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ROBUST STABILIZATION AND GUARANTEED COST CONTROL OF LARGE SCALE LINEAR SYSTEMS WITH JUMPS[1]

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In this paper we consider systems which are linear in the continuous plant state and whose mode dynamics is described via random jumps modelled by a discrete-state Markov chain. By the use of decomposition and coordination leading to a two level control system, the robustness in the sense of robust stability and guaranteed cost control is ensured for the partly unknown large scale linear system with markovian jumps. Decision makers on each level have different models of the system and instantaneous information. Two different structures are proposed: decentralized and centralized one.

In the decentralized structure control strategy combines the linear control law resulting from a solution of the JLQ problem for local decision makers and the nonlinear one of the coordinator who takes into account bounds imposed on the uncertainty disturbing the overall system and interconnections between subsystems.

In the centralized structure decision maker of the upper level has only nominal linear model of the system neglecting uncertainties. A centralized controller is found using the quadratic criterion for the system and incorporates the information about its state. The role of the local decision maker is to ensure robust stability and guaranteed cost in spite of uncertainties represented by deviation of parameters and disturbances.

1. INTRODUCTION

Design of feedback controllers for systems with uncertain parameters has been a topic of interest of system designers for many years. Parameter uncertainty can be dealt with in variety of ways. One possibility is constant parameter estimation through extensive testing or through use of real-time or nonreal time system identification. Alternatively, parameters may be accepted at their a priori levels, and a control should be designed so as to be, in some sense, robust or insensitive to their variations. It is the latter approach that is applied in this paper.

In many practical situations, the natural state space is hybrid: to the usual plant state in $\mathbb{R}^n$ we append a discrete variable taking values in $\mathcal{B} = \{1, 2, \dots s\}$ called the mode that describes sudden changes in the plant characteristics. It is typical case

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in the complex large scale systems, such as manufacturing systems (see for example [2]), power systems (see for example [11]) or redundant multiplex control systems ([9]).

In this paper we consider systems which are linear in the continuous plant state and whose mode dynamics is described via random jumps modelled by a discrete-state Markov chain. One way of stabilizing the linear stochastically stabilizable system with markovian jumps is to solve the JLQ problem (see for example [5, 8, 12]). However the optimality of the solution as well as the stability of the system is guaranteed only for the perfectly measurable state variables and complete information about the system parameters. Moreover an optimal controller uses all the state variables to construct a control vector. This is an overidealization especially in the case of a complex system containing many subsystems interconnected by incompletely known crosscoupling. The situation becomes especially complex for the piecewise deterministic processes when the controller is designed under the assumption of the complete access to the mode i.e. discrete random state variables representing the form process.

To overcome at least a part of these difficulties we propose to combine decentralized jump linear quadratic (DJLQ) approach with nonlinear control design method used by some authors (see for example [1, 6, 7]) to ensure practical stability of uncertain systems. Simply we decompose the system into subsystems and consider two level control structure. Decision makers of the lower level have only linear models of their subsystems neglecting interconnections between subsystems. A local controller is found using the quadratic criterion for the subsystem and incorporates the information about its local state. The role of the coordinator (upper level decision maker) is to ensure robust stability and guaranteed cost in spite of uncertainties represented by interconnections among subsystems and deviation of parameters. The uncertainty is described by deterministic inequality model and the main assumption is the well-known matching conditions. The coordinator uses the information about local states and bounds for uncertainties to design the robust control actions which are transmitted to the local decision makers and added to the local control variables. This control is nonlinear but it is bounded by the constraints imposed on the uncertainties. Yet another possibility is to design the JLQ controller for the overall system described by a model without uncertainty and to render this strategy robust by local nonlinear law based on the local estimation of bounds for uncertain variables. We call this approach a centralized one.

The paper is organized as follows. In Section 2, we establish a model of the system, a model of the uncertainty and a nominal model used by the local decision makers in the decentralized structure. Then, we state the control problem and we describe an information structure in the system. In Section 3, we construct the control laws of the local decision makers and the coordinator and we give the main results of this paper in the form of two theorems dealing with robust stochastic stability and guaranteed control property of the system. In Section 4 we compare the results with the ones obtained in the centralized structure and in Section 5, we present some concluding remarks.

