriterion scheduling problem. The solution algorithm then enumerates all nondominated points (see [7,8]).

A more complex situation arises when there is a multiple machine configuration. The configurations, in increasing order of their complexity, are parallel, flowshop and jobshop. Due to the difficulty in obtaining optimal solutions, a large number of problem specific heuristic techniques have been developed for one criterion multiple machine scheduling problems.

In a parallel–machine configuration there is a number of one operation jobs which can be processed on any of the machines. The paper is devoted to heuristic solving of a real life problem of inject press optimization that can be formulated as a bicriteria parallel–machine scheduling problem.

In Section 2 the inject press optimization problem is defined. The basic preliminaries and algorithms for parallel–machine scheduling problems are presented in Section 3. In Section 4 a heuristic approach for the inject press optimization problem is proposed. An example with real data is added. Concluding remarks are summarized in Section 5.

2. PROBLEM DEFINITION

The inject press optimization problem was formulated in the shoe industry. The inject press is a carousel type machine for production of sport shoes. It produces soles of shoes by injecting a plastic composition into matrices. There are m parallel matrices for m pairs of shoes on an inject press.

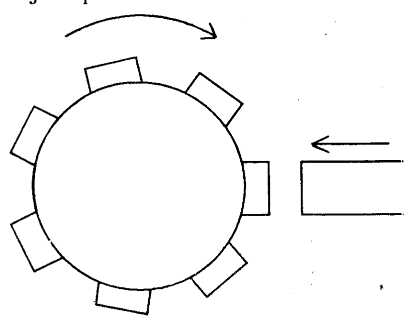

Fig. 1. Inject press machine.

The production is organized in such a way that the incoming production orders are processed as independent units. An order must be produced in a whole n size assortment and it is requested to minimize the production time T and the number of matrix exchanges E. The shoes are packed in ten pairs of the same size. Therefore the production is considered in tens of pairs of shoes.

The matrix exchange can be of three types:

1. The exchanges needed to produce the whole assortment with respect to size. The number of the exchanges is $A = n - m = \text{const}$.
2. The exchanges following from the separation of the large scale lots of the same size on two matrices. The number of the exchanges L is equal to the number of the large scale lots which are necessary to divide to two matrices.
3. The exchanges generated by job preemptions. The number of the exchanges P is to be minimized.

The number of all matrix exchanges $E = A + L + P$. The problem can be formulated as a parallel-machine scheduling problem with two objectives: to minimize the makespan and to minimize the number of the exchanges generated by job preemptions.

The whole production order is determined for one decade or can be determined for several decades. In the second case the objectives are minimized in every decade separately.

3. PRELIMINARIES AND BACKGROUND

In this paragraph we summarize some basic results for scheduling of independent jobs on parallel identical machines (see [1, 2]). The parallel-machine scheduling models are characterized by a set of m identical processors, a set of n independent single-operation jobs and a set of processing times t_i $(i = 1, 2, \ldots, n)$. It is required to find an optimal schedule with minimum makespan T.

In case that the job preemptions are permitted, the McNaughton's formula for minimum makespan $T_{\min}$ can be applied:

$$T_{\min} = \max\left\{1/m \sum_{i=1}^{n} t_i, \max_i t_i\right\}. \tag{1}$$

In the following investigations, we will use the McNaughton's algorithm which consists of these steps:

Step 1. Select some job to begin on machine 1 at time zero.

Step 2. Choose any unscheduled job and schedule it as early as possible on the same machine. Repeat this step until the machine is occupied beyond time $T_{\min}$ or until all jobs are scheduled.

Step 3. Reassign the processing scheduled beyond $T_{\min}$ to the next machine instead, starting at time zero. Return to Step 2.

In the schedules constructed by McNaughton's algorithm there can be at most $m - 1$ preemptions ($P_{\max} \leq m - 1$). The problem of minimizing the number of job preemption is NP-hard.

In case that the job preemptions are prohibited, the problem of minimizing makespan T is NP-hard also. The model can be expressed as an integer programming problem

$$\begin{aligned}
&\text{minimize} && T \\
&\text{subject to} && T - \sum_{i=1}^{n} t_i x_{ij} \geq 0, \quad j = 1, 2, \ldots, m \\
& && \sum_{j=1}^{m} x_{ij} = 1, \qquad\quad i = 1, 2, \ldots, n \\
& && x_{ij} = 1, \quad \text{if job } i \text{ is assigned to machine } j \\
& && \quad\;\; = 0, \quad \text{otherwise.}
\end{aligned}$$

The formulation of this problem contains $m + n$ constraints and $mn + 1$ integer variables.

