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STRONG DECOUPLING OF DESCRIPTOR SYSTEMS VIA PROPORTIONAL STATE FEEDBACK[1]

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The problem of strong input-output decoupling by proportional state feedback is considered for linear descriptor systems. The resulting system is required to be regular, with a diagonal transfer function matrix and an impulse-free response.

The problem is solved in two steps. First, a generalized structure algorithm is used to regularize the system. Then, another algorithm is proposed which produces a sequence of integers. These integers are invariant under restricted system equivalence and regular proportional state feedback. The second algorithm provides a condition for existence as well as a procedure for construction of a decoupling feedback law.

1. INTRODUCTION

We consider a linear, time-invariant descriptor system of the form

$$\begin{aligned} \dot{x}^1 &= A^{11}x^1 + A^{12}x^2 + B^1 u \\ 0 &= A^{21}x^1 + A^{22}x^2 + B^2 u \\ y &= C^1 x^1 + C^2 x^2 + Du \end{aligned} \tag{1}$$

where $A^{i1} \in R^{n_i \times n_1}$, $A^{i2} \in R^{n_i \times n_2}$, $B^i \in R^{n_i \times m}$, $C^i \in R^{m \times n_i}$ for $i = 1, 2$ and $D \in R^{m \times m}$.

The system (1) is said to be regular if the matrix

$$\begin{bmatrix} sI - A_{11} & -A_{12} \\ -A_{21} & -A_{22} \end{bmatrix}$$

is nonsingular. A regular system has a unique solution $x^1(t)$, $x^2(t)$ for every input u. An initial condition $x^1(0^-)$, $x^2(0^-)$ is said to be consistent if it satisfies the system equation (1).

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The problem in question is to find, if possible, a regular proportional state feedback of the form

$$u = F^1x^1 + F^2x^2 + Gv \tag{2}$$

with G being nonsingular, such that the corresponding closed-loop system

$$\begin{aligned} \dot{x}^1 &= (A^{11} + B^1F^1)x^1 + (A^{12} + B^1F^2)x^2 + B^1Gv \\ 0 &= (A^{21} + B^2F^1)x^1 + (A^{22} + B^2F^2)x^2 + B^2Gv \\ y &= (C^1 + DF^1)x^1 + (C^2 + DF^2)x^2 + DGv \end{aligned} \tag{3}$$

has the following properties

1. it is regular;
2. its solution x^1 is differentiable and x^2 is piecewise continuous for any piecewise continuous input v and any consistent initial conditions;
3. it has noninteracting property, that is, the transfer function matrix of the closed-loop system (3) is diagonal and nonsingular.

Such a problem is referred to as the strong input-output decoupling problem. It is worth noting that the definition above is somewhat different from that proposed by Dai [6] in that only consistent initial conditions are considered.

Descriptor systems (also referred to as the differential-algebraic equation, singular, implicit or semi-state systems) constitute an important class of systems of both theoretical interest and practical significance. Such systems arise naturally, among others, in robotic systems [14], chemical engineering [9], mechanical systems [22], and electrical circuits [15]. For a comprehensive introduction, see books [7] and [5], or survey papers [4] and [11].

For linear descriptor systems, input-output decoupling problems have been addressed by several authors (Dai [6], Paraskevopoulos and Koumboulis [16, 17, 18], Ailon [1], and Shayman and Zhou [19]). However, the results given in these papers are all obtained under the assumption of regularity by using transfer function methods. In fact, descriptor systems which are not regular can also be decoupled by proportional state feedback. This paper addresses the decoupling problem without the assumption of regularity; one assumes mere regularizability by proportional state feedback.

In contrast to standard state-space systems, continuous inputs to a descriptor system can give rise to discontinuities or impulsive modes in the state trajectories. Therefore it is of practical importance to design a feedback such that the corresponding closed-loop system is free of impulsive modes. Dai [6] investigated an input-output decoupling problem with impulse-free response, and called it the strong input-output decoupling problem. However, these results have the drawback that the conditions under which the strong input-output decoupling problem is solvable depend on a matrix which needs to be chosen first. In addition, these results were given under a very restrictive assumption that B^2 has full row rank. In this paper, under the condition of regularizability, necessary and sufficient conditions for the solvability of the strong input-output decoupling problem will be derived, which only depend on the parameter matrices of the original system.

The paper is organized as follows. Section 2 investigates the problem of regularization via proportional state feedback. An algorithm is proposed, which is based on the structure algorithm given in [21]. This algorithm yields a necessary and sufficient condition for the solvability of the regularization problem. In Section 3, the strong input-output decoupling problem is addressed. Another algorithm is presented, which produces a sequence of integers. It is proved that these integers are invariant under the restricted system equivalence and regular proportional state feedback. A necessary and sufficient condition is derived, under which the strong input-output decoupling problem is solvable. Technical proofs are relegated to the Appendix. An alternative approach, which is based on the standard Falb–Wolovich test [8], is discussed in the concluding section.

2. REGULARIZATION PROBLEM

The problem of finding a feedback (2) which makes the closed-loop system (3) regular is that of regularization. This problem has been investigated considerably, see [2, 3]. We address this problem by using an alternative method.

First we present an algorithm which is based on the structure algorithm of Silverman [20, 21].

