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ANTICIPATION IN DISCRETE–TIME LQ CONTROL II: Closed–Loop Control

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Following the first part of the work, this second one deals with anticipating LQ discrete-time control realized in the feedback SISO structure. Again the investigation is based on the polynomial technique the necessary survey of which can be found in the first part.

II.1. INTRODUCTION

Closed-loop structures are usually applied to stabilize and control dynamic systems and processes. A control signal U is generated by a controller (control algorithm) which operates on so far available values of the measurable process magnitudes. As a rule, the only error signal $E = W_r - Y$ enters the controller C in the usual feedback structure shown in Fig. II.1.

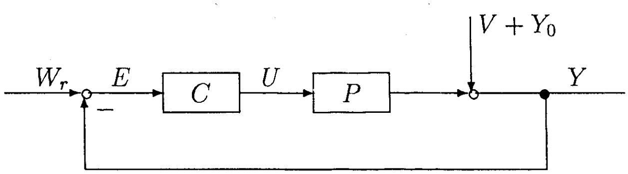

Fig. II.1.

A controlled process P is subjected to possible load disturbances as well as initial nonzero conditions affecting the output Y as V and Y_0, respectively.

The control algorithm C minimizing the performance index (I.3.1) should be determined in discrete-time, closed-loop LQ control. The external signals W_r, V and Y_0 are supposed to be deterministic; possible random components of them are reduced by feedback and their characteristics are not taken into account for the design.

Many works have dealt with the algebraic input-output approch to LQ and LQG feedback control during recent years. Basic and general results for MIMO systems can be found in [3], various types of SISO problems have been treated in [1].

This contribution is based on the known results which are for deterministic feedback control of reachable as well as observable, strictly proper SISO processes summarized in the next section. The own anticipation problem is then solved in Section 3 and the illustrating example is solved at the end.

II.2. STANDARD LQ FEEDBACK CONTROL

Returning to Fig. II.1 we assume that

$$P = \frac{b}{a}; \quad a, b \text{ coprime}, \quad a = a^c \text{ but } b = d^\beta b^c, \; \beta > 0, \tag{II.2.1}$$

and

$$W = W_r - Y_0 - V = \frac{f}{h}; \quad h, f \text{ coprime}, \quad h = h^c, \tag{II.2.2}$$

i.e., P is realized as strictly causal, reachable and observable, discrete-time system and a (generalized) reference is a causal sequence.

A controller

$$C = \frac{m}{n}; \quad (n, m)^- \sim 1, \; n = n^c, \tag{II.2.3}$$

is assumed and sought.

Generally, the minimum $\deg z$ solution $m, n, z, \deg z < \rho$, of the coupled equations

$$d^\rho s_* m + a h_a z = d^\rho b_* \psi p \tag{II.2.4}$$

and

$$d^\rho s_* n - b h_a z = d^\rho a_* \varphi p \tag{II.2.5}$$

solves the LQ problem, where $\rho = \max(\deg a, \deg b)$, $s = s^+$ follows from (I.4.7)

$$s\, s_* = a \varphi a_* + b \psi b_* \tag{II.2.6}$$

and (as in I.4.5)

$$h_a = \frac{h}{(a, h)} \quad \text{and} \quad a_h = \frac{a}{(a, h)}. \tag{II.2.7}$$

Moreover the stable polynomial

$$p = a_h^+ \widetilde{a_h^=} f^+ \widetilde{f^=} \tag{II.2.8}$$

occurs in the equations (II.2.4,5). The resulting errror and control signals are

$$E = \frac{a_h f n}{h_a s p} \quad \text{and} \quad U = \frac{a_h f m}{h_a s p}, \tag{II.2.9}$$

respectively.

Since the possible factor $p^0 = \pm a_h^0 f^0 \not\sim 1$ has been excluded from p the problem become solvable always if and only if $h_a \sim h_a^+$. The optimal controller (II.2.3) is unique.

Combining the equation (2.4) and (2.5) the so-called "implied" equation for the closed-loop pseudocharacteristic polynomial

$$c = an + bm = sp \tag{II.2.10}$$

is obtained.

If $(d^\rho s_*, a) \sim 1$, the only equation (2.4) instead of the couple may be solved for $\min \deg z$, $\deg z < \rho$, to obtain the optimal m. The remaining n then follows from (2.10) (cf. [2]).

II.3. ANTICIPATION IN LQ CLOSED-LOOP CONTROL

Let us assume that the external signals in Fig. II.1 may be determined and generated before they really occur, say ν steps in advance. Then feedback control can be improved through the additional feedforward paths according to Fig. II.2. Feedforward controllers C_W and C_V operate on signals which are constructed starting at time $-\nu T$.