For this problem a simple yet effective heuristic procedure is known (see [1]). The number of job preemption is now obviously $P_{\min} = 0$.

Heuristic procedure for minimizing makespan T

Step 1. Construct a longest processing time (LPT) sequencing of the jobs

$$\left(t_{[1]} \geq t[2] \geq \cdots \geq t_{[n]}\right).$$

Step 2. Schedule the jobs in order LPT, each time assigning a job to the machine with the least amount of processing time already assigned.

The makespan obtained by this heuristic procedure will be denoted by $T_{\max}$.

For solving the bicriteria parallel–machine scheduling problem will be this heuristic procedure more incentive than the solving of the integer programming problem.

We denote the set of feasible schedules by S and consider the parallel–machine scheduling problem with two criteria:

z_1 to minimize the makespan T,

z_2 to minimize the number of matrix exchanges E.

The schedule s^0 is nondominated if there exists no $s \in S$ such that

$$z_1(s) \leq z_1(s^0) \quad \text{and} \quad z_2(s) \leq z_2(s^0),$$

where at least one relation holds with strict inequality.

Because of using a heuristic procedure we find approximately nondominated schedules.

4. HEURISTIC ALGORITHM FOR INJECT PRESS OPTIMIZATION

In the inject press optimization problem we denote:

n the number of sizes in a production order,

t_i the number of pairs of shoes of size i,

N the whole production order for one decade $\left(N = \sum_{i=1}^{j} t_i\right)$,

m the number of places for matrices on an inject press,

r_j the number of pairs currently assigned on the place j $(j = 1, 2, \ldots, m)$,

Δ a variable toleration from the minimum makespan $T_{\min}$.

The production order must be produced in a whole assortment of sizes. For frequently appearing sizes at least two matrices are available. The number of matrices used for production can be greater than number of sizes in a production order. The minimum makespan $T_{\min}$ is now determined as follows

$$T_{\min} = 1/m \sum_{i=1}^{n} t_i. \tag{2}$$

(The second term on the right hand side of (1) disappears owing to the division of large scale lots into two parts.)

The problem can be formulated as a parallel–machine scheduling problem with two objectives: to minimize the makespan T and to minimize the number of job preemption P. Obviously, $T \in \langle T_{\min}, T_{\max} \rangle$ and $P \in \langle 0, P_{\max} \rangle$. For an ideal solution it holds $P = 0$ and $T = T_{\min}$. In general, of course, these two conditions can not be attained simultaneously. By the McNaughton's algorithm we obtain $P = P_{\max}$ and $T = T_{\min}$, using the heuristic procedure we obtain $P = 0$ and $T = T_{\max}$. If we choose suitably the toleration Δ from $T_{\min}$, we can reduce the number of job preemption. The toleration Δ is always expressed as whole multiples of ten pairs of shoes. It ranges from 0 to $T_{\max} - T_{\min}$. The algorithm that is described in what follows permits us to compute the number of job preemptions for different tolerations Δ.

The algorithm for finding one decade schedule

Step 1. Compare t_i $(i = 1, 2, \ldots, n)$ with $T_{\min} + \Delta$. If $t_i > T_{\min} + \Delta$, then the lots are produced by two matrices. One matrix produces $T_{\min} + \Delta$ pairs and the second matrix produces $p_i(2) = t_i - T_{\min} - \Delta$ pairs. Otherwise use one matrix. Remark L.

Step 2. Construct LPT sequencing of the numbers of pairs produced on different matrices.

Step 3. Schedule the jobs in the LPT order, each time assigning a job to the machine with the least amount of pairs already assigned. Check if for the current number $r_j \leq T_{\min} + \Delta$ holds. If $r_j > T_{\min} + \Delta$ then divide the assigned t_i into two parts $t_i = t_i(1) + t_i(2)$, $t_i(1)$ completes the number to $T_{\min} + \Delta$. It results in a preemption. Schedule $t_i(1)$ pairs to the place j and $t_i(2)$ pairs assign in LPT sequencing among unscheduled jobs. Continue this process until you come to $t_i(2)$. For this $t_i(2)$ use the second matrix (if available) or assign $t_i(2)$ at the beginning of the decade. In this manner we assign all pairs of shoes to the places on an inject press. Remark P.

The best thing to do is to present the set of approximately nondominated schedules to the decision maker and to let him make his own choice. The trade-offs provide useful information to the decision maker.

