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CYCLIC CONTROL OF ROBOT ARMS[1]

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The problem of moving a rigid robot arm along a finite sequence of equilibrium points, with the last point coincident with the first one, is investigated. Such a sequence, referred to as a cycle, is to be repeated over and over in time, and a controller is sought which improves system performance by using positioning errors. Differently from learning control, no system initialization is required at the end of trial. After high gain feedback linearization of the robot dynamics, it is shown that linear, robust, finite dimensional algorithms can be set up to accomplish this task for unconstrained robots and robots subject to smooth bilateral constraints for which hybrid force control is of interest. An experiment on a two-link robot arm illustrates algorithm applicability.

1. INTRODUCTION

The problem of operating robots on repetitive or periodic tasks has been largely addressed in the literature. Typically, the task considered consists in tracking a trajectory: in repetitive control [20], the periodic and continuous trajectory to be tracked is defined over the entire time axis; while in learning control [3] the trajectory to be tracked is defined over a finite time interval, at the end of which system re-initialization is allowed and the same task repeated. In the first case, one is faced with a classic control problem, in the sense that asymptotic output tracking is sought as the time goes to infinity. In the second case, one searches convergence as the number of task repetitions (trials) tends to infinity. The learning dynamics are then defined over the countable set of trials and as such are of the discrete type. In both cases, and as long as continuous time systems are considered, one has to deal with a state space which is infinite dimensional. In repetitive control, this is linked with the assumption that the periodic trajectory to be tracked may not be generated by a finite dimensional exosystem, while in learning control to the fact that the space of interest is the set of all output trajectories on the time interval considered [11]. Even if the available solutions to the repetitive [20] and learning control [1, 2, 3, 4, 5, 6, 8, 11] problems are in principle exact, proposed implementations of these algorithms are only approximate owing to the infinite dimension of their state spaces. In some instances, however, instead of trajectory tracking, repositioning is required. Moving rigid robots between equilibrium points is apparently

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a sub-task of trajectory tracking. One may argue that by tracking a trajectory connecting the equilibrium points repositioning is obtained. But this control strategy is indirect and then intrinsically not robust. For instance, it may happen that during a trial the robot reaches one of the desired equilibrium points even if the selected trajectory is not exactly tracked. This would however cause the update of the control for the next trial and the robot could no longer reach that desired equilibrium point during this new trial. In general, output tracking and state steering are quite different control problems which require different algorithms. On this basis, the problem of steering the state of a control system by learning has been investigated and some algorithms proposed [12, 13, 14, 15, 17, 18]. As opposed to algorithms for trajectory tracking, learning algorithms for state steering are finite dimensional. However system re-initialization is still needed at the end of a trial and this prevents the possibility of continuously operating the system on a task defined by a finite sequence of equilibrium points, with the first and the last one coincident. For this reason, in this paper a new type of servo-system for robot arms is introduced to specifically address this control problem, named cyclic control. A linear algorithm is presented, which asymptotically forces the robot to execute a given cycle to be repeated over and over in time. System re-initialization is not required, thus avoiding any time delay associated to this operation. Both the cases of unconstrained robots and robots subject to smooth bilateral constraints, for which hybrid force control is of interest, are considered.

To show the feasibility of the proposed control system the results of an experiment carried on a two link robot arm are reported. Has been required to the robot to move cyclically between three equilibrium points defined by having state derivatives null.

2. CYCLIC CONTROL OF UNCONSTRAINED ROBOTS

Let an open chain robot arm, with n rigid links connected by lower kinematic pairs, be given. If unconstrained, its equations of motion are of the type

$$B(q(t))\,\ddot{q}(t) + c(\dot{q}(t), q(t)) + d(\dot{q}(t), q(t), t) = f(t) \qquad (1)$$
$$q(0) = q^{\circ}, \quad \dot{q}(0) = \dot{q}^{\circ},$$

where $q(t) \in \mathbb{R}^n$ is the vector of joint variables, $B(\cdot)$ is the positive definite inertia matrix, $c(\cdot)$ is the vector of centripetal, Coriolis and gravitational terms, $d(\cdot)$ takes into account unknown disturbances and $f(t)$ is the vector of joint forces delivered by the actuators, one for each joint. All functions are assumed smooth and $d(\cdot)$ is periodic with respect to the time.

Suppose that the robot has to be operated along a finite sequence of r equilibrium points $\{q_1^d, \ldots, q_r^d\}$, to be attained at the end of the consecutive time intervals $\{\delta_1, \ldots, \delta_r\}$. From the last assigned equilibrium point, the robot has to move to the first one and re-start the "cycle". For synchronization purposes, it is also required that at the instants hT, with $h = 0, 1, 2, \ldots$, the robot equilibrium point is the one corresponding to $q = q_r^d$, that at the instants $\delta_1 + hT$ the one corresponding to

$q = q_1^d$ and so on, with T the cycle period given by

$$T = \sum_{i=1}^{r} \delta_i .$$

The period of $d(\cdot)$ is assumed to be equal to T.