Algorithm 1. (Regularization Algorithm)
Step 0. Let $q_0 = \operatorname{rank}[\,A^{22} \quad B^2\,]$ and let $[\,\bar{A}_0^{22} \quad \bar{B}_0^2\,]$ be the submatrix formed from the first q_0 independent rows of $[\,A^{22} \quad B^2\,]$. Then, there exists an $n_2 \times n_2$ nonsingular matrix S_0 such that

$$S_0\,[\,A^{22} \quad B^2\,] = \begin{bmatrix} \bar{A}_0^{22} & \bar{B}_0^2 \\ 0 & 0 \end{bmatrix}$$

For convenience, partition $S_0 A^{21}$ conformably with $S_0\,[\,A^{22} \quad B^2\,]$ as

$$S_0 A^{21} = \begin{bmatrix} \bar{A}_0^{21} \\ \tilde{A}_0^{21} \end{bmatrix}$$

where $\bar{A}_0^{21}$ has q_0 rows. If $\operatorname{rank} \tilde{A}_0^{21} = 0$, then terminate the algorithm. If $\operatorname{rank} \tilde{A}_0^{21} > 0$, then go on to next step.

Step $k+1$. Assume that $\bar{A}_i^{21}$, $\tilde{A}_i^{21}$, $\bar{A}_i^{22}$, and $\bar{B}_i^2$, $i = 1, \dots, k$, have been defined through Steps 1 to k. Calculate the matrices

$$\begin{bmatrix} \bar{A}_k^{21} \\ \tilde{A}_k^{21} A^{11} \end{bmatrix} \quad \begin{bmatrix} \bar{A}_k^{22} \\ \tilde{A}_k^{21} A^{12} \end{bmatrix} \quad \begin{bmatrix} \bar{B}_k^{2} \\ \tilde{A}_k^{21} B^{1} \end{bmatrix}.$$

Let $q_{k+1} = \operatorname{rank} \begin{bmatrix} \bar{A}_k^{22} & \bar{B}_k^2 \\ \tilde{A}_k^{21} A^{12} & \tilde{A}_k^{21} B^1 \end{bmatrix}$. If $[\,\bar{A}_{k+1}^{22} \quad \bar{B}_{k+1}^2\,]$ is the submatrix formed from the first q_{k+1} independent rows of $\begin{bmatrix} \bar{A}_k^{22} & \bar{B}_k^2 \\ \tilde{A}_k^{21} A^{12} & \tilde{A}_k^{21} B^1 \end{bmatrix}$, then there exists an $n_2 \times n_2$ nonsingular matrix S_{k+1} such that

$$S_{k+1} \begin{bmatrix} \bar{A}_k^{22} & \bar{B}_k^2 \\ \tilde{A}_k^{21} A^{12} & \tilde{A}_k^{21} B^1 \end{bmatrix} = \begin{bmatrix} \bar{A}_{k+1}^{22} & \bar{B}_{k+1}^2 \\ 0 & 0 \end{bmatrix}$$

Similarly denote

$$S_{k+1}\begin{bmatrix}\tilde{A}_k^{21}\\ \tilde{A}_k^{21}A^{11}\end{bmatrix}=\begin{bmatrix}\tilde{A}_{k+1}^{21}\\ \bar{A}_{k+1}^{21}\end{bmatrix}$$

where $\tilde{A}_{k+1}^{21}$ has q_{k+1} rows. If

$$\operatorname{rank}\begin{bmatrix}\tilde{A}_0^{21}\\ \vdots\\ \tilde{A}_k^{21}\\ \tilde{A}_{k+1}^{21}\end{bmatrix}=\operatorname{rank}\begin{bmatrix}\tilde{A}_0^{21}\\ \vdots\\ \tilde{A}_k^{21}\end{bmatrix}$$

then terminate the algorithm. Otherwise go on to next step.

Remark 1. A similar method was used in [10] for solving the problem of dynamic feedback regularization.

It follows from [21] that Algorithm 1 terminates after a finite number of steps bounded by $n_1 + 1$. The following properties of the algorithm are useful in this paper.

Lemma 1.

1. The integers q_i, $i = 0, \ldots,$ and the matrices $\tilde{A}_i^{21}$, $i = 0, \ldots,$ are invariant under the feedback of the type (2).
2. Let λ be the first integer such that $q_\lambda = q_{n_1}$. If $q_\lambda = n_2$, then the rows of L_λ are linearly independent, where

$$L_\lambda=\begin{bmatrix}\tilde{A}_0^{21}\\ \tilde{A}_1^{21}\\ \vdots\\ \tilde{A}_{\lambda-1}^{21}\end{bmatrix}.$$

For the proof of Lemma 1, see the Appendix.

The following theorem gives a necessary and sufficient condition for solvability of the regularization problem.

Theorem 1. The system (1) is regularizable if and only if $q_\lambda = n_2$.

Proof. *Sufficiency*: If $q_\lambda = n_2$, then $[\bar{A}_\lambda^{22} \;\; \bar{B}_\lambda^2]$ has full row rank, which means that there exists a matrix F^2 such that $\bar{A}_\lambda^{22} + \bar{B}_\lambda^2 F^2$ is nonsingular. The application of feedback $u = F^2x^2 + v$ to system (1) produces the following closed-loop system

$$\begin{aligned}\dot{x}^1 &= A^{11}x^1 + (A^{12} + B^1F^2)x^2 + B^1v\\ 0 &= A^{21}x^1 + (A^{22} + B^2F^2)x^2 + B^2v.\end{aligned} \tag{4}$$

It follows from Lemma 1 and [21] that the standard state-space system

$$\begin{aligned} \dot{x}^1 &= A^{11}x^1 + (A^{12} + B^1F^2)x^2 \\ y &= A^{21}x^1 + (A^{22} + B^2F^2)x^2 \end{aligned}$$

is invertible, which implies that its transfer function $A^{21}(sI - A^{11})^{-1}(A^{12} + B^1F^2) + (A^{22} + B^2F^2)$ is nonsigular. Hence the matrix $\begin{bmatrix} sI - A^{11} & -(A^{12} + B^1F^2) \\ -A^{21} & -(A^{22} + B^2F^2) \end{bmatrix}$ is nonsingular. Thus system (4) is regular.

