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CONTROLLABILITY OF RETARDED DYNAMICAL SYSTEMS

JERZY KLAMKA

In the paper linear abstract retarded dynamical systems defined in infinite-dimensional Hilbert spaces are considered. Using frequency-domain methods and spectral analysis for linear selfadjoint operators necessary and sufficient conditions for approximate relative controllability are formulated and proved. The method presented in the paper allows to verify approximate relative controllability for abstract retarded dynamical systems by consideration approximate controllability of suitable simplified abstract dynamical systems without delays. Moreover, as an illustrative example approximate relative controllability of retarded distributed parameter dynamical system is investigated. Presented results extend to more general class of retarded dynamical systems controllability theorems known in the literature.

1. INTRODUCTION

Controllability is one of the fundamental concept in modern mathematical control theory [1,4]. Roughly speaking, controllability generally means, that it is possible to steer dynamical system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. In the literature there are many different definitions of controllability which depend on the type of dynamical system [1,4,7,11,13–17,20,22], and [24]. For infinite dimensional dynamical systems it is necessary to distinguish between the notions of approximate and exact controllability [1,3,4,5,6,7,14,16,20,22], and [24]. It follows directly from the fact, that in infinite-dimensional spaces there exist linear subspaces which are not closed [4]. On the other hand, for retarded dynamical systems there exist two fundamental concepts of controllability; namely relative controllability and absolute controllability [1,4,7,11,13], and [17]. Therefore, for abstract retarded dynamical systems defined in infinite-dimensional spaces the following four main kinds of controllability are considered: approximate relative controllability, exact relative controllability, approximate absolute controllability, and exact absolute controllability. The present paper is devoted to a study of the approximate relative controllability for linear infinite-dimensional retarded dynamical systems defined in Hilbert spaces. For such dynamical systems direct verification of the approximate relative controllability is rather difficult task. Therefore, using frequency-domain methods [1,8,13] it is shown that approximate controllability of linear retarded dynamical system can be checked by approximate controllability criteria for suitable defined simplified infinite-dimensional dynamical system without delays. General results are then applied for verification of approximate relative controllability for distributed parameter dynamical system with one constant delay in the state variable. The results presented in the paper extend to a more general class of linear abstract retarded dynamical systems previous controllability theorems given in [1,3,4,8,11,13,16], and [22].

2. SYSTEM DESCRIPTION AND BASIC DEFINITIONS

First we shall give the basic notations and terminology used throughout present paper. Let X be separable Hilbert space. For a set $E \subset X$ symbol $Cl\,E$ denotes its closure. For a given real number $h > 0$ we shall denote by $L_2([-h,0], X)$ separable Hilbert space of all strongly measurable and square integrable functions from $[-h,0]$ into X. Let us introduce the space [1,4,9,11,13], $M_2([-h,0], X) = X \times L_2([-h,0], X)$ denoted shortly as M_2 which is also separable Hilbert space with standard scalar product

$$\begin{aligned}\langle g, f\rangle_{M_2} &= \langle g^0, f^0\rangle_X + \langle g^1, f^1\rangle_{L_2} = \langle g^0, f^0\rangle_X + \int_{-h}^{0} \langle g^1(s), f^1(s)\rangle_X \,\mathrm{d}s \\ &\qquad \text{for } f = (f^0, f^1) \in M_2,\ g = (g^0, g^1) \in M_2.\end{aligned}$$

Let $A_0 : X \supset D(A_0) \to X$ denote linear generally unbounded self adjoint and positive definite operator with dense domain $D(A_0)$ in X and compact resolvent $R(s; A_0)$ for all s in the resolvent set $\rho(A_0)$. Then operator A_0 has the following properties [2,4,22], and [24]:

1. Operator A_0 has only pure discrete point spectrum $\sigma_P(A_0)$ consisting entirely with isolated real positive eigenvalues

$$0 < s_1 < s_2 < \cdots < s_i < \ldots, \qquad \lim_{i\to\infty} s_i = +\infty.$$

   Each eigenvalue s_i has finite multiplicity $n_i < \infty$, $i = 1,2,3,\ldots$ equal to the dimensionality of the corresponding eigenmanifold.