Apply the following high gain control

$$f(t) = -\frac{1}{\varepsilon}(\dot q(t) - \eta(t)), \quad \varepsilon > 0$$

where the new control $\eta(t)$ has a continuous derivative. By letting $\varepsilon \longrightarrow 0$, the robot system is singularly perturbed (see e. g. [10]) and splits in a fast system given by

$$\frac{\mathrm{d}p(\tau)}{\mathrm{d}\tau} = -D(q^*(t))\, p(\tau),$$

where τ is the fast time, $p(\tau)$ is the fast transient of the velocity and $D(\cdot)$ is the inverse of the inertia matrix, and in the slow system

$$\dot q(t) = \eta(t), \quad q(0) = q^\circ,$$

whose smooth solution is denoted by $q^*(t)$.

Since $D(\cdot)$ is positive definite, the fast system is exponentially stable and Tikhonov's theorem applies. Tikhonov's theorem states that for small ε the following approximation holds

$$q(t) = q^*(t) + o(\varepsilon), \quad t \in [0, \infty). \tag{2}$$

Moreover, since $\eta(t)$ is differentiable and the robot dynamics are smooth, for a sufficiently large $t' > 0$ one has that

$$\dot q(t) = \dot q^*(t) + o(\varepsilon), \quad \ddot q(t) = \ddot q^*(t) + o(\varepsilon) \quad t \in [t', \infty). \tag{3}$$

In order to guarantee the differentiability of $\eta(t)$, a double integrator is added, that is we set

$$\ddot\eta(t) = u(t).$$

Define

$$\begin{aligned} t_{1,1} &= \delta_1 \\ t_{h,k+1} &= t_{h,k} + \delta_{k+1}, \quad h \in N, \quad k \in \{1, \ldots, r\} \\ t_{h,r+1} &= t_{h+1,1}, \qquad h \in N, \end{aligned}$$

with k the number identifying the equilibrium points and h the number of cycle repetitions. Set

$$u(t) = \lambda_k(t - t_{h,k} + \delta_k) w_{h,k}, \quad t \in [t_{h,k} - \delta_k, t_{h,k}]$$

where $w_{h,k} \in \mathbb{R}^{3\times n}$ and $\lambda_k : [0, \delta_k] \longrightarrow \mathbb{R}^n \times \mathbb{R}^{3\times n}$ is a piece-wise continuous function such that the mapping $P_k : \mathbb{R}^{3\times n} \times \mathbb{R}^{3\times n}$ defined by

$$P_k = \int_0^{\delta_k} e^{A(\delta_k - t)} V \lambda_k(t)\, \mathrm{d}t,$$

where

$$A = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix},$$

is invertible. Note that by changing the functions $\lambda_k(\cdot)$ the trajectories that the state of the system follows change.

Refer as robot position at time $t_{h,k}$

$$z_{h,k} = \begin{bmatrix} q(t_{h,k}) \\ \dot{q}(t_{h,k}) \\ \ddot{q}(t_{h,k}) \end{bmatrix}.$$

Defining

$$L_k = e^{A\delta_k} = I + A\delta_k + A^2\frac{\delta_k^2}{2},$$

one has

$$z_{h+1,1} = L_1 z_{h,r} + P_1 w_{h+1,1}, \tag{4a}$$

$$z_{h+1,k} = L_k z_{h+1,k-1} + P_k w_{h+1,k}, \quad 2 \leq k \leq r, \tag{4b}$$

This system is periodic and controllable since each P_k is invertible. This implies that there exist controls $w_{h,k}$ which guarantee the convergence of each $z_{h,k}$ on

$$z_k^d = \begin{bmatrix} q_k^d \\ 0 \\ 0 \end{bmatrix}$$

as $h \to \infty$ at the slow level.

Under the stated assumption on the unknown disturbance function $d(\cdot)$, exact tracking controls are constants. Hence, according to the new formulation of the Internal Model Principle proposed in [16], the inclusion in the closed loop of a discrete integrator for each control channel guarantees robustness with respect to T-periodic disturbances, provided that the closed loop is asymptotically stable. An example of this type of robust control law is the following

$$\begin{aligned} w_{h+1,1} &= P_1^{-1}(\alpha_{h+1,1} - L_1 z_{h,r}) \\ \alpha_{h+1,1} &= \alpha_{h,1} + E_1(z_1^d - z_{h,1}) \end{aligned} \tag{5a}$$

$$\begin{aligned} w_{h+1,k} &= P_k^{-1}(\alpha_{h+1,k} - L_k z_{h+1,k-1}) \\ \alpha_{h+1,k} &= \alpha_{h,k} + E_k(z_k^d - z_{h,k}), \\ &|I - E_k| < 1, \quad 1 \leq k < r. \end{aligned} \tag{5b}$$

Other geometrically stable control laws, incorporating an integrator for each control, can be derived by using a time invariant reformulation of the system. Set

$$\varphi_h = \begin{bmatrix} z_{h,1} \\ \vdots \\ z_{h,r} \end{bmatrix}, \quad \nu_h = \begin{bmatrix} w_{h,1} \\ \vdots \\ w_{h,r} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} P_1 & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & P_r \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0 & \cdots & L_1 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} I & 0 & 0 & \cdots & 0 & 0 \\ -L_2 & I & 0 & \cdots & 0 & 0 \\ 0 & -L_3 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -L_r & I \end{bmatrix},$$

$$\phi, \nu \in \mathbb{R}^{3\times n\times r}, \quad \Omega, \Phi, \Gamma \in \mathbb{R}^{(3\times n\times r)\times(3\times n\times r)}$$

one has

$$\Gamma\varphi_{h+1} = \Phi\varphi_h + \Omega\nu_{h+1}.$$

Since both Γ and Ω are invertible, this is a well defined time-invariant controllable linear system. For another time-invariant reformulation, which makes use of the state transition matrix of the discrete-time periodic system, the interested reader may refer to [7] and to the references therein quoted.