Necessity: Suppose that system (1) is regularizable. Then there exists a feedback (2) such that the corresponding closed-loop system (3) is regular, that is, the system

$$\begin{aligned} \dot{x}^1 &= (A^{11} + B^1F^1)x^1 + (A^{12} + B^1F^2)x^2 \\ y &= (A^{21} + B^2F^1)x^1 + (A^{22} + B^2F^2)x^2 \end{aligned}$$

with input x_2 and output y is invertible. So, performing Algorithm 1 for (3) gives $\operatorname{rank}(\bar{A}^{22}_\lambda + \bar{B}^2_\lambda F^2) = n_2$, which implies that $\operatorname{rank}[\bar{A}^{22}_\lambda \ \ \bar{B}^2_\lambda] = n_2$, i.e. $q_\lambda = n_2$. □

Throughout the paper, we assume that system (1) is regularizable. Then, for the closed-loop system to be free of impulsive modes, x^1 must be in the null space of L_λ. In addition, it is easily seen that for any $x^1 \in \operatorname{Ker} L_\lambda$ the regularizable system can be equivalently described as follows:

$$\begin{aligned} \dot{x}^1 &= A^{11}x^1 + A^{12}x^2 + B^1u \\ 0 &= \bar{A}^{21}_\lambda x^1 + \bar{A}^{22}_\lambda x^2 + \bar{B}^2_\lambda u \end{aligned}$$

where $[\bar{A}^{22}_\lambda \ \ \bar{B}^2_\lambda]$ has full row rank.

3. DECOUPLING PROBLEM

It follows from Section 2 that for any consistent initial condition a regularizable system can be equivalently described as (1) with $[A^{22} \ \ B^2]$ having full row rank. Therefore, it is convenient to make the following assumption.

Assumption 1. The matrix $[A^{22} \ \ B^2]$ has full row rank.

Now let us introduce another algorithm which will play an important role in solving the strong input-output decoupling problem.

Algorithm 2. (Decoupling Algorithm)
Step 1. If

$$\operatorname{rank}\begin{bmatrix} A^{22} & B^2 \\ C_i^2 & D_i \end{bmatrix} = n_2$$

then there exists a unique vector E_i^1 of dimension n_2 such that

$$[C_i^2 \quad D_i] = E_i^1 [A^{22} \quad B^2]$$

where C_i^1 and D_i are the i–th rows of C^1 and D, respectively. Then let $T_i^1 = C_i^1 - E_i^1 A^{21}$. Otherwise, set $r_i = 0$ and terminate the algorithm.

Step k. Assume that we have defined a sequence of $T_i^1, \ldots, T_i^{k-1}$. If

$$\operatorname{rank}\begin{bmatrix} A^{22} & B^2 \\ T_i^{k-1}A^{12} & T_i^{k-1}B^1 \end{bmatrix} = n_2$$

then there exists a unique vector E_i^k of dimension n_2 such that

$$[T_i^{k-1}A^{12} \quad T_i^{k-1}B^1] = E_i^k [A^{22} \quad B^2].$$

Then let $T_i^k = T_i^{k-1}A^{11} - E_i^k A^{21}$. Otherwise, set $r_i = k - 1$ and terminate the algorithm.

Performing Algorithm 2 for $i = 1, \ldots, m$ produces m integers, say $r_1, \ldots, r_m$. Now let us introduce the following matrices

$$\hat{C}^1 = \begin{bmatrix} \hat{C}_1^1 \\ \vdots \\ \hat{C}_m^1 \end{bmatrix} \quad \hat{C}^2 = \begin{bmatrix} \hat{C}_1^2 \\ \vdots \\ \hat{C}_m^2 \end{bmatrix} \quad \hat{D} = \begin{bmatrix} \hat{D}_1 \\ \vdots \\ \hat{D}_m \end{bmatrix}$$

with

$$\hat{C}_i^1 = \begin{cases} C_i^1 & r_i = 0 \\ T_i^{r_i}A^{11} & r_i \neq 0 \end{cases} \quad \hat{C}_i^2 = \begin{cases} C_i^2 & r_i = 0 \\ T_i^{r_i}A^{12} & r_i \neq 0 \end{cases} \quad \hat{D}_i = \begin{cases} D_i & r_i = 0 \\ T_i^{r_i}B^1 & r_i \neq 0. \end{cases}$$

Remark 2.

1. If $n_2 = 0$, i.e. there is no algebraic equation in the system, then E_i^k must be considered as a vector with no entries and T_i^k is equal to $T_i^{k-1}A^{11}$. In this case, the integers r_i, $i = 1, \ldots, m$, are the same as those given by Falb and Wolovich [8]. On the other hand, the algorithm runs even in the case $n_1 = 0$, i.e. there are no differential equations in the system.
2. If Algorithm 2 does not terminate at step n_1, then it never stops, i.e. $r_i = \infty$. In this case, it is not necessary to continue the algorithm further, so let $r_i = n_1$.

The following two properties of Algorithm 2 are useful in the sequel.

Lemma 2. The integers $r_1, \ldots, r_m$ are invariant under feedback (2) as well as the restricted equivalent transformation

$$Q = \begin{bmatrix} Q_1^{-1} & P_2 \\ 0 & P_3 \end{bmatrix} \quad P = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix} \tag{5}$$

whose definition can be found in [6].

Lemma 3. If the matrix $\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}$ is nonsingular, then the vectors

$$T_1^1, \ldots, T_1^{r_1}, \ldots, T_m^1, \ldots, T_m^{r_m}$$

are linearly independent.

The proofs of Lemma 2 and Lemma 3 can be found in the Appendix.