2. DECENTRALIZED INFORMATION STRUCTURE

We consider a decentralized system composed of L interconnected subsystems described in the state form by the following differential equation:

$$\begin{aligned} \dot{x}^i(t) &= A^{ii}(\xi^i(t))\, x^i(t) + B^i(\xi^i(t)) \left[u^i(t) + v^i(t) + e^i(\xi(t), x(t), t)\right] \\ &+ \sum_{j=1, j\neq i}^{L} A^{ij}(\xi(t))\, x^j(t) \end{aligned} \tag{1}$$

$$x^i(0) = x_0^i \tag{2}$$

where x^i is a local state vector of the ith subsystem, $x^i(t) \in \mathbb{R}^{n_i}$, u^i is a local control, $u^i(t) \in \mathbb{R}^{m_i}$, v^i is a coordinator control for the ith subsystem, $v^i(t) \in \mathbb{R}^{m_i}$, $A^{ii}(\xi^i(t))$, $B^i(\xi^i(t))$ are local system and input matrices respectively, $A^{ij}(\xi(t))$ represents crosscouplings, $e^i(\xi(t), x(t), t)$ are model uncertainties resulting from parameter deviations and bounded nonlinearities acting in the range of the local input for the ith subsystem, $e^i(\xi(t), x(t), t) \in \mathbb{R}^{m_i}$. $\xi^i(t)$ is an irreducible and continuous time discrete state Markov process representing a local mode of the ith subsystem and taking values in a finite set $\mathcal{B}^i = \{1, 2, \ldots, s^i\}$ with transition probability matrix $P = \{p_{\alpha^i\beta^i}\}$ from mode α^i to mode β^i during the time interval $[t, t+\delta t]$, given by:

$$p_{\alpha^i\beta^i} = Pr\left\{\xi^i(t+\delta t) = \beta^i | \xi^i(t) = \alpha^i\right\} = \begin{cases} q^i_{\alpha^i\beta^i}\delta t + o(\delta t), & \text{if } \alpha^i \neq \beta^i \\ 1 + q^i_{\alpha^i\alpha^i}\delta t + o(\delta t), & \text{if } \alpha^i = \beta^i. \end{cases} \tag{3}$$

In this relation, $q^i_{\alpha^i\beta^i}$ stands for the transition probability rate from mode α^i to mode β^i and satisfies the following relations:

$$q^i_{\alpha^i\beta^i} \geq 0 \tag{4}$$

$$q^i_{\alpha^i\alpha^i} = -\sum_{\beta^i\in\mathcal{B}^i, \alpha^i\neq\beta^i} q^i_{\alpha^i\beta^i} \tag{5}$$

$x(t)$ is an overall system state vector composed of the subsystem state vectors $x^i(t)$, $i = 1, 2, \ldots, L$ taking values in $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \ldots \times \mathbb{R}^{n_L}$, while the mode $\xi(t)$ of the overall system contains modes of the subsystems $\xi^i(t)$ and takes values in the product set $\mathcal{B} = \mathcal{B}^1 \times \mathcal{B}^2 \times \ldots \times \mathcal{B}^L$.

It is assumed that the unknown cross-couplings satisfy the following matching conditions (see for example [1, 6])

$$A^{ij}(\xi(t)) = B^i(\xi^i(t))\, D^{ij}(\xi(t)) \tag{6}$$

where the matrix $D^{ij}(\xi(t))$ for each $\xi(t) = \alpha = [\alpha^1, \ldots, \alpha^L]'$ satisfies the following relation:

$$||D^{ij}(\alpha)|| \leq d^{ij}(\alpha) \tag{7}$$

and $d^{ij}(\alpha)$ is a known scalar. Uncertainty $e^i(\xi(t), x(t), t)$ is assumed to be bounded for each $\xi(t) = \alpha = [\alpha^1, \ldots, \alpha^L]'$ ($'$ represents the transpose operation; this notation will be used subsequently in text) by:

$$\|e^i(\alpha, x(t), t)\| \leq f^i(\alpha)\,\|x(t)\| \tag{8}$$

where $f^i(\alpha)$ is a known scalar.