For what concerns asymptotic stability for $\varepsilon \neq 0$ but small, from Tikhonov's theorem, (eqs. (2) and (3)), one has that, for sufficiently large t', the differences between slow position, velocity and acceleration solutions and the actual ones are smooth functions of ε, vanishing at $\varepsilon = 0$. The perturbation due to the fast dynamics is then small. In addition, under the hypothesis that a control

$$\nu = \begin{bmatrix} w_1 \\ \vdots \\ w_r \end{bmatrix} \in \mathbb{R}^{3\times n},$$

solving the cyclic control problem exists, this perturbation is not persistent, since if the robot exactly execute the cycle the control is no longer updated. Now, the geometric convergence to zero of the error sequences $(z_d^k - z_{h,k})$, for $\varepsilon = 0$ and $h \to \infty$, implies that sufficiently small nonpersistent perturbations are rejected [9]. Hence, provided that a control ν exists, the algorithm is also convergent for sufficiently small ε.

A proof of the existence of such a control can be given by using the implicit function theorem. Suppose that the robot is exactly initialized at time $t = 0$, that is that $q(0) = q_1^d$ and that velocity and acceleration are zero at $t = 0$. Then condition (3) holds with $t' = 0$. Chosen the functions λ_k, the mapping $\Lambda : \mathbb{R} \times \mathbb{R}^{3\times n\times r} \to \mathbb{R}^{3\times n\times r}$, which assigns to a pair (ε, ν) the position, velocity and acceleration of the robot at the instants t_k, is well defined and smooth. Now, by construction the derivative of Λ with respect to ν is full rank for $\varepsilon = 0$. Hence there exists a neighborhood of $\varepsilon = 0$ such that Λ is one to one for each given ε. In particular this is true for a vector ν of components w_k^0 satisfying the equations

$$\begin{aligned} z_1^d &= L_1 z_r^d + P_1 w_1^0, \\ z_k^d &= L_1 z_{k-1}^d + P_k w_k^0, \quad 2 \le k \le r. \end{aligned}$$

But for this value of ν, repositioning is accomplished for $\varepsilon = 0$. Hence, for sufficiently small ε the required control exists.

As the selection of the small parameter ε and of the mappings λ_k, which affect system performance, is concerned, it is suggested to select mappings λ_k such that a given performance index, smoothly dependent on system trajectories, is satisfactory, eventually optimal, with respect to the slow motion. The smallest is ε, compatibly with hardware limitations, the closest is the actual value of the performance index to the one computed using the slow solution. Indices which take into account the torque can also be considered. For, the applied torque and the one computed by substituting the slow solution in the equation of motion are within an ε-approximation, owing to the fact that the ε–approximation holds for the acceleration too. This in particular allows to check control torque feasibility by means of the slow solution.

Given mappings λ_k and ε, it could be of interest to compute the region of convergence, that is the neighborhood of the zero in the error space such that convergence takes place for all initial errors in it. Since for small ε the error dynamics are of the perturbed linear type, the region of convergence can be estimated [9] by using the Lyapunov function associated to the linear system. Among other calculations, this approach requires the computation of the map Λ. This map can be developed in a power series of ε and, in addition to the term of (2.4), only the linear term retained for estimating the region of convergence.

3. CONSTRAINED ROBOTS

Suppose now that the robot interacts with the environment and let the model of this interaction be given by the geometric bilateral constraint

$$v(q(t)) = 0, \quad t \in \mathbb{R}, \tag{6}$$

with $v : \mathbb{R}^n \to \mathbb{R}^m$, $m < n$, smooth and $\partial v / \partial q$ full rank, $\forall\, q(t) \in \mathbb{R}^n$. Under these hypothesis, the implicit function theorem guarantees the existence of a smooth function $s : \mathbb{R}^{n-m} \to \mathbb{R}^n$, such that $s(0) = 0$ and $v(s(\zeta)) = 0$, $\forall\, \zeta \in \mathbb{R}^{n-m}$.

In addition to ζ, we also wish to control the reaction force component which does not make work on δq, i.e. the force normal to the surface S defined by (6). In the sequel, the formulation for hybrid force control given in [19] is used. When no confusion is possible, functional dependency is omitted for notational simplicity.

