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A SIMPLE ROBUST ESTIMATOR OF CORRELATIONS FOR GAUSSIAN STATIONARY RANDOM SEQUENCES

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The paper deals with a robust estimation of correlations for a Gaussian stationary sequence. The investigated estimator is based on signs of the original series and is easy to compute. The asymptotic normality of the estimator and the boundedness of the influence functional under the assumption of the pure replacement outlier model has been proved. A consistent estimate of the variance of the asymptotic distribution has been suggested.

1. INTRODUCTION

Suppose that $\{X_i\}_{i=-\infty}^{\infty}$ is a zero mean stationary Gaussian discrete process. Let $\{\rho_j\}_{j=0}^{\infty}$ be the correlation function of $\{X_i\}_{i=-\infty}^{\infty}$, i.e.,

$$\rho_j = \frac{E(X_1 X_{1+j})}{E(X_1^2)}, \qquad j = 1, 2, \dots .$$

Estimation of the correlation function has been studied by various authors. A simple estimator has been suggested in Hurt [2]. Put

$$Z_i = \operatorname{sign} X_i, \quad T_{ij} = Z_i Z_{i+j}, \quad i = \dots, -1, 0, 1, \dots, \quad j = 1, 2, \dots .$$

Then the proposed estimator of ρ_j based on the observations $X_1, X_2, \dots, X_n$ is

$$\widehat{\rho}_j = \sin\left(\frac{\pi}{2}\overline{T}_j\right), \tag{1}$$

where

$$\overline{T}_j = \frac{1}{n-j}\sum_{i=1}^{n-j} T_{ij}.$$

Recall that $E(T_{1j}) = \frac{2}{\pi}\arcsin\rho_j$, so that (1) is a natural estimator based on $T_{1j}, \dots, T_{n-j,j}$.

Kedem [3] considered an equivalent estimator ρ_j^* which is constructed with the sequence $\{Y_t\}_{t=-\infty}^{\infty}$ defined by clipping X_t at level zero:

$$Y_t = \begin{cases} 1 & X_t \ge 0, \\ 0 & X_t < 0. \end{cases}$$

Then $\rho_j^* = \sin[\pi(\widehat{\lambda}_j - \frac{1}{2})]$ where

$$\widehat{\lambda}_j = 2\overline{U}_j - \frac{2n\overline{Y}}{n-j} + 1$$

$$\overline{U}_j = \frac{1}{n-j} \sum_{t=j+1}^{n} Y_t Y_{t-j}$$

$$\overline{Y} = \frac{1}{n} \sum_{t=1}^{n} Y_t.$$

The sequence Z_t is equivalent to the binary series $2(Y_t - \frac{1}{2})$. It follows that $\widehat{\lambda}_j$ is asymptotically equal to the random variable $\frac{1}{2}(\overline{T}_j + 1)$.

In Kedem [3] it is shown how the estimator $\widehat{\lambda}_j$ (or $\overline{T}_j$) can be used to construct estimates for the parameters $\phi_1, \ldots, \phi_k$ of an autoregressive process of order k (AR(k)-process). For example, $\widehat{\phi} = \sin[\pi(\widehat{\lambda}_1 - \frac{1}{2})] = \widehat{\rho}_1$ is an estimator for the parameter ϕ of an AR(1)-process $X_t - \phi X_{t-1} = \varepsilon_t$, where ε_t, $t \in (-\infty, \infty)$, are independent Gaussian variables with $E(\varepsilon_t) = 0$, $\operatorname{var} \varepsilon_t = \sigma^2$.

For large samples the estimators $\widehat{\rho}_j$ or $\widehat{\lambda}_j$ are more economical than the usual maximum likelihood estimators

$$\widetilde{\rho}_j = \frac{\sum_{t=1}^{n-j} X_t X_{t-j}}{\sum_{t=1}^{n-j} X_t^2}, \qquad \widetilde{\lambda}_j = \frac{1}{\pi} \arcsin \widetilde{\rho}_j + \frac{1}{2}$$

since their computation is very fast. That's why they can be used as simple initial estimates for more complicated estimation procedures. The main advantage is their robustness which is considered in Section 3 by means of the influence functional. Martin and Yohai [5] mentioned that the usual estimators $\widetilde{\rho}_j$ are not always robust in this sense.