A nominal model of the ith local decision maker has a simplified form:

$$\dot{x}^i(t) = A^{ii}(\xi^i(t))\,x^i(t) + B^i(\xi^i(t))\,u^i(t) \tag{9}$$

and is used to find a control $u^i(t)$ minimizing a local quadratic performance index:

$$J^i = \mathsf{E}\left\{\int_0^\infty x^{i\prime}(t)\,Q^i(\xi^i(t))\,x^i(t) + u^{i\prime}(t)\,R^i(\xi^i(t))\,u^i(t)\,\mathrm{d}t\right\} \tag{10}$$

where the cost weighting matrices $R^i(\xi^i(t))$ and $Q^i(\xi^i(t))$ are symmetric respectively positive definite and positive semidefinite for each $\xi(t)$.

Each ith nominal model is assumed to be stochastically stabilizable [5] and each pair $(A^{ii}(\alpha^i), C^i(\alpha^i))$ is observable for all $\alpha^i \in \mathcal{B}^i$ where $C^{i\prime}(\alpha^i)\,C^i(\alpha^i) = Q^i(\alpha^i)$. It is also assumed that all state variables of the ith subsystem are perfectly measurable. The information transmitted to the coordinator at each time t consists of values of the state vector norm $\|x^i(t)\|$, the mode $\xi^i(t)$ and the control $u^i(t)$ for each $i = 1, \ldots, L$. Based on this information, the control $v^i(t), i = 1, \ldots, L$, which will render the system robust, is evaluated by the coordinator and then transmitted to the ith subsystem. The design objective is to find a feedback control law that guarantees robust stability of each subsystem. Moreover it will be shown that the control ensures robustness of the overall system in the sense of guaranteed cost property [4, 10] given by the inequality:

$$J = \sum_{i=1}^{L} J^i \leq \sum_{i=1}^{L} x^{i\prime}_0 K^i(\alpha^i)\,x^i_0 \tag{11}$$

where $K^i(\alpha^i)$, $(\alpha^i \in \mathcal{B}^i)$ is the set of the unique positive solutions of the coupled Riccati equations corresponding to the local JLQ problem (9) – (10) given by:

$$A^{ii\prime}(\alpha^i)\,K^i(\alpha^i) + K^i(\alpha^i)\,A^{ii}(\alpha^i) - K^i(\alpha^i)\,B^i(\alpha^i)R^{i^{-1}}(\alpha)B^{i\prime}(\alpha)\,K^i(\alpha) + \sum_{\beta^i=1}^{s^i} q_{\alpha^i\beta^i} K^i(\beta^i) + Q^i(\alpha^i) = 0; \quad \alpha^i \in \mathcal{B}^i. \tag{12}$$

The coordinator uses his own resources to realize his control policy $v^i(t)$ thus its cost is not included in the local performance index.

3. CONTROL LAW DESIGN

The feedback control used in each subsystem is the sum of the local decision maker strategy and the coordinator's one. The local control law is found by minimizing

(10) for the nominal model of the ith subsystem (9) and has for any given $\xi^i(t) = \alpha^i$ the following form [5]:

$$u^i(\alpha^i, t) = -R^{i^{-1}}(\alpha^i)\, B^{i'}(\alpha^i)\, K^i(\alpha^i)\, x^i(t). \tag{13}$$

The corresponding optimal cost for the nominal model is given by (for $\xi^i(0) = \alpha^i$):

$$J^{i^o} = x^{i'}_0 K^i(\alpha^i)\, x^i_0. \tag{14}$$

The coordinator has an information about the structure of the overall system including bounds on the incompletely known crosscouplings (6), (7) and uncertainties (8), and the actual information about the values of $u^i(\xi^i(t), t)$, $||x^i(t)||$ and $\xi^i(t)$ from all subsystems. This information is used to construct the control law v^i defined for each $\xi(t) = \alpha$ as follows:

$$v^i(\alpha, t) = \begin{cases} \frac{R^i(\alpha^i)u^i(\alpha^i,t)}{||R^i(\alpha^i)\,u^i(\alpha^i,t)||}\rho^i(\alpha, ||x(t)||) & \text{if } u^i(\alpha^i, t) \neq 0 \\ 0 & \text{if } u^i(\alpha^i, t) = 0 \end{cases} \tag{15}$$

where $\rho^i(\alpha, ||x(t)||)$ is an upper bound on the entire uncertainty $\eta^i(\alpha, x(t), t)$ for the ith system, defined as:

$$\eta^i(\alpha, x(t), t) = \sum_{j=1, j\neq i}^{L} D^{ij}(\alpha)\, x^j(t) \; + e^i(\alpha, x(t), t)$$

where the matrice $D^{ij}(\alpha^i)$ comes from the use of the Eq. (6).