Let $\chi \in \mathbb{R}^m$ denote the normal force at a point q of S, the corresponding joint force is given by (the apostrophe denotes transposition)

$$r^n = \left[\frac{\partial v}{\partial q}\right]' \chi.$$

The control objective is the control of the pair (χ, ζ) and is a well posed problem. Indeed, decomposes the total reaction force as $r = r^n + r^t$, with r^t the joint force component due to the reaction force tangent to S, one has

$$B(q)\,\ddot{q} + c(\dot{q}, q) = f + r^n + r^t,$$

and, by substitution of $q(t) = s(\zeta(t))$,

$$B\frac{\partial s}{\partial \zeta}\ddot{\zeta} + \gamma = f + r^n + r^t, \tag{7}$$

with

$$\gamma_i = c_i + s_{i,jk}\dot{\zeta}_j\dot{\zeta}_k,$$

where a subscript denotes a vector component, the indexes after a comma denote partial derivatives with respect to components of ζ, and the Einstein summation convention has been adopted. Pre-multiply (7) by $[\partial s/\partial z]'$ to obtain

$$\left[\frac{\partial s}{\partial \zeta}\right]' B \left[\frac{\partial s}{\partial \zeta}\right]\ddot{\zeta} + \left[\frac{\partial s}{\partial \zeta}\right]'\gamma = \left[\frac{\partial s}{\partial \zeta}\right]' f + \left[\frac{\partial s}{\partial \zeta}\right]' r^t, \tag{8}$$

where the matrix

$$\left[\frac{\partial s}{\partial \zeta}\right]' B \left[\frac{\partial s}{\partial \zeta}\right] = M$$

is positive definite and hence invertible, and by $[\partial v/\partial q]$ to get

$$\begin{aligned} &\left[\frac{\partial v}{\partial q}\right] B \left[\frac{\partial s}{\partial \zeta}\right]\ddot{\zeta} + \left[\frac{\partial v}{\partial q}\right]\gamma \\ = &\left[\frac{\partial v}{\partial q}\right]' f + \left[\frac{\partial v}{\partial q}\right] r^t + \left[\frac{\partial v}{\partial q}\right]\left[\frac{\partial v}{\partial q}\right]' \chi. \end{aligned} \tag{9}$$

Let

$$N = \left[\frac{\partial v}{\partial q}\right] B \left[\frac{\partial s}{\partial \zeta}\right], \quad R = \left[\frac{\partial v}{\partial q}\right]\left[\frac{\partial v}{\partial q}\right]', \quad G = \begin{bmatrix} \left[\frac{\partial s}{\partial \zeta}\right]' \\ \frac{\partial v}{\partial q} \end{bmatrix},$$

and combine (8) and (9) to get

$$\begin{bmatrix} M & 0 \\ N & R \end{bmatrix}\begin{bmatrix} \ddot{\zeta} \\ \chi \end{bmatrix} + G(\gamma - r^t) = Gf.$$

Notice that G is invertible as well as the matrix

$$\begin{bmatrix} M & 0 \\ N & R \end{bmatrix}$$

since the matrix R is positive definite. This proves that the control problem addressed is well posed. Consider the application of the following high gain feedback

$$f(t) = -\frac{1}{\varepsilon}\left[\frac{\partial s(\zeta(t))}{\partial \zeta}(\dot{\zeta}(t) - u_\zeta(t)) + \left[\frac{\partial v(q(t))}{\partial q}\right]'(\chi(t) - u_\chi(t))\right], \quad \varepsilon > 0. \quad (10)$$

By letting $\varepsilon \to 0$, one obtains the fast system

$$\begin{aligned} \frac{d\xi(\tau)}{d\tau} &= -M^{-1}(\zeta^*(t))\,Q(\zeta^*(t))\,\xi(\tau), \\ Q &= \left[\frac{\partial s}{\partial \zeta}\right]'\left[\frac{\partial s}{\partial \zeta}\right], \end{aligned}$$

where $\zeta^*(t)$ is the solution of the slow system. Since both M and Q are positive definite, the fast system is globally exponentially stable and Tikhonov's theorem applies. The slow system is given by

$$\frac{\partial s(\zeta(t))}{\partial \zeta}(\dot{\zeta}(t) - u_\zeta(t)) + \left[\frac{\partial v(q(t))}{\partial q}\right]'(\chi(t) - u_\chi(t)) = 0,$$

which is equivalent to

$$\dot{\zeta}(t) = u_\zeta(t), \quad \chi(t) = u_\chi(t),$$

since the matrix

$$G' = \begin{bmatrix} \frac{\partial s}{\partial \zeta} & \left[\frac{\partial v}{\partial q}\right]' \end{bmatrix}$$

is full rank.