Since for fixed j, $\{T_{i,j}\}_{i=-\infty}^{\infty}$, is a stationary random process, the covariance $\operatorname{cov}(T_{i,j}, T_{i+k,j})$ depend on k only. Denote

$$R_j(k) = \operatorname{cov}(T_{i,j}, T_{i+k,j}), \quad k = 0, 1, \ldots$$

and

$$\sigma_{T_j}^2 = R_j(0) + 2\sum_{k=1}^{\infty} R_j(k).$$

2. ASYMPTOTIC NORMALITY

Theorem 1. (Kedem [3], Corollary 7.2.) Let $\{X_t\}_{t=-\infty}^{\infty}$ be a zero mean stationary Gaussian process. If $\sum_{j=-\infty}^{\infty} |\, E(X_1 X_{1+j}) \,| < \infty$ then

$$\sqrt{n}(\widehat{\rho}_j - \rho_j) \xrightarrow{\mathcal{D}} N\left(0, \frac{1}{4}\pi^2(1-\rho_j^2)\,\sigma_{T_j}^2\right) \quad \text{as} \quad n \to \infty.$$

A sufficient condition for the assumption $\sum_{j=-\infty}^{\infty} |\, E(X_1 X_{1+j}) \,| < \infty$ is given in Hurt [2] (Theorem 1) which follows.

Theorem 2. Suppose that the spectral density $f(\lambda)$ of a zero mean stationary Gaussian sequence is twice differentiable and that there exist constants m, M_2 such that $f(\lambda) \geq m > 0$, $|f''(\lambda)| \leq M_2$ for $\lambda \in [-\pi, \pi]$. Then for fixed $j \geq 1$,

$$\sqrt{n}(\widehat{\rho}_j - \rho_j) \xrightarrow{\mathcal{D}} N\left(0, \frac{1}{4}\pi^2(1-\rho_j^2)\,\sigma_{T_j}^2\right) \quad \text{as} \quad n \to \infty.$$

Some remarks on the method of proof applied in [2] are given in Section 4. The conditions of Theorem 2 above are fulfilled e. g. for autoregressive stationary random sequences. (See Hurt [2] for details.)

What about the efficiency of the estimator $\widehat{\rho}_j$ with respect to $\widetilde{\rho}_j$? Denote

$$EFF(\widehat{\rho}_j) = \frac{\lim\limits_{n\to\infty} n \operatorname{var} \widetilde{\rho}_j}{n \operatorname{var} \widehat{\rho}_j}.$$

As an example we consider a first order autoregression with parameter $\phi = 0.6$ and the estimators $\widehat{\rho}_1$, $\widehat{\rho}_2$, $\widehat{\rho}_3$. Then we have

$$\lim_{n\to\infty} n \operatorname{var} \widetilde{\rho}_j = \frac{(1+\phi^2)(1-\phi^{2j})}{1-\phi^2} - 2j\phi^{2j}.$$

For estimating the variance of $\widehat{\rho}_j$, $j = 1,2,3$ we refer to a simulation result in Hurt [2], Table 1 $(n = 20)$:

Table 1.

j	$\lim n \operatorname{var} \widetilde{\rho}_j$	$n \operatorname{var} \widehat{\rho}_j$	$EFF(\widehat{\rho}_j)$
1	0.64	1.14	0.5614
2	1.3312	1.82	0.7314
3	1.7459	2.56	0.6820

Other experimental results in Kedem [3] concerning the efficiency of the parametric estimators in a second order autoregression support that $EFF(\widehat{\rho}_j)$ is approximately 0.5. The binary series should be roughly twice longer as the original series to obtain a similar accuracy.

Theorem 3. In addition to the assumptions of Theorem 1 let us suppose that $\widehat{\sigma}_{T_j}^2$ is a consistent estimator of $\sigma_{T_j}^2$. Then

$$\frac{2\sqrt{n}\,(\widehat{\rho}_j - \rho_j)}{\pi\sqrt{(1-\widehat{\rho}_j^2)\widehat{\sigma}_{T_j}^2}} \xrightarrow{\mathcal{D}} N(0,1) \quad \text{as} \quad n \to \infty.$$