According to Lemma 3, if $\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}$ is nonsingular, then $r_1 + \cdots r_m \leq n_1$. As a consequence, it is always possible to chooce $n_1 - r$ linearly independent vectors $T_1, \ldots, T_r$ such that $T_1, \ldots, T_{n_1-r}, T_1^1, \ldots, T_1^{r_1}, \ldots, T_m^1, \ldots, T_m^{r_m}$ are linearly independent, where $r = r_1 + \cdots + r_m$. Therefore one can choose the following coordinate transformation

$$\begin{aligned} \eta &= (\eta_1, \ldots, \eta_{n_1-r})' \\ \xi &= (\xi_1^1, \ldots, \xi_1^{r_1}, \ldots, \xi_m^1, \ldots, \xi_m^{r_m})' \end{aligned}$$

with $\xi_j^i = T_j^i x^1$, $i = 1, \ldots, r_j$, $j = 1, \ldots, m$, and $\eta_i = T_i x$, $i = 1, \ldots, n_1 - r$. It is easily seen that the description of system (1) in the new coordinates takes the form of

$$\begin{aligned} \dot{\eta} &= \tilde{A}^1 \eta + \tilde{A}^2 \xi + \tilde{A}^3 x^2 + \tilde{B}^1 u \\ 0 &= A^{21} x^1 + A^{22} x^2 + B^2 u \\ \dot{\xi}_i^1 &= \xi_i^2 \\ &\vdots \\ \dot{\xi}_i^{r_i - 1} &= \xi_i^{r_i} \\ \dot{\xi}_i^{r_i} &= \hat{C}_i^1 x^1 + \hat{C}_i^2 x^2 + \hat{D}_i u \\ y_i &= \xi_i^1, \qquad i = 1, \ldots, m \end{aligned} \tag{6}$$

where

$$\begin{aligned} x^1 &= T^{-1} \begin{bmatrix} \eta \\ \xi \end{bmatrix} \\ \begin{bmatrix} \tilde{A}^1 & \tilde{A}^2 \end{bmatrix} &= \begin{bmatrix} T_1 \\ \vdots \\ T_{n_1-r} \end{bmatrix} A^{11} T^{-1} \\ \begin{bmatrix} \tilde{A}^3 & \tilde{B}^1 \end{bmatrix} &= \begin{bmatrix} T_1 \\ \vdots \\ T_{n_1-r} \end{bmatrix} \begin{bmatrix} A^{12} & B^1 \end{bmatrix} \end{aligned}$$

with $T = [T_1, \ldots, T_{n_1-r}, T_1^1, \ldots, T_1^{r_1}, \ldots, T_m^1, \ldots, T_m^{r_m}]'$.

Since the relation

$$\operatorname{rank} \begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}$$

$$
\begin{aligned}
&= \operatorname{rank}\left\{\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}\begin{bmatrix} I & 0 \\ F^2 & I \end{bmatrix}\begin{bmatrix} I & -(A^{22}+B^2F^2)^{-1}B^2 \\ 0 & I \end{bmatrix}\right\} \\
&= \operatorname{rank}\begin{bmatrix} A^{22}+B^2F^2 & 0 \\ \hat{C}^2+\hat{D}F^2 & \hat{D}-(\hat{C}^2+\hat{D}F^2)(A^{22}+B^2F^2)^{-1}B^2 \end{bmatrix}
\end{aligned}
$$

holds for any F^2 such that $A^{22}+B^2F^2$ is nonsingular, the application of feedback (2) with such F^2s, and

$$
\begin{aligned}
F^1 &= -G[\hat{C}^1 - (\hat{C}^2+\hat{D}F^2)(A^{22}+B^2F^2)^{-1}A^{21}] \\
G &= [\hat{D} - (\hat{C}^2+\hat{D}F^2)(A^{22}+B^2F^2)^{-1}B^2]^{-1}
\end{aligned} \tag{7}
$$

to system (6) gives

$$
\begin{aligned}
\dot{\eta} &= \bar{A}^1\eta + \bar{A}^2\xi + \bar{A}^3x^2 + \bar{B}^1v \\
0 &= (A^{21}+B^2F^1)x^1 + (A^{22}+B^2F^2)x^2 + B^2Gv \\
\dot{\xi}_i^1 &= \xi_i^2 \\
&\vdots \\
\dot{\xi}_i^{r_i-1} &= \xi_i^{r_i} \\
\dot{\xi}_i^{r_i} &= v_i \\
y_i &= \xi_i^1, \qquad i=1,\dots,m.
\end{aligned} \tag{8}
$$

The structure of these equations shows that the noninteraction requirements have been achieved. As a matter of fact, the input v_1 controls only the output y_1 through a chain of r_1 integrators, the input v_2 controls only the output y_2 through a chain of r_2 integrators, etc. In addition, the nonsingularity of the matrix $A^{22}+B^2F^2$ guarantees that system (8) has Properties 1 and 2 given in Introduction. Therefore the nonsigularity of the matrix $\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}$ is a sufficient condition for the solvability of the strong input-output decoupling problem. In fact, it is also necessary.

Theorem 2. The Strong Input–Output Decoupling Problem is solvable if and only if $\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}$ is nonsingular.