The upper bound $\rho^i(\alpha, ||x(t)||)$ is defined by the following formula for any $\xi(t) = \alpha$:

$$\begin{aligned} ||\eta^i(\alpha, x(t), t)|| &= \left\| \sum_{j=1, j\neq i}^{L} D^{ij}(\alpha)\, x^j(t) \; + e^i(\alpha, x(t), t) \right\| \\ &\leq \sum_{j=1, j\neq i}^{L} ||D^{ij}(\alpha)\, x^j(t)|| + ||e^i(\alpha, x(t), t)|| \\ &\leq \sum_{j=1, j\neq i}^{L} d^{ij}(\alpha)\, ||x^j(t)|| + f^i(\alpha) \sum_{j=1}^{L} ||x^j(t)|| = \rho^i(\alpha, ||x(t)||). \end{aligned} \tag{16}$$

If we define $d^{ii} = 0$, then

$$\rho^i(\alpha, ||x(t)||) = \sum_{j=1}^{L} (d^{ij}(\alpha) + f^i(\alpha))\, ||x^j(t)||. \tag{17}$$

Thus the coordinator control law is also bounded.

To find the sufficient conditions for robust stochastic stability, let assume Lyapunov function candidate for each subsystem i in the form:

$$V(x^i, \alpha^i) = x^{i'} K^i(\alpha^i)\, x^i = S^i(x^i, \alpha^i) \tag{18}$$

where $S^i(x^i, \alpha^i)$ is the optimal cost to go for the nominal model of the ith subsystem (9) starting from x_0^i, $\xi^i(0) = \alpha^i$. Its expression is given by (14).

The following theorem gives the required conditions.

Theorem 3.1. Assume that the ith subsystem described by the state equation (2) meets the matching conditions (6) – (8) and is governed by the control law (13) and coordinator control (15). Then the overall system remains stochastically stable in the whole ranges of uncertainty.

Proof. Consider the weak infinitesimal operator $\tilde{A}$ of the joint process ($\xi^i(t)$, $x^i(t)$) which is the natural analogue of the deterministic derivative of the Lyapunov function and is defined as follows:

$$\tilde{A}V(x^i, \alpha^i) \tag{19}$$
$$= \lim_{h\to 0} \frac{1}{h}\Big[\mathsf{E}\Big\{V(x^i(t+h), \xi^i(t+h))\,|x(t), \xi(t) = \alpha\Big\} - V(x^i(t), \xi^i(t) = \alpha^i)\Big].$$

The weak infinitesimal operator is then given by:

$$\begin{aligned}\tilde{A}V(x^i, \alpha^i) \;=\;& x^{i'}(t)\Big\{[A^{ii}(\alpha^i) - B^i(\alpha^i)\,R^{i^{-1}}(\alpha^i)\,B^{i'}(\alpha^i)\,K^i(\alpha^i)]'\,K^i(\alpha^i)\\ &+ K^i(\alpha^i)\,[A^{ii}(\alpha^i) - B^i(\alpha^i)\,R^{i^{-1}}(\alpha^i)\,B^{i'}(\alpha^i)\,K^i(\alpha^i)]\\ &+ \sum_{\beta^i\in\mathcal{B}^i} q^i_{\alpha^i\beta^i}\,K^i(\beta^i)\Big\}x^i(t)\\ &+ x^{i'}(t)\,K^i(\alpha^i)\,B^i(\alpha^i)\,[\eta^i(\alpha, x(t), t) + v^i(\alpha, t)]\\ &+ [\eta^i(\alpha, x(t), t) + v^i(\alpha, t)]'\,B^{i'}(\alpha^i)\,K^i(\alpha^i)\,x^i(t).\end{aligned} \tag{20}$$