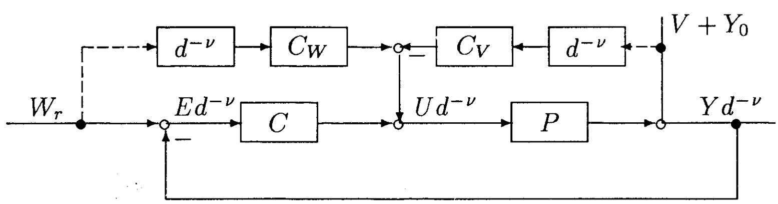

Fig. II.2.

The equivalent block diagram in Fig. II.3 may be considered if the problem is restricted to the case $C_V = C_W$ and time steps are numbered by zero at time $-\nu T$.

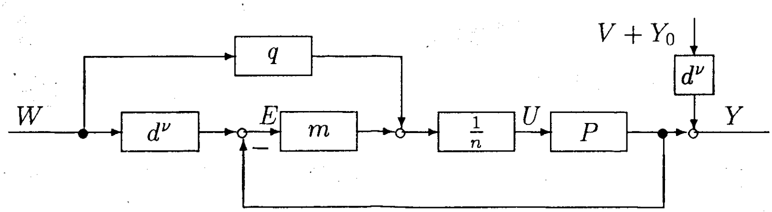

Fig. II.3.

The entire control algorithm realized in one-system fashion is described in the form

$$nU = mE + qW \tag{II.3.1}$$

where W stands in (2.2). The using (2.1), (2.2) and (3.1) yields

$$E = Wd^\nu - bMW \tag{II.3.2}$$

and

$$U = aMW \tag{II.3.3}$$

where

$$M = \frac{md^\nu + q}{c} \tag{II.3.4}$$

with c given by (2.10).

Then the optimal solution of LQ feedback discrete-time control with anticipation is given by the following theorem.

Theorem 2. Given a process (2.1) subjected to the equivalent input (2.2), LQ closed-loop control minimizing the performance index (I.3.1) and using ν-steps anticipation results in the control algorithm (3.1), where polynomials n, m and $q = aq_0$ along with z satisfy the equations

$$d^{\rho+\nu} s_* n - d^\rho s_* b q_0 - b h_a z = d^{\rho+\nu} a_* \varphi p \tag{II.3.5}$$

and

$$d^{\rho+\nu} s_* m + d^\rho s_* a q_0 + a h_a z = d^{\rho+\nu} b_* \psi p \tag{II.3.6}$$

with the minimum $\deg z$.

In (3.5) and (3.6) there is s the stable polynomial following from (2.6), a_h and h_a stand in (2.7), $\rho = \max(\deg a,\, \deg b)$ and p stands in (2.8).

The resulting error signal

$$E = \frac{a_h f(nd^\nu - bq_0)}{h_a s p} \tag{II.3.7}$$

and the control signal

$$U = \frac{a_h f(md^\nu + aq_0)}{h_a s p} \tag{II.3.8}$$

are unique while the optimal controller (3.1) is not. The problem is solvable if and only if $h_a \sim h_a^+$.

Proof.

1. It has been shown in the first part that $s = s^+$.

2. To investigate solvability of the equations (3.5) and (3.6) we write them in the form

$$C[n; m; q_0; z]^T = D$$

where

$$C = \begin{bmatrix} d^{\rho+\nu} s_* & 0 & -d^\rho s_* b & -b h_a \\ 0 & d^{\rho+\nu} s_* & d^\rho s_* a & a h_a \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} d^{\rho+\nu} a_* \varphi p \\ d^{\rho+\nu} b_* \psi p \end{bmatrix}.$$

The equations are solvable if and only if the greatest common divisors (GCD) of all nonzero minors of C and $[C; D]$ are identical. Finding the respective GCD of nonzero first and second order minors of C to be

$$g_1 = (d^\rho s_*, h_a) \quad \text{and} \quad g_2 = d^{\rho+\nu} s_* g_1$$

and the respective GCD of nonzero first and second order minors of $[C; D]$

$$g_3 = (g_1, d^{\rho+\nu} p) \quad \text{and} \quad g_4 = d^{\rho+\nu} s_* g_3$$

we obtain $g_3 = g_1 \sim 1$ and $g_4 = g_2 = d^{\rho+\nu} s_*$ if $h_a \sim h_a^+$ since $d^\rho s_*$ is unstable polynomial. Therefore (3.5) and (3.6) are solvable if $h_a \sim h_a^+$.