Proof. Sufficiency has already been verified. Now let us prove the necessity. To this end, we assume that the closed-loop system (3) has been rendered noninteractive, i.e. possesses Properties 1-3 stated in Introduction. It follows from Lemma 1 that a necessary condition under which system (3) has Property 1 is that the matrix $A^{22}+B^2F^2$ is nonsingular. As a result, x^2 can be uniquely determined from the second equation of (3) as

$$x^2 = -(A^{22}+B^2F^2)^{-1}(A^{21}+B^2F^1)x^1 - (A^{22}+B^2F^2)^{-1}B^2Gv.$$

Substituting this into the first and third equations of (3) yields

$$
\begin{aligned}
\dot{x}^1 &= (\tilde{A}+\tilde{B}F^1)x^1 + \tilde{B}Gv \\
y &= (\tilde{C}+\tilde{D}F^1)x^1 + \tilde{D}Gv
\end{aligned} \tag{9}
$$

with

$$\begin{aligned}
\tilde{A} &= A^{11} - (A^{12} + B^1F^2)(A^{22} + B^2F^2)^{-1}A^{21} \\
\tilde{B} &= B^1 - (A^{12} + B^1F^2)(A^{22} + B^2F^2)^{-1}B^2 \\
\tilde{C} &= C^1 - (C^2 + DF^2)(A^{22} + B^2F^2)^{-1}A^{21} \\
\tilde{D} &= D - (C^2 + DF^2)(A^{22} + B^2F^2)^{-1}B^2.
\end{aligned}$$

According to the noninteractive property of system (3), it follows that system (9) has the same property, that is, the system

$$\begin{aligned}
\dot{x}^1 &= \tilde{A}x^1 + \tilde{B}w \\
y &= \tilde{C}x^1 + \tilde{D}w
\end{aligned} \tag{10}$$

can be decoupled by the feedback $w = F^1x^1 + Gv$.

We need the following lemmas whose proof can be found in the Appendix.

Lemma 4. For any matrix F^2 which renders $A^{22} + B^2F^2$ invertible, $d_i = r_i - 1$ for $i = 1, \ldots, m$ where

$$d_i = \begin{cases} 0 & \tilde{D}_i \neq 0 \\ \min\{j : \tilde{C}_i\tilde{A}^j\tilde{B} \neq 0, j = 0, 1, \ldots, n_1 - 1\} & \tilde{D}_i = 0 \\ n_1 - 1 & \tilde{C}_i\tilde{A}^j\tilde{B} = 0,\ j = 0, 1, \ldots, n_1 \end{cases}$$

where $\tilde{C}_i$ and $\tilde{D}_i$ being the i–th row of $\tilde{C}$ and $\tilde{D}$, respectively.

Lemma 5. For any matrix F^2 which renders $A^{22} + B^2F^2$ invertible, B^* is nonsingular if and only if the matrix $\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix}$ is nonsingular where

$$B^* = \begin{bmatrix} \tilde{C}_1\tilde{A}^{d_1}\tilde{B} \\ \vdots \\ \tilde{C}_m\tilde{A}^{d_m}\tilde{B} \end{bmatrix}.$$

According to the results on the input-output decoupling of standard state-space systems [20], it follows from Lemma 5 that the condition of the theorem is necessary.

4. CONCLUSIONS

The strong input-output decoupling problem has been considered for linear descriptor systems under the assumption of regularizability. Two algorithms have been proposed, one answering the existence of a regularizing feedback while the other one the existence of a decoupling feedback. Both algorithms are constructive.

The method used in this paper is different from those found in the literature. The results of the paper are given under the assumption of regularizability, so they are more general than the existing ones. Compared with the results by Dai [6], the condition for the solvability of the strong input-output decoupling problem is less restrictive and easier to check.

For descriptor systems (1) with the matrix $[A^{22}\ \ B^2]$ having full row rank, an alternative solution of the strong input–output decoupling problem is available. Let F^2 be a matrix such that $A^{22}+B^2F^2$ is nonsingular. The application of feedback $u=F^2x^2+v$ to system (1) produces a descriptor system in which

$$x^2=-(A^{22}+B^2F^2)^{-1}(A^{21}x^1+B^2v).$$

On eliminating x^2, one obtains a system in standard state-space form. Thus the effect of F^2 has been a shift of all infinite eigenvalues of (1) to finite positions. For the resulting system the standard Falb–Wolovich test [8] is applicable, thus providing an alternative design. The result, however, may depend on the choice of matrix F^2 and it is difficult to interpret in terms of the original system matrices. The method proposed in this paper avoids these difficulties.

Perhaps the main advantage of the method of this paper is that it can be generalized to linear time-varying and nonlinear descriptor systems. An indication of how such a generalization to the time-varying case can be obtained is given in [20]; the generalization to the nonlinear case can be found in [12, 13].

APPENDIX

Proof of Lemma 1

For convenience, let "$\longrightarrow$" denote the application of feedback (2) to the system. For $i=0$,

$$\begin{bmatrix} A^{22} & B^2 \end{bmatrix} \longrightarrow \begin{bmatrix} A^{22}+B^2F^2 & B^2G \end{bmatrix} = \begin{bmatrix} A^{22} & B^2 \end{bmatrix}\begin{bmatrix} I & 0 \\ F^2 & G \end{bmatrix}$$

which implies that $q_0 \longrightarrow q_0$, as a result, $S_0 \longrightarrow S_0$. From the relation

$$\begin{bmatrix} \bar{A}^{21} \\ \tilde{A}_0^{21} \end{bmatrix} = S_0A^{21} \longrightarrow S_0[A^{21}+B^2F^1] = \begin{bmatrix} \bar{A}_0^{21}+\bar{B}_0^2F^1 \\ \tilde{A} \end{bmatrix}$$

it follows that $\tilde{A}_0^{21} \longrightarrow \tilde{A}_0^{21}$. Now assuming that the result holds for $i=k$, then it is easily seen that the following relation is satisfied.

$$\begin{aligned} &\begin{bmatrix} \bar{A}_k^{22} & \bar{B}_k^2 \\ \tilde{A}_k^{21}A^{12} & \tilde{A}_k^{21}B^1 \end{bmatrix} \\ \longrightarrow &\begin{bmatrix} \bar{A}_k^{22}+\bar{B}_k^2F^2 & \bar{B}_k^2G \\ \tilde{A}_k^{21}A^{12}+\tilde{A}_k^{21}B^1F^2 & \tilde{A}_k^{21}B^1G \end{bmatrix} = \begin{bmatrix} \bar{A}_k^{22} & \bar{B}_k^2 \\ \tilde{A}_k^{21}A^{12} & \tilde{A}_k^{21}B^1 \end{bmatrix}\begin{bmatrix} I & 0 \\ F^2 & G \end{bmatrix}. \end{aligned}$$