Theorem 4. Let $\{X_t\}_{t=-\infty}^{\infty}$ be a Gaussian stationary random sequence satisfying the strong mixing condition with coefficients $\alpha_X(\tau)$. Suppose $\sum_{\tau=1}^{\infty} \alpha_X^{\gamma}(\tau) < \infty$ for some $\frac{1}{2} < \gamma < 1$. For j fixed natural let $\widehat{\sigma}_{T_j}^2$ be defined by

$$\widehat{\sigma}_{T_j}^2 = \frac{1}{n-j} \sum_{t=1}^{n-j} (T_{tj} - \overline{T}_j)^2 + 2 \sum_{k=2}^{m} \frac{1}{\left[\frac{n-j}{k}\right]} \sum_{l=0}^{\left[\frac{n-j}{k}\right]-1} (T_{lk+1,j} T_{(l+1)\,k,j} - \overline{T}_j^2), \qquad (2)$$

where $m = o(\sqrt[3]{n-j})$. Then $\widehat{\sigma}_{T_j}^2$ converges to $\sigma_{T_j}^2$ in probability as $n \to \infty$.

3. ROBUSTNESS PROPERTIES OF THE ESTIMATOR

To investigate the robustness properties of statistics from stationary processes several concepts have been developed (see e.g. [4], [5]). Here we consider the *influence functional (IF)* defined by Martin and Yohai [5]. It is an indicator for the sensitivity of an estimator against departures from the basic model. Assume that the observable contaminated process $\{y_i^{\gamma}\}_{i=-\infty}^{\infty}$ takes the form

$$y_i^{\gamma} = (1 - v_i^{\gamma})\, x_i + v_i^{\gamma} w_i .$$

Let $\{v_i^{\gamma}\}_{i=-\infty}^{\infty}$ be a 0-1 process with

$$P(v_i^{\gamma} = 1) = \gamma + o(\gamma) \quad \text{as} \quad \gamma \to 0, \qquad (3)$$

and let $\{x_i\}_{i=-\infty}^{\infty}$, $\{w_i\}_{i=-\infty}^{\infty}$ be stationary ergodic processes with the associated ergodic measures μ_v^{γ}, μ_x, and μ_w on $(\mathbb{R}^{\infty}, \mathbb{B}^{\infty})$, respectively. In the pure replacement model $\{v_i^{\gamma}\}$, $\{x_i\}$, and $\{w_i\}$ are mutually independent processes, i.e., $\mu_{vxw}^{\gamma} = \mu_v^{\gamma} \mu_x \mu_w$. Martin and Yohai [5] defined the *IF* of a functional $T = T(\mu)$ as

$$IF(\mu_w, T, \mu_y^{\gamma}) = \lim_{\gamma \to 0} \frac{T(\mu_y^{\gamma}) - T(\mu_y^0)}{\gamma}$$

provided that the limit exists.

If we have a contaminated process $\{y_i^{\gamma}\}_{i=-\infty}^{\infty}$ then the estimator $t = \widehat{\rho}_j$ given by (1) can be considered as a solution to the equation

$$\sum_{i=1}^{n} \widetilde{\psi}_i(y_i^{\gamma}, \ldots, y_1^{\gamma}, t) = 0, \qquad (4)$$

where

$$\widetilde{\psi}_i(y_i^{\gamma}, \ldots, y_1^{\gamma}, t) = \begin{cases} 0 & i \le j, \\ \operatorname{sign} y_i^{\gamma} \operatorname{sign} y_{i-j}^{\gamma} - \frac{2}{\pi} \arcsin t & i > j. \end{cases}$$

Denote $\underline{x}_1 = (x_1, x_0, x_{-1}, \ldots)$. From the ergodic theorem it follows that

$$P\text{-}\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \widetilde{\psi}_i(x_i^{\gamma}, \ldots, x_1^{\gamma}, t) = E\, \psi(\underline{x}_1, t),$$

where $\psi(\underline{x}_1, t) = \operatorname{sign} x_1 \operatorname{sign} x_{1-j} - \frac{2}{\pi} \arcsin t$. Hence $P\text{-}\lim_{n\to\infty} \widehat{\rho}_j = T(\mu_y^\gamma)$ and $\lim_{\gamma\to 0} T(\mu_y^\gamma) = T(\mu_x) = \rho_j =: T_0$. Put $m(\gamma, t) = E\,\psi(\underline{y}_1^\gamma, t)$. Assume that $D(\gamma, t) = \frac{\partial}{\partial t} m(\gamma, t)$ exists for $0 \le \gamma \le \varepsilon$, $t = T_0$, and $D(0, T_0)$ is greater than zero. Then it has been proved in [5] that

$$IF(\mu_w, T, \mu_y^\gamma) = -\frac{1}{D(0, T_0)} \lim_{\gamma\to 0} \frac{1}{\gamma} \left[m(\gamma, T_0) - m(0, T_0)\right].$$