The problem to be solved consists of finding a control law such that the robot executes a cycle characterized by given value of ζ and χ:

$$\{\zeta_1^d, \ldots, \zeta_r^d\}, \quad \{\chi_1^d, \ldots, \chi_r^d\}.$$

As for the unconstrained case, a control scheme working at the slow level is developed. In order to guarantee that the slow solutions are within an ε-approximation of the actual ones, set

$$\ddot{u}_\zeta(t) = \sigma_\zeta(t), \quad \dot{u}_\chi(t) = \sigma_\chi(t).$$

Next, set

$$\sigma_\zeta(t) = \lambda_k(t - t_{h,k} + \delta_k)w_{h,k}, \quad t \in [t_{h,k} - \delta_k, t_{h,k}],$$

where $\lambda_k : [0, \delta_k] \longrightarrow \mathbb{R}^{n-m} \times \mathbb{R}^{3\times n-m}$ is a piece-wise continuous function such that for each mapping $P_k : \mathbb{R}^{3\times n-m} \longrightarrow \mathbb{R}^{3\times n-m}$ defined by

$$P_k = \int_0^{\delta_k} e^{A(\delta_k - t)} V \lambda_k(t)\, \mathrm{d}t,$$

is invertible. Here A and V, given the appropriate dimensions, are defined as in the previous section. Similarly, set

$$\sigma_\chi(t) = \mu_k(t - t_{h,k} + \delta_k)\omega_{h,k}, \quad t \in [t_{h,k} - \delta_k, t_{h,k}],$$

where $\omega_{h,k} \in \mathbb{R}^{3\times m}$, $\mu_k : [0, \delta_k] \in \mathbb{R}^m \times \mathbb{R}^{3\times m}$ is piece-wise continuous and such that each mapping $\Pi_k : \mathbb{R}^{3\times m} \longrightarrow \mathbb{R}^{3\times m}$ defined by

$$\Pi_k = \int_0^{\delta_k} \mu_k(t)\, \mathrm{d}t,$$

is invertible. Set

$$\rho_k = \begin{bmatrix} \zeta_k \\ \dot{\zeta}_k \\ \ddot{\zeta}_k \end{bmatrix}, \quad \rho_k^d = \begin{bmatrix} \zeta_k^d \\ 0 \\ 0 \end{bmatrix}, \quad \phi_k = \begin{bmatrix} \chi_k \\ \dot{\chi}_k \\ \ddot{\chi}_k \end{bmatrix}, \quad \phi_k^d = \begin{bmatrix} \chi_k^d \\ 0 \\ 0 \end{bmatrix}.$$

At the slow level, once more we obtain a periodic and controllable, discrete time, linear system:

$$\begin{aligned}
\rho_{h+1,1} &= L_1 \rho_{h,r} + P_1 w_{h+1,1} \\
\rho_{h+1,k} &= L_k \rho_{h+1,k-1} + P_k w_{h+1,k} \quad 2 \le k \le r, \\
\phi_{h+1,1} &= \phi_{h,r} + \Pi_1 \omega_{h+1,1} \\
\phi_{h+1,k} &= \phi_{h+1,k-1} + \Pi_k \omega_{h+1,k} \quad 2 \le k \le r
\end{aligned}$$

with

$$L_k = e^{A\delta_k} = I + A\delta_k + A^2 \frac{\delta_k^2}{2}.$$

Hence, there exist controls $w_{h,k}$ and $\omega_{h,k}$ such that convergence is achieved at the slow level.

Under the stated hypothesis of smoothness of the constraining function $v(\cdot)$, convergence of an algorithm defined by a robust control law of the type given in the previous section can be proven, for sufficiently small ε, by using the same arguments of the previous section. If the constraining surface is not exactly known, the high gain control (10) looks like

$$f = -\frac{1}{\varepsilon}\left[\frac{\partial \tilde{s}}{\partial \zeta}(\dot{\zeta} - u_\zeta) + \left[\frac{\partial \tilde{v}}{\partial q}\right](\chi - u_\chi)\right], \quad \varepsilon > 0,$$

where the tilde denotes an approximation of the true function. Even if these functions approximate true ones, for consistency they must satisfy the requirement that the matrix

$$\tilde{G}' = \left[\frac{\partial \tilde{s}}{\partial z} \; \left[\frac{\partial \tilde{v}}{\partial q}\right]'\right] \tag{11}$$

is full rank. The fast system is now given by

$$\begin{aligned}\frac{\mathrm{d}\xi(\tau)}{\mathrm{d}\tau} &= -M^{-1}(\zeta^*(t))\,Q^{\circ}(\zeta^*(t))\,\xi(\tau),\\ Q^{\circ} &= \left[\frac{\partial s}{\partial \zeta}\right]'\left[\frac{\partial \tilde{s}}{\partial \zeta}\right],\end{aligned}$$

and, as long as Q° is positive definite, it is exponentially stable and Tikhonov's theorem applies. Stability then depends on a sufficient good knowledge of the constraining surface. At the slow level one has

$$\frac{\partial \tilde{s}}{\partial \zeta}(\dot{\zeta} - u_{\zeta}) + \left[\frac{\partial \tilde{v}}{\partial q}\right]'(\chi - u_{\chi}) = 0,$$

and, since the matrix $\tilde{G}$ is full rank, the same algorithm is still convergent.

4. EXPERIMENTAL RESULTS ON A TWO-LINK ARM

A scheme of the experimental robot used is shown in Figure 1.