2. The eigenvectors $x_{ik} \in D(A_0)$, for $i = 1,2,3,\ldots$, $k = 1,2,3,\ldots,n_i$, form orthonormal complete set in the separable Hilbert space X.

3. A_0 has spectral representation

$$A_0 x = \sum_{i=1}^{i=\infty} s_i \sum_{k=1}^{k=n_i} \langle x, x_{ik}\rangle_X \, x_{ik} \quad \text{for } x \in D(A_0).$$

4. Fractional power $A_0^{1/2}$ of the operator A_0 can be defined as follows

$$A_0^{1/2} x = \sum_{i=1}^{i=\infty} s_i^{1/2} \sum_{k=1}^{k=n_i} \langle x, x_{ik}\rangle_X \, x_{ik} \quad \text{for } x \in D(A_0^{1/2})$$

$$D(A_0^{1/2}) = \left\{ x \in X : \sum_{i=1}^{i=\infty} s_i \sum_{k=1}^{k=n_i} |\langle x, x_{ik} \rangle_X|^2 < \infty \right\}.$$

Operator $A_0^{1/2}$ is selfadjoint and positive-definite with dense domains in X.

5. Operators $-A_0$ and $-A_0^{1/2}$ generate analytic semigroups on X.

We shall consider linear abstract retarded dynamical control system described by functional differential equation [9-13]

$$x'(t) = -A_0\, x(t) + c\, A_0^{1/2}\, x(t-h) + \sum_{j=1}^{j=m} b_j\, u_j(t) \tag{2.1}$$

with initial conditions

$$x_0 = g^0 \in X, \quad x(t) = g^1(t) \in L_2([-h,0], X) \tag{2.2}$$

where $h > 0$ is a constant delay and $c \in R$ is a given constant,

$$b_j \in X, \quad \text{for } j = 1, 2, 3, \ldots, m.$$

It is generally assumed that admissible controls $u_j(t) \in L_2([0,\infty), R)$ for $j = 1, 2, 3, \ldots, m$. It is well known that retarded system (2.1) with initial conditions (2.2) has for $t > 0$ unique so called mild solution $x(t; g, u) \in X$ [9, 18, 19]. In dynamical system (2.1) the space of control values is finite-dimensional and control operator $B : R^m \to X$ is given by

$$Bu = \sum_{j=1}^{j=m} b_j\, u_j(t). \tag{2.3}$$

Since X is a Hilbert space, then adjoint operator $B^* : X \to R^m$ is defined as follows [13]

$$B^* x = (\langle b_1, x \rangle_X,\ \langle b_2, x \rangle_X, \ldots, \langle b_j, x \rangle_X, \ldots, \langle b_m, x \rangle_X). \tag{2.4}$$

In what follows we shall give short comments on spectral decomposition of the retarded dynamical system (2.1). The detail analysis of this problem can be found in [11, 12, 25]. First of all, for each $z \in C$ we introduce the densely defined closed linear operator

$$\Delta(z; A_0, A_0^{1/2}) = zI - c\exp(-zh)\, A_0^{1/2} \tag{2.5}$$

where I denotes the identity operator on X. The retarded resolvent set $\rho(A_0, A_0^{1/2})$ we understand as the set of all values $z \in C$ for which the operator $\Delta(z; A_0, A_0^{1/2})$ has a bounded inverse with dense domain in X. In this case $\Delta(z; A_0, A_0^{1/2})^{-1}$ is so called retarded resolvent and denoted by $R(z; A_0, A_0^{1/2})$. The complement of $\rho(A_0, A_0^{1/2})$ in the complex plane is called the retarded spectrum and is denoted by $\rho(A_0, A_0^{1/2})$. It is well known that the retarded resolvent set $\rho(A_0, A_0^{1/2})$ is open in