We consider two cases:

(**A**) – isolated outliers, when v_i^γ are i.i.d. random variables as in (3),

(**B**) – patchy outliers of length $\widetilde{k}$ when the random variables v_i^γ are assumed to be dependent in the following way:

$$v_i^\gamma = \begin{cases} 1 & \text{if } \ \widetilde{v}_{i-k} = 1 \ \text{ for any } \ k = 0, 1, \ldots, \widetilde{k} - 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\widetilde{v}_i$ are i.i.d. random variables possessing a binomial distribution with parameters 1 and $p = \gamma/\widetilde{k}$, $\widetilde{k} \in \{1, 2, \ldots\}$.

Theorem 5. Let $\{x_t\}_{t=-\infty}^{\infty}$ be a zero mean Gaussian stationary random sequence and assume that the contaminated process $\{y_t^\gamma\}_{t=-\infty}^{\infty}$ fulfills the pure replacement model. Suppose that T is a $\widetilde{\psi}$-estimate defined by (4). Then

$$IF(\mu_w, T, \mu_y^\gamma) = \begin{cases} -\frac{2}{D(0,T_0)} \left(-\frac{2}{\pi} \arcsin T_0\right) & \text{for case (A) and case (B), } \widetilde{k} \le j \\ -\frac{1}{D(0,T_0)} \Big[(1 - \frac{j}{\widetilde{k}}) E\left(\operatorname{sign} w_1 \operatorname{sign} w_{1-j}\right) & \\ \qquad -(1 + \frac{j}{\widetilde{k}}) \frac{2}{\pi} \arcsin T_0\Big] & \text{for case (B), } \widetilde{k} > j, \end{cases}$$

where $D(0, T_0) = -\frac{2}{\pi\sqrt{1 - T_0^2}}$.

Hence the influence functional remains bounded in every case.

4. PROOFS

Proof of Theorem 2.

From the central limit theorem for a strictly stationary random sequence of uniformly bounded random variables (see Theorem 3 in Hurt [2], e. g.) it follows that

$$\sqrt{n}(\overline{T}_j - E\,T_{1j}) \xrightarrow{\mathcal{D}} N(0, \sigma_{T_j}^2) \quad \text{as} \quad n \to \infty$$

(Theorem 4' in Hurt [2]). Put $g(t) = \sin\left(\frac{1}{2}\pi t\right)$ and $\theta = \frac{2}{\pi} \arcsin \rho_j$ in (6a.2.5), Rao [6]. We have $g'(\theta) = \frac{\pi}{2} \cos\left(\arcsin \rho_j\right)$ so that the asymptotic variance is

$$[g'(\theta)]^2 \sigma_{T_j}^2 = \frac{\pi^2}{4} \cos^2(\arcsin \rho_j)\, \sigma_{T_j}^2 = \frac{\pi^2}{4} (1 - \rho_j^2)\, \sigma_{T_j}^2.$$

The assertion now follows.

Proof of Theorem 3.

Theorem 3 follows from Theorem 1 and the limit theorem 6.14.1 in Fisz [1].

Proof of Theorem 4.

(i) Applying the Chebyschev inequality we obtain for any $\varepsilon > 0$

$$P(\,|\widehat{\sigma}^2_{T_j} - \sigma^2_{T_j}| \geq \varepsilon) \leq \frac{1}{\varepsilon^2} E\,|\widehat{\sigma}^2_{T_j} - \sigma^2_{T_j}|^2 \,.$$