Using the form of the control law (15) and assumptions regarding the bounds imposed on the uncertainty, the last term in (20) can be estimated as follows:

$$\begin{aligned}&[\eta^i(\alpha, x(t), t) + v^i(\alpha, t)]'\,B^{i'}(\alpha^i)\,K^i(\alpha^i)\,x^i(t)\\ =\;& \Big[\eta^i(\alpha, x(t), t) + \frac{R^i(\alpha^i)\,u^i(\alpha^i, t)}{\|R^i(\alpha^i)\,u^i(\alpha^i, t)\|}\rho^i(\alpha, \|x(t)\|)\Big]'\,B^{i'}(\alpha^i)\,K^i(\alpha^i)\,x^i(t)\\ =\;& \eta^{i'}(\alpha, x(t), t)B^{i'}(\alpha^i)\,K^i(\alpha^i)x^i(t) - \|B^{i'}(\alpha^i)\,K^i(\alpha^i)\,x^i(t)\|\,\rho^i(\alpha, \|x(t)\|)\\ \le\;& \|\eta^{i'}(\alpha, x(t), t)\|\,\|B^{i'}(\alpha^i)\,K^i(\alpha^i)\,x^i(t)\|\\ &- \|B^{i'}(\alpha^i)\,K^i(\alpha^i)\,x^i(t)\|\rho^i(\alpha, \|x(t)\|) \;\le\; 0.\end{aligned} \tag{21}$$

Notice that we have the same results for the term

$$x^{i'}(t)\,K^i(\alpha^i)\,B^i(\alpha^i)[\eta^i(\alpha, x(t), t) + v^i(\alpha, t)]$$

which is just the transpose of the previous one. Thus from (15), (20) and (21), it follows that $\tilde{A}V(x^i, \alpha^i)$ satisfies:

$$\tilde{A}V(x^i, \alpha^i) \le -x^{i'}(t)\,[Q^i(\alpha^i) + K^i(\alpha^i)\,B^i(\alpha^i)\,R^{i^{-1}}(\alpha^i)\,B^{i'}(\alpha^i)K^i(\alpha^i)]\,x^i(t). \tag{22}$$

Since

$$V(x^i, \alpha^i) \quad \leq \quad \lambda_{\max}[K^i(\alpha^i)] \, ||x^i||^2 \tag{23}$$

and

$$x^{i'}(t) \, [Q^i(\alpha^i) + K^i(\alpha^i) \, B^i(\alpha^i) \, R^{i^{-1}}(\alpha^i) \, B^{i'}(\alpha^i) \, K^i(\alpha^i)] \, x^i(t)$$
$$\geq \lambda_{\min}[Q^i(\alpha^i) + K^i(\alpha^i) \, B^i(\alpha^i) \, R^{i^{-1}}(\alpha^i) \, B^{i'}(\alpha^i) \, K^i(\alpha^i)] \, ||x^i||^2 \tag{24}$$

hold then

$$\frac{\tilde{A}V(x^i, \alpha^i)}{V(x^i, \alpha^i)} \leq -\frac{\lambda_{\min}[Q^i(\alpha^i) + K^i(\alpha^i) \, B^i(\alpha^i) \, R^{i^{-1}}(\alpha^i) B^{i'}(\alpha^i) \, K^i(\alpha^i)]}{\lambda_{\max}(K^i(\alpha^i))} \tag{25}$$

Then by Dynkin's formula and the Bellman–Gronwall lemma for all $\alpha^i \in \mathcal{B}^i$, it follows that:

$$\mathsf{E}[V(x^i, \alpha^i)] \quad \leq \quad \exp(-\gamma t) \, V(x_0^i, \alpha^i) \tag{26}$$

where

$$\gamma \quad = \quad \min_{i \in \mathcal{B}} \left[\frac{\lambda_{\min}[Q^i(\alpha^i) + K^i(\alpha^i) \, B^i(\alpha^i) R^{i^{-1}}(\alpha^i) B^{i'}(\alpha^i) \, K^i(\alpha^i)]}{\lambda_{\max}[K^i(\alpha^i)]} \right] > 0. \tag{27}$$