C and retarded resolvent $R(A_0, A_0^{1/2})$ is analytic function for $z \in \rho(A_0, A_0^{1/2})$. Let us denote by $\rho_0(A_0, A_0^{1/2})$ the connected component of the resolvent set $\rho(A_0, A_0^{1/2})$ which contains the right half-plane of the complex plane. Let $x(t; g, 0)$ for $g \in M_2([-h, 0], X)$ be mild solution of homogeneous dynamical system (2.1). Moreover, define family of linear bounded operators $S(t) : M_2 \to M_2$, for $t \geq 0$ and $g \in M_2$ by

$$S(t)\, g = (x(t; g, 0),\ x_t(s; g, 0)) \tag{2.6}$$

where

$$x_t(s; g, 0) = x(t + s; g, 0), \quad s \in [-h, 0]. \tag{2.7}$$

Then $S(t)$ is strongly continuous semigroup of linear bounded operators on M_2. Let A be the infinitesimal generator of semigroup $S(t)$. Since operator A_0 has compact resolvent then spectrum $\sigma(A)$ is pure discrete point spectrum consisting entirely with countable set of eigenvalues. In fact we have

$$\sigma(A) = \bigcup_{i=1}^{i=\infty} \sigma_i, \tag{2.8}$$

where

$$\sigma_i = \left\{ z \in C : \Delta_i = z - s_i - c\exp(-zh)\, s_i^{1/2} = 0 \right\} \quad \text{for } i = 1, 2, 3, \ldots \tag{2.9}$$

Now, we shall introduce various concepts of controllability for retarded dynamical system (2.1). It is well known that for retarded dynamical systems there exist two fundamental notions of controllability namely, related controllability and absolute controllability. In the present paper we shall concentrate on the relative controllability. Since dynamical system (2.1) is defined in infinite-dimensional space X then it is necessary to distinguish between exact relative controllability and approximate relative controllability. However, since control operator B is finite dimensional and therefore compact, then dynamical system (2.1) cannot be exactly relatively controllable for infinite-dimensional space X [21, 23]. Thus, in the sequel we shall concentrate on approximate relative controllability. First of all, let R_t and R_∞, $t > 0$ denote attainable sets given by

$$R_t = \{x(t; 0, u) \in X : u \in L_2([0, t]\, R^m)\} \quad \text{and} \quad R_\infty = \bigcup_{t>0} R_t. \tag{2.10}$$

Definition 2.1. Dynamical system (2.1) is said to be *approximately relatively controllable in time* $t > 0$ if $Cl(R_t) = X$.

Definition 2.2. Dynamical system (2.1) is said to be *approximately relatively controllable in finite time* if $Cl(R_\infty) = X$.

Several others definitions of controllability for retarded dynamical systems can be found in monographs [1] and [4].

3. APPROXIMATE CONTROLLABILITY

In this section we shall formulate and prove criteria for approximate relative controllability in finite time of retarded dynamical system (2.1). First of all we shall introduce the following notation [1,4,22]

$$B_i = \begin{vmatrix} \langle b_1, x_{i1}\rangle_X & \langle b_2, x_{i1}\rangle_X & \cdots & \langle b_m, x_{i1}\rangle_X \\ \langle b_1, x_{i2}\rangle_X & \langle b_2, x_{i2}\rangle_X & \cdots & \langle b_m, x_{i2}\rangle_X \\ \cdots & \cdots & \cdots & \cdots \\ \langle b_1, x_{in_i}\rangle_X & \langle b_2, x_{in_i}\rangle_X & \cdots & \langle b_m, x_{in_i}\rangle_X \end{vmatrix}, \quad \text{for } i = 1,2,3,\ldots \tag{3.1}$$

Let us recall modified version of necessary and sufficient condition for approximate relative controllability in finite time.