This implies that $S_{k+1} \longrightarrow S_{k+1}$ and $q_{k+1} \longrightarrow q_{k+1}$. In addition,

$$\begin{aligned} \begin{bmatrix} \bar{A}_{k+1}^{21} \\ \tilde{A}_{k+1}^{21} \end{bmatrix} &= S_{k+1}\begin{bmatrix} \bar{A}_k^{21} \\ \tilde{A}_k^{21}A^{11} \end{bmatrix} \\ &\longrightarrow S_{k+1}\begin{bmatrix} \bar{A}_k^{21} \\ \tilde{A}_k^{21}A^{11} \end{bmatrix} + S_{k+1}\begin{bmatrix} \bar{B}_k^2 \\ \tilde{A}_k^{21}B^1 \end{bmatrix}F^1 = \begin{bmatrix} \bar{A}_{k+1}^{21}+\bar{B}_{k+1}^2F^1 \\ \tilde{A}_{k+1}^{21} \end{bmatrix} \end{aligned}$$

which means $\tilde{A}^{21}_{k+1} \to \bar{A}^{21}_{k+1}$. By induction, it is no hard to prove the first part of the lemma. The second part is proved in [21]. □

Proof of Lemma 2

In order to prove the result, let us denote the system formed after applying feedback (2) and restricted equivalent transformation (5) as follows:

$$\begin{aligned} \dot{\bar{x}}^1 &= \bar{A}^{11}\bar{x}^1 + \bar{A}^{12}\bar{x}^2 + \bar{B}^1 v \\ 0 &= \bar{A}^{21}\bar{x}^1 + \bar{A}^{22}\bar{x}^2 + \bar{B}^2 v \\ y &= \bar{C}^1\bar{x}^1 + \bar{C}^2\bar{x}^2 + \bar{D}v \end{aligned} \tag{11}$$

where

$$\begin{aligned} \bar{A}^{11} &= Q_1^{-1}(A^{11} + B^1F^1)Q_1 + P_2(A^{21} + B^2F^1)Q_1 \\ &\quad + Q_1^{-1}(A^{12} + B^1F^2)Q_2 + P_2(A^{22} + B^2F^2)Q_2 \\ \bar{A}^{12} &= Q_1^{-1}(A^{12} + B^1F^2)Q_3 + P_2(A^{22} + B^2F^2)Q_3 \\ \bar{A}^{21} &= P_3(A^{21} + B^2F^1)Q_1 + P_3(A^{22} + B^2F^2)Q_2 \\ \bar{A}^{22} &= P_3(A^{22} + B^2F^2)Q_3 \\ \bar{B}^1 &= (Q_1)^{-1}B^1G + P_2B^2G \\ \bar{B}^2 &= P_3B^2G \\ \bar{C}^1 &= (C^1 + DF^1)Q_1 + (C^2 + DF^2)Q_2 \\ \bar{C}^2 &= (C^2 + DF^2)Q_3 \\ \bar{D} &= DG. \end{aligned} \tag{12}$$

Now it is sufficient to prove that $r_i = \bar{r}_i$, $i = 1, \ldots, m$, where $\bar{r}_i$ is associated with system (11).

First, let us verify the following relations by induction.

$$\begin{aligned} \bar{E}_i^k &= (E_i^k + T_i^{k-1}Q_1P_2)P_3^{-1} \\ \bar{T}_i^k &= T_i^kQ_1 \end{aligned} \tag{13}$$

for $k = 1, \ldots, r_i$ and $i = 1, \ldots, m$, where $T_i^0 = 0$.

It is easily seen that relations (13) hold for $k = 1$. Assume that relations (13) hold for $k = j$. Then according to (12) and $[T_i^jA^{12} \quad T_i^jB^1] = E_i^{j+1}[A^{22} \quad B^2]$, it follows that

$$[\bar{T}_i^j\bar{A}^{12} \quad \bar{T}_i^j\bar{B}^1] = (E_i^{j+1} + T_i^jQ_1P_2)P_3^{-1}[\bar{A}^{22} \quad \bar{B}^2]$$

which implies $\bar{E}_i^{j+1} = (E_i^{j+1} + T_i^jQ_1P_2)P_3^{-1}$. Moreover, it is easily seen that

$$\bar{T}_i^{j+1} = \bar{T}_i^j\bar{A}^{11} - \bar{E}_i^{j+1}\bar{A}^{21} = (T_i^jA^{11} - E_i^{j+1}A^{21})Q_1 = T_i^{j+1}Q_1.$$

This proves (13). Then, from

$$\operatorname{rank}\begin{bmatrix} A^{22} & B^2 \\ T_i^{r_i}A^{12} & T_i^{r_i}B^1 \end{bmatrix} = n_2 + 1$$

it follows that

$$
\begin{aligned}
&\operatorname{rank}\begin{bmatrix} \bar{A}^{22} & \bar{B}^2 \\ \bar{T}_i^{r_i}\bar{A}^{12} & \bar{T}_i^{r_i}\bar{B}^1 \end{bmatrix} \\
&= \operatorname{rank}\begin{bmatrix} P_3(A^{22}+B_2F^2)Q^3 & P_3B^2G \\ T_i^{r_i}(A^{12}+B^1F^2)Q_3+T_i^{r_i}Q_1P_2(A^{22}+B^2F^2)Q_3 & T_i^{r_i}B^1G+T_i^{r_i-1}Q_1P_2B^2G \end{bmatrix} \\
&= \operatorname{rank}\left\{\begin{bmatrix} P_3 & 0 \\ T_i^{r_i}Q_1P_2 & I \end{bmatrix}\begin{bmatrix} A^{22} & B^2 \\ T_i^{r_i}A^{12} & T_i^{r_i}B^1 \end{bmatrix}\begin{bmatrix} Q_3 & 0 \\ F^2Q_3 & G \end{bmatrix}\right\} \\
&= \operatorname{rank}\begin{bmatrix} A^{22} & B^2 \\ T_i^{r_i}A^{12} & T_i^{r_i}B^1 \end{bmatrix} \\
&= n_2+1
\end{aligned}
$$