Thus

$$\lim_{T \to \infty} \mathsf{E} \left[\int_0^T x^{i'}(t) \, K^i(\alpha^i) x^i(t) \, \mathrm{d}t \, | x_0, \, \xi(0) = \alpha \right] \quad \leq \quad \frac{1}{\gamma} x^{i'}_0 K^i(\alpha^i) \, x_0^i. \tag{28}$$

Since $K^i(\alpha^i) > 0$ for each $\alpha^i \in \mathcal{B}^i$, thus

$$\lim_{T \to \infty} \mathsf{E} \left[\int_0^T x^{i'}(t) \, x^i(t) \, \mathrm{d}t \, | x_0, \, \xi(0) = \alpha \right] \quad \leq \quad x^{i'}_0 \max_{\alpha^i} \frac{K^i(\alpha^i)}{\gamma ||K^i(\alpha^i)||} \, x_0^i \tag{29}$$

which proves the theorem. □

Similar arguments could be used to demonstrate guaranteed cost property of the proposed control strategy assuming that the "cost" of the coordination is not included in the performance index. The following theorem states the conditions for this property.

Theorem 3.2. Assume that the *i*th subsystem described by the state equation (2) meets the matching conditions (6) – (8) and is governed by the control law (13) and coordinator control (15). Then the value of the performance index (10) for the subsystem does not exceed the optimal cost for the nominal model (9) given by the function:

$$S^i(x_0^i, \, \xi^i(0) = \alpha^i) \quad = \quad x^{i'}_0 K^i(\alpha^i) \, x_0^i. \tag{30}$$

Proof. Using Lyapunov function (18) and taking into account inequality (22), the value of the performance index (10) for the subsystem (2) may be estimated as follows:

$$\begin{aligned}
&\mathsf{E}\Big\{\int_0^\infty [x^{i'}(t)\, Q^i(\xi^i(t))\, x^i(t) + u^{i'}(t)\, R^i(\xi^i(t))\, u^i(t)]\, \mathrm{d}t\Big\} \\
=\;& \mathsf{E}\Big\{\int_0^\infty [x^{i'}(t)\, \{Q^i(\xi^i(t)) \\
&\quad + K^i(\xi^i(t)) B^i(\xi^i(t))\, R^{i^{-1}}(\xi^i(t))\, B^{i'}(\xi(t))\, K^i(\xi^i(t))\} x^i(t)]\, \mathrm{d}t\Big\} \\
\le\;& -\int_0^\infty \tilde{A} V(x^i(t), \xi^i(t))\, \mathrm{d}t \\
=\;& \mathsf{E}\int_0^\infty -\lim_{h\to 0} \frac{1}{h}\Big\{\mathsf{E}\Big[V(x^i(t+h), \xi^i(t+h))\,|x(t), \xi(t)\Big] \\
&\quad -V(x^i(t), \xi^i(t))\Big\}\, \mathrm{d}t \\
=\;& \mathsf{E}\Big\{-\lim_{h\to 0} \sum_{k=0}^{\infty} h \frac{1}{h}\Big\{\mathsf{E}\Big[V(x^i((k+1)h), \xi^i((k+1)h))\,|x(kh), \xi(kh)\Big] \\
&\quad -V(x^i(kh), \xi^i(kh))\Big\}\Big\} \\
=\;& -\mathsf{E}\Big\{V(x^i(t=\infty), \xi^i(t=\infty)\,|x_0^i, \xi^i(0)=\alpha^i\Big\} + V(x_0^i, \alpha^i). \qquad (31)
\end{aligned}$$

From inequality (26), the first term on the right hand side of (31) is zero. Thus, since the left hand side of inequality (31) is the cost to go from (x_0^i, $\xi^i(0) = \alpha^i$) for the system (2) and the right hand side equals the Lyapunov function:

$$V(x_0^i, \xi^i(0) = \alpha^i) \;=\; x^{i'}_0 K^i(\alpha^i)\, x_0^i \qquad (32)$$

the theorem is proven. □

If the performance index of the overall system is defined as a sum of the local indices then the proposed control strategy guarantees the cost given by the formula (11).