Lemma 3.1. [13] Dynamical system (2.1) is approximately relatively controllable in finite time if and only if

$$\bigcap_{z\in\rho_0(A_0,A_0^{1/2})} Ker\, B^*\, R(z; A_0, A_0^{1/2}) = \{0\}. \tag{3.2}$$

Theorem 3.1. Dynamical system (2.1) is approximately relatively controllable in finite time if and only if

$$\operatorname{rank} B_i = n_i \quad \text{for each } i = 1,2,3,\ldots \tag{3.3}$$

P r o o f. *Necessity.* By contradiction. Suppose that there exists index $i_0 \geq 1$ such that

$$B_{i_0} < n_{i_0}. \tag{3.4}$$

Therefore, since the rows of (3.1) are linearly dependent, then there exist real coefficients γ_k, $k = 1,2,3,\ldots,n_{i_0}$, $\sum_{k=1}^{k=n_{i_0}} \gamma_k^2 > 0$ such that

$$\sum_{k=1}^{k=n_{i_0}} \langle b_j, x_{i_0k}\rangle_X = \sum_{k=1}^{k=n_{i_0}} \langle b_j, \gamma_k\, x_{i_0k}\rangle_X = \left\langle b_j, \sum_{k=1}^{k=n_{i_0}} \gamma_k\, x_{i_0k} \right\rangle_X = \langle b_j, x^0\rangle_X = 0$$
$$\text{for } j = 1,2,3,\ldots,m \tag{3.5}$$

where the nonzero element $x^0 = \sum_{k=1}^{k=n_{i_0}} \gamma_k\, x_{i_0k}$. Therefore, by formulas (2.4), (2.5), (2.9) and (3.5) we immediately deduce that there exist an eigenvalue $z_0 \in \sigma_{i_0}$ and nonzero element $x^0 \in Ker\, \Delta(z_0; A_0, A_0^{1/2})$ such that

$$B^*\, x^0 = \left(\langle b_1, x^0\rangle_X, \langle b_2, x^0\rangle_X, \cdots, \langle b_m, x^0\rangle_X\right) = 0. \tag{3.6}$$

Let $z \in \rho(A_0, A_0^{1/2})$. Since operator $A_0^{1/2}$ is selfadjoint, then by formula (2.5) operator $\Delta(z; A_0, A_0^{1/2})$ is normal and moreover its inverse operator $\Delta(z; A_0, A_0^{1/2})^{-1} = R(z; A_0, A_0^{1/2})$ is also normal operator for all $z \in \rho(A_0, A_0^{1/2})$. Furthermore, let us observe, that by formulas (2.6) and (2.9) for a given $z \in \rho(A_0, A_0^{1/2})$ eigenvalues of the retarded resolvent $R(z; A_0, A_0^{1/2})$ are equal to $\Delta_i(z)^{-1} \in C$, for $i = 1,2,3,\ldots$ Therefore, for $x \in X$ we have the following equalities

$$\begin{aligned} R(z; A_0, A_0^{1/2})\, x_0 &= (zI - A_0 - c\exp(-zh)\, A_0^{1/2})^{-1}\, x_0 \\ &= (zI - A_0 - c\exp(-zh)\, A_0^{1/2})^{-1} \sum_{k=1}^{k=n_{i_0}} \gamma_k\, x_{ik} \\ &= (zI - s_{i_0} - c\exp(-z_0 h)\, s_{i_0}^{1/2})^{-1} \sum_{k=1}^{k=n_{i_0}} \gamma_k\, x_{ik} = ((\Delta_{i_0}(z_0)))^{-1}\, x_0. \end{aligned} \tag{3.7}$$

Thus, from (3.5), (3.6) and (3.7) follows that

$$B^*\, R(z; A_0, A_O^{1/2})\, x_0 = B^*\, ((\Delta_{i_0}(z_0)))^{-1}\, x_0 = ((\Delta_{i_0}(z_0)))^{-1}\, B^*\, x_0 \quad \text{for each } z \in \rho(A_0, A_0^{1/2}). \tag{3.8}$$

This contradicts (3.2) and therefore, by Lemma 3.1 dynamical system (2.1) is not approximately relatively controllable in finite time. Hence necessity follows.