This completes the proof. □

Proof of Lemma 3

It easily follows from Algorithm 2 that the following relations hold.

$$
\begin{aligned}
[C_i^2 \quad D_i] &= E_i^1\,[A^{22} \quad B^2] \\
[T_i^{k-1}A^{12} \quad T_i^{k-1}B^1] &= E_i^k\,[A^{22} \quad B^2], \quad k=2,\ldots,r_i,\ i=1,\ldots,m
\end{aligned}
\tag{14}
$$

These are equivalent to the following relations

$$
\begin{aligned}
[C_i^2 \quad D_i]\begin{bmatrix} I & 0 \\ F^2 & I \end{bmatrix} &= E_i^1\,[A^{22} \quad B^2]\begin{bmatrix} I & 0 \\ F^2 & I \end{bmatrix} \\
[T_i^{k-1}A^{12} \quad T_i^{k-1}B^1]\begin{bmatrix} I & 0 \\ F^2 & I \end{bmatrix} &= E_i^k\,[A^{22} \quad B^2]\begin{bmatrix} I & 0 \\ F^2 & I \end{bmatrix}, \\
& \qquad k=2,\ldots,r_i,\ i=1,\ldots,m
\end{aligned}
\tag{15}
$$

that is,

$$
\begin{aligned}
[C_i^2+D_iF^2 \quad D_i] &= E_i^1\,[A^{22}+B^2F^2 \quad B^2] \\
[T_i^{k-1}(A^{12}+B^1F^2) \quad T_i^{k-1}B^1] &= E_i^k\,[A^{22}+B^2F^2 \quad B^2], \\
& \qquad k=2,\ldots,r_i,\ i=1,\ldots,m.
\end{aligned}
\tag{16}
$$

Therefore, for any F^2 which makes $A^{22}+B^2F^2$ nonsingular, E_i^k can be uniquely determined as follows:

$$
\begin{aligned}
E_i^1 &= (C_i^2+D_iF^2)(A^{22}+B^2F^2)^{-1} \\
E_i^k &= T_i^{k-1}(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}, \quad k=2,\ldots,r_i,\ i=1,\ldots,m.
\end{aligned}
\tag{17}
$$

Assume that $T_1^1,\ldots,T_1^{r_1},\ldots,T_m^1,\ldots,T_m^{r_m}$ are linearly dependent. Then,

$$0=\sum_{i=1}^{m}\sum_{j=1}^{r_i-1} c_i^jT_i^j+\sum_{i=1}^{m} c_i^{r_i}T_i^{r_i}$$

with c_i^j being constant, $j = 1, \ldots, r_i, i = 1, \ldots, m$. As a result, one can get

$$
\begin{aligned}
0 &= \left\{ \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j T_i^j + \sum_{i=1}^{m} c_i^{r_i} T_i^{r_i} \right\} [A^{12} \quad B^1] \\
0 &= \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j [T_i^j A^{12} \quad T_i^j B^1] + \sum_{i=1}^{m} c_i^{r_i} [T_i^{r_i} A^{12} \quad T_i^{r_i} B^1] \\
&= \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j E_i^{j+1} [A^{22} \quad B^2] + \sum_{i=1}^{m} c_i^{r_i} [T_i^{r_i} A^{12} \quad T_i^{r_i} B^1] .
\end{aligned}
$$

Therefore, from the nonsingularity of the matrix

$$
\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix} = \begin{bmatrix} A^{22} & B^2 \\ T_1^{r_1} A^{12} & T_1^{r_1} B^1 \\ \vdots & \vdots \\ T_m^{r_m} A^{12} & T_m^{r_m} B^1 \end{bmatrix}
$$

it follows that $c_i^{r_i} = 0$ for $i = 1, \ldots, m$. As a result,

$$
0 = \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j T_i^j .
$$

Considering (17), it is easily seen that

$$
\begin{aligned}
0 &= \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j T_i^j \left[A^{11} - (A^{12} + B^1 F^2)(A^{22} + B^2 F^2)^{-1} A^{21}\right] [A^{12} \quad B^1] \\
&= \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j \left[T_i^j A^{11} - E_i^{j+1} A^{21}\right] [A^{12} \quad B^1] \\
&= \sum_{i=1}^{m} \sum_{j=1}^{r_i - 1} c_i^j T_i^{j+1} [A^{12} \quad B^1] \\
&= \sum_{i=1}^{m} \sum_{j=1}^{r_i - 2} c_i^j E_i^{j+2} [A^{22} \quad B^2] + \sum_{i=1}^{m} c_i^{r_i - 1} [T_i^{r_i} A^{12} \quad T_i^{r_i} B^1] .
\end{aligned}
$$

It follows that $c_i^{r_i - 1} = 0$ for $i = 1, \ldots, m$.