4. CENTRALIZED JLQ PROBLEM AND ITS ROBUSTNESS

In the centralized structure decision maker of the upper level has only linear model of the overall system neglecting uncertainties resulting from imprecisely known parameters of the subsystems and local environmental disturbances.

$$\begin{aligned}
\dot{x}(t) &= A(\xi(t))\, x(t) + B(\xi(t))\, u(t) \\
x(0) &= x_0. \qquad (33)
\end{aligned}$$

The quadratic criterion for the system which is a sum of the local cost functions (10) and incorporates the information about its state is defined by:

$$J = \mathsf{E}\left\{\int_0^\infty x'(t)\, Q(\xi(t))\, x(t) + u'(t)\, R(\xi(t))\, u(t)\, \mathrm{d}t\right\} \qquad (34)$$

where the cost weighting matrices $R(\xi(t))$ and $Q(\xi(t))$ are symmetric block diagonal respectively positive definite and positive semidefinite for each $\xi(t)$.

The central control law is found by minimizing (34) for the nominal model of the system (33) and has for any given $\xi(t) = \alpha$ the following form [5]:

$$u(\alpha, t) = -R^{-1}(\alpha)\, B'(\alpha)\, K(\alpha)\, x(t). \tag{35}$$

The corresponding optimal cost for the nominal model is given by (for $\xi(0) = \alpha$):

$$J^o = x_0' K(\alpha)\, x_0. \tag{36}$$

The feedback control used in the ith subsystem is the sum of the particular part of the coordinator's control (35) and the local decision maker strategy. The role of the local decision maker is to ensure robust stability and guaranteed cost in spite of the uncertainties. The local uncertainty in the subsystem is described by deterministic inequality model:

$$||e^i(\alpha^i, x^i(t), t)|| \leq f^i(\alpha^i)\, ||x^i(t)|| + d^i(\alpha^i) \tag{37}$$

where $f^i(\alpha^i)$, $d^i(\alpha^i)$ are known scalars.

The local decision maker has an information on the bound of the norm of the uncertainty and the part of the coordinator control used to control the particular subsystem. Since the bound imposed on the uncertainty is a function of the norm of the local state and the local mode these two numbers should be also known in each time moment by the local decision maker. This information is used to construct the control law v^i defined for each $\xi(t) = \alpha$ as follows:

$$v^i(\alpha, t) = \begin{cases} \frac{R^i(\alpha^i)u^i(\alpha^i,t)}{||R^i(\alpha^i)u^i(\alpha^i,t)||}\, \phi^i(\alpha^i, ||x^i(t)||) & \text{if } u^i(\alpha^i, t) \neq 0 \\ 0 & \text{if } u^i(\alpha^i, t) = 0 \end{cases} \tag{38}$$

where $\phi^i(\alpha^i, ||x^i(t)||)$ is an upper bound on the uncertainty (37) and u^i is the part of the centralized control used to drive the ith subsystem.

This control is nonlinear but it is bounded by the constraints imposed on the uncertainties.

Although the idea presented in this section is a complete contrast to the decentralized one described before. Nevertheless the same properties of the closed-loop system i.e. robust stochastic stability and guaranteed cost property are ensured and their proof follows almost the same line (see [3]). The main differences between the two approaches are in the complexity of computation performed by the specific levels and the amount of information required by decision makers in order to design their strategies. In the decentralized structure the coordinator should only be endowed in the sufficient amount of additional resources to support the action of local decision makers which should be able to solve the quite complex local system of the coupled Riccati equations.The information processed by the coordinator is very simple and could be easily obtained from the subsystems. In the centralized structure the complexity of the full order coupled Riccati equation solved by the coordinator may be really huge. On the other hand local decision makers have very simple task and should only care to compensate the effect of uncertainty disturbing locally their subsystems.