Sufficiency. Since operator $-A_0$ generates an analytic semigroup $T(t)$ for $t > 0$, then (3.2) is necessary and sufficient condition for approximate controllability in any time interval of dynamical system without delays [20, 22]

$$x'(t) = -A_0\, x(t) + \sum_{j=1}^{j=m} b_j\, u_j(t). \tag{3.9}$$

From (2.10) follows that attainable sets for dynamical systems (2.1) and (3.9) are the same for $t \in [0, h]$, then by Definitions 2.1 and 2.2 follows approximate relative controllability in finite time for dynamical system (2.1). Thus Theorem 3.1 follows. □

Corollary 3.1. Suppose that all the eigenvalues s_i, $i = 1,2,3\ldots$ are simple, i.e. $n_i = 1$ for $i = 1,2,3,\ldots$ Then the dynamical system (2.1) is approximately relatively controllable in finite time interval if and only if

$$\sum_{j=1}^{j=m} \langle b_j, x_i\rangle_X^2 \neq 0 \quad \text{for } i = 1,2,3\ldots \tag{3.10}$$

Proof. From Theorem 3.1 immediately follows that for the case when multiplicities $n_i = 1$ for $i = 1,2,3,\ldots$ dynamical system (2.1) is approximately relatively controllable in finite time if and only if m-dimensional row vectors

$$B_i = |\langle b_1, x_i\rangle_X \quad \langle b_2, x_i\rangle_X \quad \cdots \quad \langle b_m, x_i\rangle_X| \neq 0 \quad \text{for } i = 1,2,3,\ldots \tag{3.11}$$

Since relations (3.10) and (3.11) are equivalent then, Corollary 3.1 immediately follows. □

Corollary 3.2. Dynamical system (2.1) is approximately relatively controllable in finite time if and only if dynamical system without delays

$$x'(t) = -A_0 + \sum_{j=1}^{j=m} b_j\, u_j(t) \tag{3.12}$$

is approximately controllable in finite time.

Proof. Comparing approximate controllability results given in [1, 4] and [22] with equalities (3.3) in Theorem 3.1 immediately follows that retarded dynamical system (2.1) is approximately relatively controllable in finite time if and only if dynamical system without delays (3.12) is approximately controllable. Hence, Corollary 3.2 follows. □

4. EXAMPLE

Let us consider retarded dynamical system with distributed parameters described by the following functional partial differential equation

$$w_t(t,y) = -w_{yyyy}(t,y) + w_{yy}(t-h,y) + \sum_{j=1}^{j=m} b_j(y)\, u_j(t) \tag{4.1}$$

defined for $t > 0,\ y \in [0, L]$, with the homogeneous boundary conditions

$$w(t,0) = w(t,L) = w_{yy}(t,0) = w_{yy}(t,L) = 0 \tag{4.2}$$

and with the following initial conditions

$$w(0,y) = g^0(y) \in L_2([0,L],\, R) = X, \quad w(t,y) = g^1(t,y) \in L_2([-h,0],\, X). \tag{4.3}$$

Moreover,

$b_j(y) = b_j \in L_2([0,L], R) = X, \quad j = 1,2,3,\ldots,m,$ are given functions,

$u_j(t) \in L_2([0,\infty), R), \quad j = 1,2,3,\ldots,m,$ are scalar control functions,

$h > 0$ is a constant delay.

Retarded linear partial differential equation (4.1) can be expressed in abstract form (2.1) by substituting $w(t,y) = x(t) \in X$ and using linear unbounded differential operator $A_0 : X \supset D(A_0) \to X$ defined as follows

$$A_0\, x = A_0\, w(y) = w_{yyyy}(y) \tag{4.4}$$

$$D(A_0) = \left\{x = w(y) \in H^4([0,L],\, R) : w(0) = w(L) = w_{yy}(0) = w_{yy}(L) = 0\right\} \tag{4.5}$$

where symbol $H^4([0,L],\, R)$ denotes fourth-order Sobolev space.

The linear unbounded differential operator A_0 besides the behaviour stated in Section 2 has the following properties [1, 4], and [22]

1. Operator A_0 is self-adjoint and positive-definite operator with dense domain $D(A_0)$ in Hilbert space X.

2. There exists a compact inverse A_0^{-1} and consequently, the resolvent $R(s;A_0)$ of A_0 is a compact operator for all $s \in \rho(A_0)$.