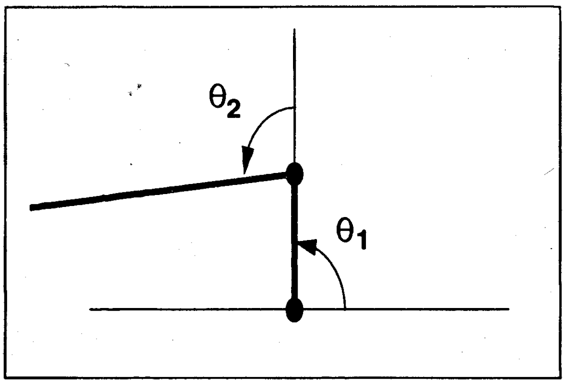

Fig. 1. The two-link robot arm.

It is an open chain planar arm with two links and two revolute joints. The lengths of the links are equal to 0.3 m and 0.7 m, respectively for the first and the second one. The moments of inertia of the links are equal to $J_1 = 0.447$ Kg m^2 and $J_2 = 0.303$ Kg m^2, the static moments to 0.1114 Kg m and 0.5369 Kg m, respectively for the first and the second link. The mass of the second link is equal to 1.8 Kg.

Each joint is actuated by a direct drive dc motor and is equipped with an encoder and a tachometer. The encoders resolution is equal to $\pi/10000$ rad.

The robot is digitally controlled by means of a personal computer using a sampling frequency of 200 Hz for each signal. Analog feedbacks from the tachometers signals are closed at the joints. Denoting by $f(t)$ the motor torques, by $\theta_1(t)$ and $\theta_2(t)$ the components of vector $q(t)$ (see Fig. 1), and with $\eta(t)$ the control input generated by the computer, one has

$$f(t) = -\begin{bmatrix} k_{d1} & 0 \\ 0 & k_{d2}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1(t)\\ \dot{\theta}_2(t)\end{bmatrix} + \eta(t), \tag{12}$$

with $k_{d1} = 2$ Nm sec/rad and $k_{d2} = 0.8$ Nm sec/rad. In addition, a proportional loop has been implemented using the computer to stabilize the robot around a desired reference signal

$$\eta_1(t) = -K_{p1}(\theta_1(t) - r_1(t)),$$
$$\eta_2(t) = -K_{p2}(\theta_2(t) - r_2(t)),$$

with $K_{p1} = 20$ Nm sec/rad and $K_{p2} = 2$ Nm sec/rad. An integrator for each channel has been added to smooth the control $r(t)$.

Since the robot is moving on a plane orthogonal to the gravity vector, instead of requiring that the velocities and the accelerations are null, it is sufficient to impose a zero value for the velocities and the control torques. This implies a change of coordinates and then a straightforward modification of the algorithm presented.

The algorithm has been tested on the following cycle

$$q_1 = \begin{bmatrix} \frac{\pi}{4} \\ -\frac{\pi}{4} \end{bmatrix}, \quad q_2 = \begin{bmatrix} \frac{\pi}{4} \\ 0 \end{bmatrix}, \quad q_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\delta_1 = 1\,\text{s}, \quad \delta_2 = 1\,\text{s}, \quad \delta_3 = 1\,\text{s}.$$

as depicted in Figure 2.

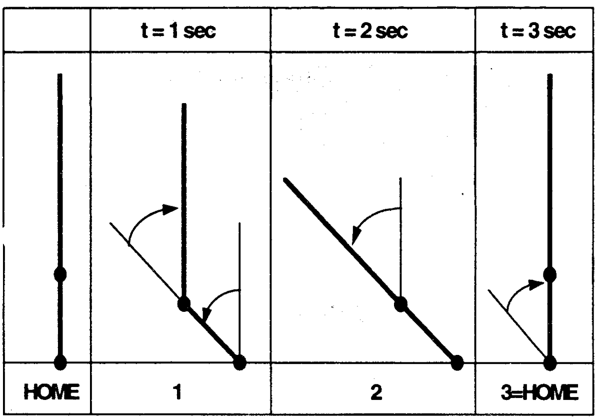

Fig. 2. The test cycle.

The control law (5) has been used with $E_k = 0.5 \;\forall k$, and by setting $L_1 = 0$ in (5a) to simplify calculations. This approximation has not destroyed the stability of the algorithm. The mappings $P_1 = P_2 = P_3$ and $L_2 = L_3$ have been calculated by using a simple linear model of the robot in which the coupling terms between the two links have been neglected:

$$\begin{bmatrix} J_1 + J_2 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}.$$

For each equilibrium point six state variables are to be steered. Denoting with z the state vector and with z_1^d, z_2^d, z_3^d the three desired equilibrium points, the defined cycle implies

$$z = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ r_1 \\ \theta_2 \\ \dot{\theta}_2 \\ r_2 \end{bmatrix}, \quad z_1^d = \begin{bmatrix} \pi/4 \\ 0 \\ \pi/4 \\ -\pi/4 \\ 0 \\ -\pi/4 \end{bmatrix}, \quad z_2^d = \begin{bmatrix} \pi/4 \\ 0 \\ \pi/4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad z_3^d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Then, as functions $\lambda_k(t)$, the following six polynomials have been chosen

$$[\lambda_1(t) \quad \dots \quad \lambda_6(t)] = \begin{bmatrix} t & t^2 & t^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & t & t^2 & t^3 \end{bmatrix}.$$

The joint velocities have been estimated by a high gain differentiation of the encoders signals, which at steady state leads to a negligible error.