Similarly, it is easily proved that $c_i^j = 0$ for $j = 1, \ldots, r_i - 2, \ i = 1, \ldots, m$. This completes the proof. □

Proof of Lemma 4

Substituting (17) into (14) yields

$$\begin{aligned} C_i^2-(C_i^2+D_iF^2)(A^{22}+B^2F^2)^{-1}A^{22} &= 0\\ D_i-(C_i^2+D_iF^2)(A^{22}+B^2F^2)^{-1}B^2 &= 0\\ T_i^{k-1}[A^{11}-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}A^{22}] &= 0, \quad k=2,\dots,r_i\\ T_i^{k-1}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2] &= 0, \quad k=2,\dots,r_i,\ i=1,\dots,m. \end{aligned} \tag{18}$$

By construction of T_i^k, it is easily seen that

$$\begin{aligned} T_i^1 &= C_i^1-E_i^1A^{21}=C_i^1-(C_i^2+D_iF^2)(A^{22}+B^2F^2)^{-1}A^{21}\\ T_i^k &= T_i^{k-1}-E_i^kA^{21}=T_i^{k-1}[A^{11}-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}A^{21}],\\ &\qquad k=2,\dots,r_i,\ i=1,\dots,m. \end{aligned} \tag{19}$$

In addition, for any F^2 which renders $A^{22}+B^2F^2$ nonsingular, we have

$$\begin{aligned} &\operatorname{rank}\begin{bmatrix} A^{22} & B^2\\ T_i^{r_i}A^{12} & T_i^{r_i}B^1\end{bmatrix}\\ =\ &\operatorname{rank}\left\{\begin{bmatrix} A^{22} & B^2\\ T_i^{r_i}A^{12} & T_i^{r_i}B^1\end{bmatrix}\begin{bmatrix} I & 0\\ F^2 & I\end{bmatrix}\begin{bmatrix} I & -(A^{22}+B^2F^2)^{-1}B^2\\ 0 & I\end{bmatrix}\right\}\\ =\ &\operatorname{rank}\begin{bmatrix} A^{22}+B^2F^2 & 0\\ T_i^{r_i}(A^{12}+B^1F^2) & T_i^{r_i}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2]\end{bmatrix}. \end{aligned}$$

As a result, $\operatorname{rank}\begin{bmatrix} A^{22} & B^2\\ T_i^{r_i}A^{12} & T_i^{r_i}B^1\end{bmatrix}=n_2+1$ if and only if

$$T_i^{r_i}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2]\neq 0. \tag{20}$$

From equations (14) – (20), one can easily deduce the following relations.

$$\begin{aligned} \tilde{D}_i &= D_i-(C_i^2+D_iF^2)(A^{22}+B^2F^2)^{-1}B^2=0\\ \tilde{C}_i &= C_i^1-(C_i^2+D_iF^2)(A^{22}+B^2F^2)^{-1}A^{21}=T_i^1\\ \tilde{C}_i\hat{B} &= T_i^1[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2]=0\\ \tilde{C}_i\hat{A} &= T_i^1[A^{11}-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}A^{21}]=T_i^2\\ \tilde{C}_i\hat{A}\hat{B} &= T_i^2[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2]=0\\ \tilde{C}_i\hat{A}^2 &= T_i^2[A^{11}-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}A^{21}]=T_i^3\\ &\ \ \vdots\\ \tilde{C}_i\hat{A}^{r_i-2}\hat{B} &= T_i^{r_i-1}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2]=0\\ \tilde{C}_i\hat{A}^{r_i-1} &= T_i^{r_i-1}[A^{11}-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}A^{21}]=T_i^{r_i}\\ \tilde{C}_i\hat{A}^{r_i-1}\hat{B} &= T_i^{r_i}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2]\neq 0, \quad i=1,\dots,m \end{aligned} \tag{21}$$

which means that $d_i = r_i - 1$. □

Proof of Lemma 5

According to the definitions of $\hat{C}^2$ and $\hat{D}$, it follows that

$$\begin{aligned}
&\operatorname{rank}\begin{bmatrix} A^{22} & B^2 \\ \hat{C}^2 & \hat{D} \end{bmatrix} \\
= \; &\operatorname{rank}\begin{bmatrix} A^{22} & B^2 \\ T_1^{r_1}A^{12} & T_1^{r_1}B^1 \\ \vdots & \vdots \\ T_m^{r_m}A^{12} & T_m^{r_m}B^1 \end{bmatrix} \\
= \; &\operatorname{rank}\left\{ \begin{bmatrix} A^{22} & B^2 \\ T_1^{r_1}A^{12} & T_1^{r_1}B^1 \\ \vdots & \vdots \\ T_m^{r_m}A^{12} & T_m^{r_m}B^1 \end{bmatrix} \begin{bmatrix} I & 0 \\ F^2 & I \end{bmatrix} \begin{bmatrix} I & -(A^{22}+B^2F^2)^{-1}B^2 \\ 0 & I \end{bmatrix} \right\} \\
= \; &\operatorname{rank}\begin{bmatrix} A^{22}+B^2F^2 & 0 \\ T_1^{r_1}(A^{12}+B^1F^2) & T_1^{r_1}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2] \\ \vdots & \vdots \\ T_m^{r_m}(A^{12}+B^1F^2) & T_m^{r_m}[B^1-(A^{12}+B^1F^2)(A^{22}+B^2F^2)^{-1}B^2] \end{bmatrix} \\
= \; &\operatorname{rank}\begin{bmatrix} A^{22}+B^2F^2 & 0 \\ T_1^{r_1}(A^{12}+B^1F^2) & \tilde{C}_1\tilde{A}^{r_1-1}\tilde{B} \\ \vdots & \vdots \\ T_m^{r_m}(A^{12}+B^1F^2) & \tilde{C}_m\tilde{A}^{r_m-1}\tilde{B} \end{bmatrix} \\
= \; &\operatorname{rank}\begin{bmatrix} A^{22}+B^2F^2 & 0 \\ X & B^* \end{bmatrix}
\end{aligned}$$

which proves the claim. □

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