3. Considering the expression (I.3.1) and using gradually (3.2), (3.3) and (2.2) we get

$$J = \psi E_* E + \psi U_* U =$$

$$= \frac{f_* f}{h_* h}(\psi - b_* M_* d^\nu \psi - bMd^{-\nu}\psi + s_* s M_* M) = \frac{f_* f}{h_* h} J_0. \tag{II.3.9}$$

Multiplying (3.9) by $\frac{a_{h*} a_h}{a_{h*} a_h} = 1$ and applying (2.8) yields

$$J = \frac{f_* f a_{h*} a_h}{h_* h a_{h*} a_h} J_0 = p_*^0 p^0 \frac{p_* p}{h_* h a_{h*} a_h} J_0 =$$

$$= p_*^0 p^0 \left[\psi \frac{p_* p}{h_* h a_{h*} a_h} - \psi^2 \frac{b_* b p_* p}{s_* s h_* h a_{h*} a_h} + \right.$$

$$\left. + \left(\frac{b_* p \psi}{s_* h a_h} - \frac{sp}{h a_h} M d^{-\nu} \right)_* \left(\frac{b_* p \psi}{s_* h a_h} - \frac{sp}{h a_h} M d^{-\nu} \right) \right] = J_A + J_B$$

where $p^0 = \pm a_h^0 f^0$ and $pp^0 = a_h^* f^*$.

Obviously

$$J_A = \psi p_*^0 p^0 \frac{p_* p (s_* s - \psi b_* b)}{s_* s h_* h a_{h*} a_h} = \frac{p_*^0 p^0 \varphi \psi p_* p}{s_* s h_{a*} h_a}$$

does not dep nd on M and is stable and

$$J_B = p_*^0 p^0 \left(\frac{b_* p \psi}{s_* h a_h} - \frac{sp}{h a_h} M d^{-\nu} \right)_* \left(\frac{b_* p \psi}{s_* h a_h} - \frac{sp}{h a_h} M d^{-\nu} \right) =$$

$$= p_*^0 p^0 \left(\frac{b_* p \psi d^\nu}{s_* h a_h} - \frac{sp}{h a_h} M \right)_* \left(\frac{b_* p \psi d^\nu}{s_* h a_h} - \frac{sp}{h a_h} M \right). \tag{II.3.10}$$

Using the decomposition

$$\frac{b_* p \psi d^\nu}{s_* h a_h} = \frac{d^{\rho+\nu} b_* p \psi}{d^\rho s_* h a_h} = \frac{d^\rho s_* y + h a_h z}{d^\rho s_* h a_h} = \frac{y}{h a_h} + \frac{z}{d^\rho s_*} \tag{II.3.11}$$

in (3.10) we get

$$J_B = p_*^0 p^0 \left(X + \frac{z}{d^\rho s_*} \right)_* \left(X + \frac{z}{d^\rho s_*} \right) = p_*^0 p^0 \bar{J}_B$$

where

$$X = \frac{y}{h a_h} - \frac{sp}{h a_h} M. \tag{II.3.12}$$

Let us assume $p^0 = 1$ at first, i.e., $a_h(d)$ and $f(d)$ having no zeros on the unit circle in d plane. It has been proved in [3] and applied in the proof of Theorem 1 in the first part that $\langle \bar{J}_B \rangle$ obtains its minimum for $X = 0$ provided $\deg z < \rho$. Hence $X = 0$ being optimal for all zeros of a_h and f inside as well outside the unit circle must be also optimal for $p^0 \neq 1$, i.e., if there are zeros just on the unit circle, seeing that both X and $\langle \bar{J}_B(X) \rangle$ are continuous regarding the zeroes.

Then

$$M = \frac{y}{sp} \tag{II.3.13}$$

follows from (3.12) and

$$c = sp \quad \text{and} \quad y = md^\nu + q \tag{II.3.14}$$

from the comparison (3.13) and (3.4).

If M given by (3.13) and the relation

$$d^{\rho+\nu} b_* p \psi - a h_a z = d^\rho s_* y \tag{II.3.15}$$

following from the decomposition (3.11) are substituted into (3.2) we have

$$E = \frac{f d^\nu}{h} - \frac{b f y}{sph} = \frac{d^\rho f (s_* spd^\nu - s_* by)}{hd^\rho s_* sp} = \frac{af(d^{\rho+\nu} a_* \varphi p + b h_a z)}{hd^\rho s_* sp} = \frac{a_h f x}{h_a sp}$$

where the denotation

$$d^{\rho+\nu} a_* \varphi p + b h_a z = d^\rho s_* x \tag{II.3.16}$$

has been introduced.

Now adding (3.15) multiplied by b and (3.16) by a yields

$$d^\rho s_* (ax + by) = d^{\rho+\nu} s_* sp \quad \text{or} \quad ax + by = spd^\nu = (an + bm)d^\nu .$$

Hence using (3.14) and seeing that x must be a polynomial the relations

$$y = md^\nu + aq_0 \quad \text{and} \quad x = nd^\nu - bq_0 \tag{II.3.17}$$

have been found.