In Figures 3 and 4 positions and velocities of the two joints during 47th iteration are reported. Note that the trajectory followed are quite smooth as a consequence of the polynomial class of functions chosen for the control input. In Figure 5, the torques applied during the same cycle are shown. These are zero in correspondence of the time instants 1,2 and 3 seconds that, together with the zero values of velocities at the same instants, imply the equilibrium of the robot at the three points of the cycle.

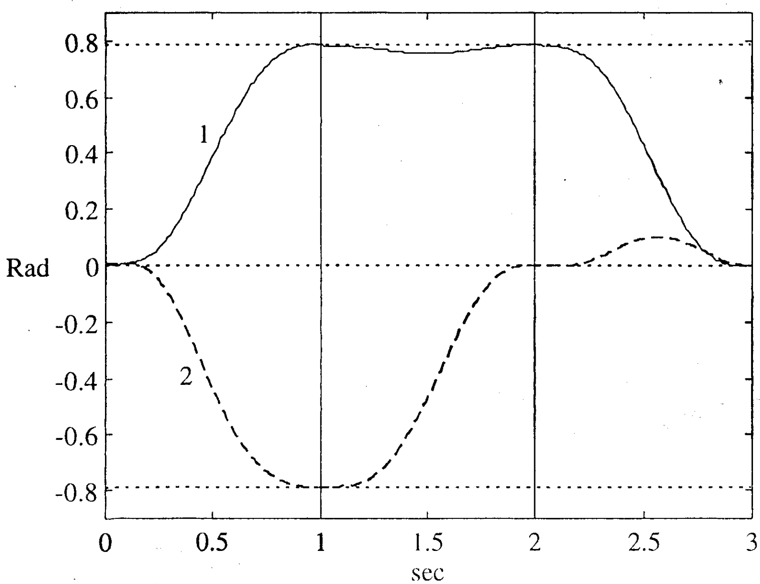

Fig. 3. Link positions during 47th cycle.

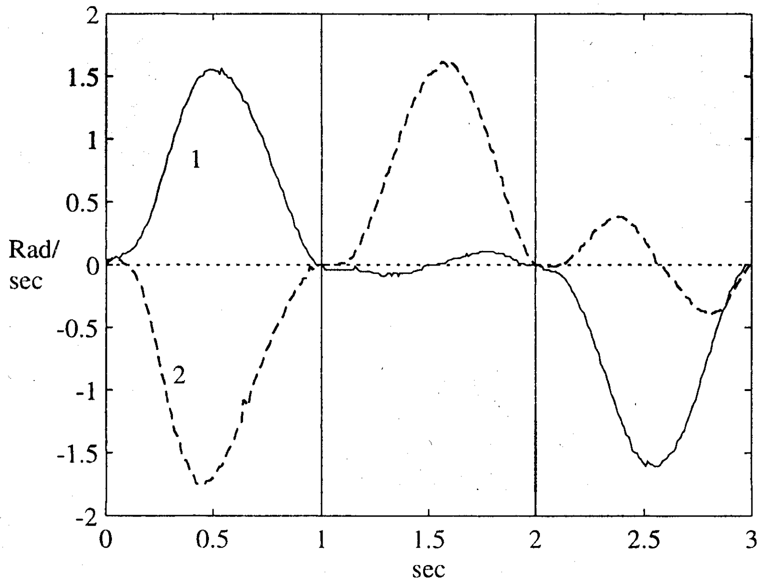

Fig. 4. Link velocities during 47th cycle.

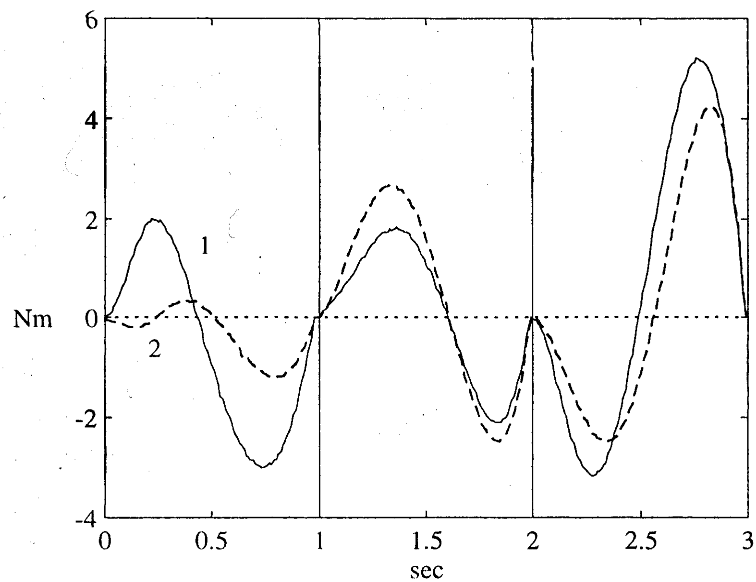

Fig. 5. Motor torques during 47th cycle.