We set $\Delta_1 = 0$, $T_1 = T_{\min}$ and compute the first approximately nondominated schedule s_1 by the heuristic procedure. The number of approximately nondominated schedules is equal to $q = L + P + 1$, where the number of the exchanges L is equal to the number of the large scale lots which are necessary to divide into two matrices and the number of the exchanges P is generated by job preemptions.

The other approximately nondominated schedules s_k, $k = 2, \ldots, q$, are computed by the heuristic procedure setting

$$\Delta_k = \min_i \left[p_i^{k-1}(2), t_i^{k-1}(2) \right], \quad T_k = T_{\min} + \Delta_k,$$

where $p_i^{k-1}(2)$ is the number of pairs produced by a second matrix in the previous schedule s_{k-1}, $t_i^{k-1}(2)$ is the number of pairs of the second part generated by job preemptions in the previous schedule s_{k-1}.

Example. We demonstrate the heuristic procedure on the real data for a one decade schedule.

Table 1. The order.

Size of shoes	Number of matrices	Number of pairs
250	1	600
255	1	600
260	1	800
265	2	900
270	2	1200
275	3	1070
280	2	970
285	2	570
290	1	280
295	1	150
300	1	40
310	1	20

$N = 7200$, $m = 7$, $T_{\min} = 1030$ (round on ten pairs).
By the heuristic algorithm for finding a one decade schedule we get a schedule s_1:
We set $\Delta_1 = 0$, $T_1 = T_{\min} = 1030$

1. [270] – 1200 we divide for two matrices [270–1] – 1030
   [270–2] – 170
   [275] – 1070 we divide for two matrices [275–1] – 1030
   [275–2] – 40

2. LPT sequencing

Table 2. LPT sequencing.

Matrices	Number of pairs
[270–1]	1030
[275–1]	1030
[280]	970
[265]	900
[260]	800
[250]	600
[255]	600
[285]	570
[290]	280
[270–2]	170
[295]	150
[300]	40
[275–2]	40
[310]	20

3. The job [285]–570 we divide into two parts: [285]–430 (1)
[285]–140 (2).

We assign the job [285]–140 (2) at the beginning of the decade to the place 4.

The job [265]–900 we divide into two parts: [265]–890 (1)
[265]–10 (2).

We assign the job [265]–10 (2) at the beginning of the decade to the place 5.

Table 3. The schedule s_1.

Place j	Schedule	Current number r_j
1	[270–1]—1030	1030
2	[275–1]—1030	1030
3	[280]–970, [300]–40, [310]–20	970, 1010, 1030
4	[285]–140 (2), [265]–890 (1)	140, 1030
5	[265]–10 (2), [260]–800, [270–2]—170, [275–2]—40	800, 970, 1010, 1020
6	[255]–600, [290]–280, [295]–150	600, 880, 1030
7	[255]–600, [285]–430 (1)	600, 1030

By the heuristic algorithm for finding one decade schedule we get a schedule s_2:

$$\begin{aligned} \Delta_2 &= \min_i[p_i(2),\, t_i(2)] = \min(170,\, 40,\, 140,\, 10) = 10 \\ T_2 &= T_{\min} + \Delta_2 = 1030 + 10 = 1040. \end{aligned}$$

We know that $A_2 = 5,\ L_2 = 2,\ P_2 = 1,\ E_2 = 8$.

For variable tolerations Δ we get other approximately nondominated schedules:

Table 4. The set of approximately nondominated schedules.

Schedule	Δ_k	T_k	A_k	L_k	P_k	E_k
s_1	0	1030	5	2	2	9
s_2	10	1040	5	2	1	8
s_3	40	1070	5	1	1	7
s_4	140	1170	5	1	0	6
s_5	170	1200	5	0	0	5

5. CONCLUSIONS

The inject press optimization problem was formulated in the shoe industry but these types of problems are very often found in other industries too (for example the machine industry, the chemical industry or the glass industry). For this type of NP–hard problems we propose an easy heuristic algorithm.

This approach can be modified in the case of the transition from one decade to another decade. By this transition we must consider the occupation of places by matrices from the previous decade.

Real life scheduling problems require us to consider a number of criteria for correct decision making in general. Some results are known for bicriteria or multicriteria single–machine scheduling problems. But only a few papers have considered multiple machines in their models. The lack of results in this case has been due the complex nature of scheduling problems. Traditional optimization techniques have not been very successful in solving real life scheduling problems and hence new techniques need to be developed. It is hoped that using problem specific heuristics in conjunction with generic techniques it will be possible to solve more complex multicriteria scheduling problems.

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