The unique minimum $\deg z$ solution x, y, z of the equations (3.15) and (3.16) results in the unique optimal signals E and U. Using the obtained x and y in the equations (3.17) and solving them generally by

$$m = m_p - at, \quad n = n_p + bt \quad \text{and} \quad q_0 = q_{0p} + d^\nu t$$

where m_p, n_p and q_{0p} is a particular solution and t an arbitrary polynomial, yields the optimal control algorithm (3.1) which therefore is not unique.

Combining (3.15) to (3.17) we obtain the equations (3.5) and (3.6). Seeing that the sequences (3.7) and (3.8) must be stable the problem is solvable for $h_a \sim h_a^+$ only.

II.4. EXAMPLE

One small example is given to illustrate the described approach.

Given

$$P = \frac{b}{a} = \frac{d}{1-d}, \quad W_r = \frac{1}{1-d} \quad \text{and} \quad V = Y_0 = 0.$$

Let us determine the optimal LQ control algorithm for $\psi = \varphi = 1$ assuming W_r is known ν steps in advance, alternatively $\nu = 0; 1; 2$.

At first we find that the problem is solvable and

$$W = \frac{f}{h} = W_r, \quad a_h = h_a = 1, \quad s = 1.618 - 0.618d, \quad \rho = 1 \quad \text{and} \quad p = 1.$$

The equations (3.5) and (3.6) are

$$(-0.618 + 1.618d)\, d^\nu n - (-0.618 + 1.618d)\, dq_0 - dz = (-1+d)d^\nu$$

and

$$(-0.618 + 1.618d)\, d^\nu m + (-0.618 + 1.618d)\,(1-d)\, q_0 + (1-d)\, z = d^\nu$$

The way solving the equations (3.15) and (3.16) at first and then (3.17) separately will be shown.

1. For $\nu = 0$ (control without anticipation) (3.15) and (3.16) are

$$(-0.618 + 1.618d)\, y + (1-d)\, z = 1$$

and

$$(-0.618 + 1.618d)\, x - dz = -1 + d$$

which solved for the minimum deg$z$ by $x = 1.618$, $y = 1$ and $z = 1.618$.

Hence

$$E = \frac{1.618}{1.618 - 0.618d}, \quad U = \frac{1}{1.618 - 0.618d} \quad \text{and} \quad \vartheta = 1.618.$$

The equations (3.17)

$$m + (1-d)\, q_0 = 1 \quad \text{and} \quad n - dq_0 = 1.618$$

are solved by $m = 1-(1-d)\,t$, $n = 1.618 + dt$ and $q_0 = t$. Obviously the feedforward path can be ignored here using $t = 0$ and consequently $q = aq_0 = 0$.

3. For $\nu = 1$ we solve the equations

$$(-0.618 + 1.618d)\, y + (1-d)\, z = d$$

and

$$(-0.618 + 1.618d)\, x - dz = (-1+d)\, d$$

for the minimum deg z with the resulting $x = 1.618d$, $y = 1$ and $z = 0.618$.

Hence

$$E = \frac{0.618d}{1.618 - 0.618d}, \qquad U = \frac{1}{1.618 - 0.618d} \quad \text{and} \quad \vartheta = 0.618.$$

Then the equations

$$md + (1-d)\,q_0 = 1 \quad \text{and} \quad nd - dq_0 = 0.618d$$

are solved to reach $m = 1-(1-d)\,t$, $n = 1.618 + dt$ and $q_0 = 1 + dt$ and $m = 1$, $n = 1.618$ and $q_0 = 1$ for the simplest controller.

3. Finally for $\nu = 2$ the equations (3.15) and (3.16)

$$(-0.618 + 1.618d)\,y + (1-d)\,z = d^2$$

and

$$(-0.618 + 1.618d)\,x - dz = (-1+d)\,d^2$$

have the minimum deg z solution $x = -0.382d + 0.618d^2$, $y = 0.382 + 0.618d$ and $z = 0.236$. Then

$$E = \frac{-0.382d + 0.618d^2}{1.618 - 0.618d}, \qquad U = \frac{0.382 + 0.618d}{1.618 - 0.618d} \quad \text{and} \quad \vartheta = 0.472.$$

Writing (3.17)

$$md^2 + (1-d)\,q_0 = 0.382 + 0.618d \quad \text{and} \quad nd^2 - dq_0 = -0.382d + 0.618d^2$$

and solving then generally yields $m = 1-(1-d)t$, $n = 1.618 + dt$ and $q_0 = 0.382 + d + d^2t$ with the simplest $(t = 0)$ $m = 1, n = 1.618, q_0 = 0.382 + d$.

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