Finally in Figures 6, 7 and 8 the sum of square of the positions, velocities and torques errors during the iterations is reported for the three equilibrium points.

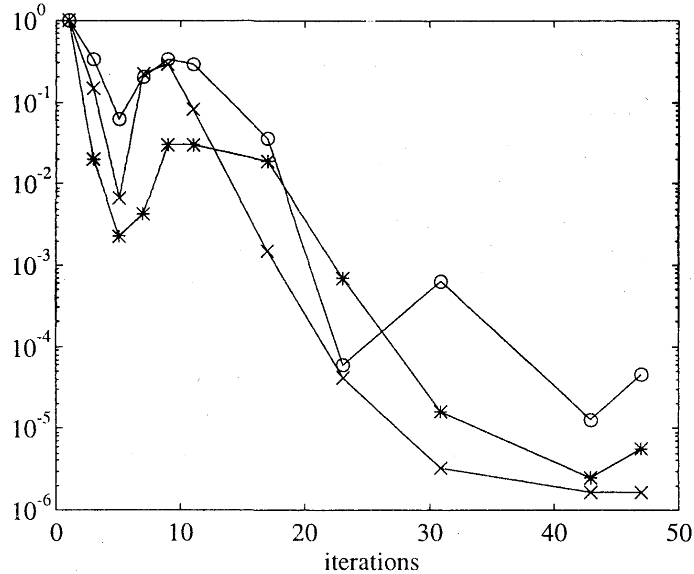

Fig. 6. Sum of square of position errors at the three equilibrium points: $* = 1$, $x = 2$, $o = 3$.

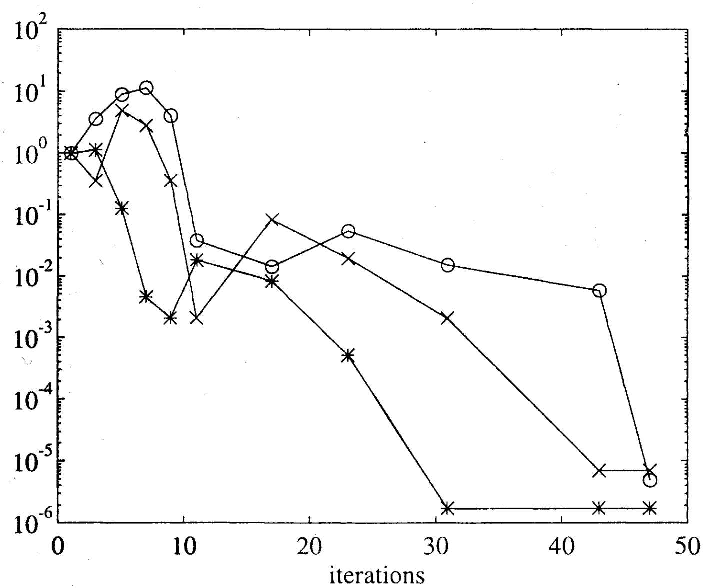

Fig. 7. Sum of square velocity errors at the three equilibrium points: $* = 1$, $x = 2$, $o = 3$.

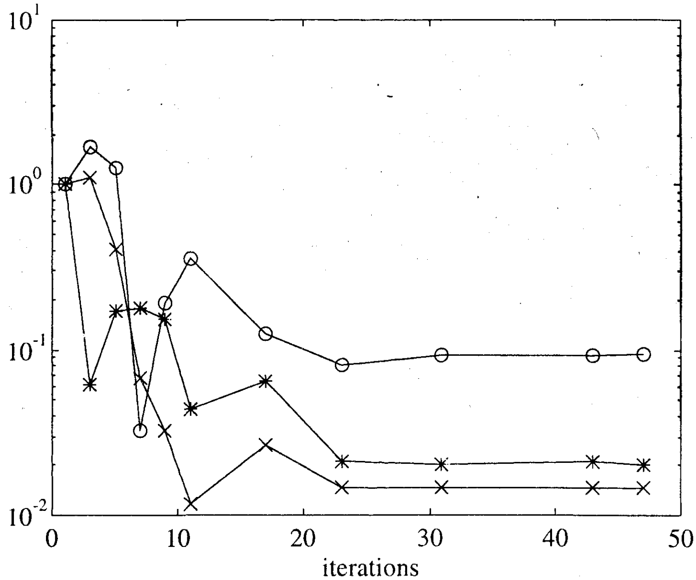

Fig. 8. Sum of square torque errors at the three equilibrium points: $* = 1$, $x = 2$, $o = 3$.

5. CONCLUSIONS

A new type of servo system has been introduced to deal with cyclic control of robot arms. A finite dimensional linear algorithm has been developed which asymptotically forces the robot to execute a cycle defined by a sequence of equilibrium points to be attained at assigned time instants. As opposed to learning algorithms, no system initialization is needed at the end of a cycle, and continuous system operation is allowed. No prior knowledge of robot's parameters is required for controller design. Complete rejection of periodic plant disturbance of period equal to the cycle period has been proven and illustrated by means of an experiment on a two link robot arm. Robustness with respect to other type of disturbances is the one typical of high gain feedback.

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