5. CONCLUSIONS

The main idea of the paper is to decompose the complex system into subsystems and to use hierarchical structure to ensure robustness in the sense of robust stability and guaranteed cost property. This purpose can be realized in to different structure. In the decentralized approach the control law minimizing the quadratic cost is decentralized while the effect of imprecisely known crosscouplings and uncertainties disturbing the subsystems is compensated by the coordinator. Although the control law depends on perfectly measurable state variables and modes but due to decentralization the local decision maker needs only to measure the local state variables. On the other hand the coordinator utilizes only an aggregated information from the subsystems in the form of the local control actions and the norm of the local state vectors. In the case of MIMO large scale system decomposed into SISO subsystems it enables to transmit only three numbers from each subsystem to the coordinator at each time t and only one number from the coordinator to each subsystem. Moreover in this case the matching conditions imposed on the uncertain crosscoupling are not restricting at all. The centralized structure leads to the huge computational effort at the coordinator level where the coupled Riccati equation system should be solved. On the other hand the coordinator must only use the nominal model of the system while a knowledge of the uncertainties is used by local decision makers to design a simple local strategy which robustifies the closed-loop system.

To compare more precisely both structures we may gather their characteristic features in the following table:

structure	*centralized*	*decentralized*
coordinator model	nominal model of the entire system	global model of uncertainty
local model of ith subsystem	local uncertainty model	local nominal model (without crosscouplings)
coordinator apriori information	nominal parameters and structure of the overall system	bounds on the uncertain parameters, disturbances and crosscouplings
local apriori information	bounds on the local uncertain parameters and disturbances	local nominal parameters and structure
current coordinator information	entire system state	local control, norm of local state, local mode
local current information	respective component of the coordinator law, norm of local state, local mode	local state and mode
coordinator control law	linear JLQ for the overall system	nonlinear control law of the general sign type based on the local control
local control law	nonlinear control law of general sign type	linear JLQ
numerical complexity at the coordinator level	huge, coupled Riccati equations dim. $n \times n \times s$	low, composition of local state norms, calculation of general sign
numerical complexity at the local level	very low, calculation of general signs	medium, coupled Riccati equations dim. $n^i \times n^i \times s^i$

The results have been obtained under the assumption that the state of the system is is measured. Nevertheless is worth noting that the state which is considered is the one of the nominal model that implies its relatively low dimension especially in the decentralized structure. On the other hand an information about the state used to robustify JLQ control law has an aggregated form for example only the norm of the state vector is needed.

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REFERENCES

[1] B. R. Barmish, M. Corless G. and Leitmann: A new class of stabilizing controllers for uncertain dynamical systems. SIAM J. Control Optim. *21* (1983), 2, 95–102.

[2] E. K. Boukas and A. Haurie: Manufacturing flow control and preventive maintenance: A stochastic control approach. IEEE Trans. Automat. Control *AC-35* (1990), 9, 1024–1031.

[3] E. K. Boukas, A. Swierniak, K. Simek and H. Yang: Robust control of complex piecewise deterministic linear systems – CJLQ problem. In: System Modelling Control 8 (E. Kacki, ed.), 1, Lodz 1995, pp. 134–138.

[4] S. S. Chang and T. K. Peng: Adaptive guaranteed cost control of systems with uncertain parameters. IEEE Trans. Automat. Control *AC-17* (1972), 474–481.

[5] Y. Ji and H. Chizek: Controllability, stability, and continuous-time Markovian jump linear quadratic control. IEEE Trans. Automat. Control *AC-35* (1990), 7, 777–788.

[6] G. Leitmann: Guaranteed asymptotic stability for some linear systems with bounded uncertainties. Trans. ASME *101* (1979), 212–216.

[7] G. Leitmann: On the efficacy of nonlinear control in uncertain linear system. ASME J. Dynam. Systems, Measurement and Control *102* (1981), 95–102.

[8] M. Mariton: Jump Linear Control Systems. Marcel-Dekker, New York 1990.

[9] D. D. Siljak: Reliable control using multiple control systems. Internat. J. Control *31* (1980), 2, 303–329.

[10] A. Swierniak and K. Simek: Guaranteed exponential stability in partly unknown large scale systems. SAMS *12* (1993), 159–165.

[11] D. D. Sworder and R. O. Rogers: An LQ-solution to a control problem associated with a solar thermal central receiver. IEEE Trans. Automat. Control *AC-28* (1983), 5, 971–978.

[12] W. M. Wonham: Random differential equations in control theory. In: Probab. Methods in Appl. Math. 2 (A. T. Bharucha-Reid, ed.), N. Y. 1970, pp. 131–212.

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