3. Operator A_0 has a spectral representation

$$A_0\,x = A_0\,w(y) = \sum_{i=1}^{i=\infty} s_i \langle x, x_i\rangle_X x_i = \sum_{i=1}^{i=\infty} \left(\int_0^L w(y)\,x_i(y)\,\mathrm{d}y\right) x_i(y)$$

where $s_i > 0$ and $x_i(y) \in D(A_0)$, $i = 1,2,3,\ldots$ are simple (multiplicities $n_i = 1$) eigenvalues and corresponding eigenfunctions of A_0, respectively. Moreover

$$s_i = \left(\frac{\pi i}{L}\right)^4, \quad x_i(y) = \left(\frac{2}{L}\right)^{\frac{1}{2}} \sin\left(\frac{\pi i y}{L}\right) \quad \text{for } y \in [0, L]$$

and the set $\{x_i(y),\ i = 1,2,3,\ldots\}$ forms a complete orthonormal system in X.

4. Fractional power $A_0^{1/2}$ can be defined by

$$A_0^{1/2}\,x = \sum_{i=1}^{i=\infty} s_i^{1/2} \langle x, x_i\rangle_X\, x_i, \quad \text{for } x \in D(A_0^{1/2})$$

which is also a selfadjoint and positive-definite operator with a dense domain in X.

Moreover, it should be remark that, although operator A_0 being a differential operator does not at all ensure that fractional power is also a differential operator. However, particularly for the operator $A_0^{1/2}$ we have

$$\begin{aligned} A_0^{1/2}\,x &= A_0^{1/2}\,w(y) = -w_{yy}(y) \\ D(A_0^{1/2}) &= \{x = w(y) \in X : w(0) = w(1) = 0\}\,. \end{aligned} \tag{4.6}$$

Therefore, linear unbounded differential operator A_0 defined by the formulas (4.4) and (4.5) satisfies all the assumptions stated in Section 3. Hence linear retarded partial differential equation (4.1) has the following abstract representation

$$x'(t) = -A_0\,x(t) - A_0^{1/2}\,x(t-h) + \sum_{j=1}^{j=m} b_j\,u_j(t). \tag{4.7}$$

Therefore, using general results stated in Section 3 it is possible to formulate necessary and sufficient condition for approximate relative controllability of retarded dynamical system (4.1).

Theorem 4.1. Dynamical system (4.1) is approximately relatively controllable in finite time if and only if

$$\sum_{j=1}^{j=m} \left(\int_0^L \sqrt{\frac{2}{L}}\, b_j(y) \sin\left(\frac{\pi i y}{L}\right) \mathrm{d}y \right)^2 \neq 0 \quad \text{for } i = 1, 2, 3, \ldots \tag{4.8}$$

Proof. Let us observe, that retarded dynamical system (4.1) satisfies all the assumptions of Corollary 3.1. Therefore, taking into account analytic formula for the eigenvectors $x_i(y) \in L_2([0, L], R)$, $i = 1, 2, 3, \ldots$ and the form of scalar product in Hilbert space $L_2([0, L], R)$ from relation (3.1) we directly obtain inequalities (4.6). Hence, Theorem 4.1 immediately follows. □

5. FINAL REMARKS

In the present paper some controllability problems for linear abstract retarded dynamical systems have been considered. Using frequency-domain methods and spectral analysis of linear unbounded selfadjoint operators necessary and sufficient conditions for approximate relative controllability in finite time have been formulated and proved. These conditions allows us to investigate approximate relative controllability in finite time for linear abstract retarded dynamical systems by checking approximate controllability of abstract dynamical systems without delays. The presented results can be extended to cover more general types of linear abstract retarded dynamical systems.

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Prof. Dr. Jerzy Klamka, Institute of Automation, Technical University, ul. Akademicka 16, 44-100 Gliwice. Poland.