For j fixed, $\{T_{tj}\}_{t=-\infty}^{\infty}$ is a stationary strong mixing sequence with coefficients $\alpha_{T_j}(\tau + j) \leq \alpha_X(\tau)$ (see Lemma 5 in Hurt [2]). From the pointwise ergodic theorem (see Stout [7]) it follows that $P\text{-}\lim_{n\to\infty} \overline{T}_j = E\,T_{0j}$. Put $n_k = [\frac{n-j}{k}]$ and

$$\widetilde{R}_j(k) = \frac{1}{n_k} \sum_{l=0}^{n_k - 1} [T_{l(k+1)+1,j} T_{(l+1)\,(k+1),j} - (E\,T_{0,j})^2],$$

$$\widehat{R}_j(k) = \frac{1}{n_k} \sum_{l=0}^{n_k - 1} [T_{l(k+1)+1,j} T_{(l+1)\,(k+1),j} - \overline{T}_j^2].$$

Hence $\widetilde{R}_j(k)$ is an unbiased estimator of $R_j(k)$ and

$$P\text{-}\lim_{n\to\infty} [\widetilde{R}_j(k) - \widehat{R}_j(k)] = 0, \quad k = 0, 1, \ldots, m-1.$$

(ii) We now consider

$$E\,(\widehat{\sigma}^2_{T_j} - \sigma^2_{T_j})^2$$

$$= \sum_{k=1}^{m}\sum_{s=1}^{m} E\,[(\widetilde{R}_j(k-1) - R_j(k-1))\,(\widetilde{R}_j(s-1) - R_j(s-1))\,\gamma_k] + 4\left[\sum_{k=m+1}^{\infty} R_j(k-1)\right]^2$$

where $\gamma_k = 1$ for $k = 1$ and $\gamma_k = 2$ for $k > 1$. Since

$$E\,|(\widetilde{R}_j(k-1) - R_j(k-1))\,(\widetilde{R}_j(s-1) - R_j(s-1))\,|$$

$$\leq [E\,(\widetilde{R}_j(k-1) - R_j(k-1))^2\, E\,(\widetilde{R}_j(s-1) - R_j(s-1))^2\,]^{1/2},$$

it is sufficient to estimate

$$E\,(\widetilde{R}_j(k-1) - R_j(k-1))^2$$

$$= n_{k-1}^{-2} \sum_{l=0}^{n_{k-1}-1} \sum_{r=0}^{n_{k-1}-1} E\,[T_{lk+1,j} T_{(l+1)\,k,j} T_{rk+1,j} T_{(r+1)\,k,j} - (E\,T_{1,j} T_{k,j})^2]$$

$$= n_{k-1}^{-2} \sum_{l=0}^{n_{k-1}-1} \sum_{r=0}^{n_{k-1}-1} [(R_j^2((r-l)\,k)$$

$$+R_j((r-l+1)\,k-1)\,R_j((r-l-1)\,k+1)$$
$$+k^4_{T_j}(lk+1,(l+1)\,k,rk+1,(r+1)\,k)],$$

where $k^4_{T_j}$ is the cumulant function of order 4. Substituting $u=r-l$, $v=lk+1$ we obtain

$$E\left[\widetilde{R}_j(k-1)-R_j(k-1)\right]^2$$

$$=\frac{1}{n_{k-1}}\sum_{u=-(n_{k-1}-1)}^{n_{k-1}-1}\frac{n_{k-1}-|u|}{n_{k-1}}\left[R_j^2(uk)+R_j((u+1)\,k-1)\,R_j((u-1)\,k+1)\right.$$

$$\left.+\frac{1}{n_{k-1}-|u|}\sum_{v=1}^{n_{k-1}-|u|}k^4_{T_j}(v,v+k-1,v+uk,v+(u+1)\,k-1)\right]. \qquad (5)$$

Suppose that $\sum_{u=-\infty}^{\infty}R_j^2(uk)<\infty$ and $\sum_{u=-\infty}^{\infty}k^4_{T_j}(0,k-1,uk,uk+k-1)<\infty$; then

$$\lim_{n\to\infty}E\,[\widetilde{R}_j(k-1)-R_j(k-1)]^2=O\left(\frac{1}{n_{k-1}}\right)\quad\text{for every}\quad k\in\{1,2,\ldots,m\}.$$

(iii) The convergence of the series in (5) can be proved with the help of a result in Sujev [8]. It follows for the stationary strong mixing sequence $\{T_{t,j}\}_{t=-\infty}^{\infty}$ that

$$k^4_{t_j}(0,k-1,uk,(u+1)\,k-1)\;\le\;C\;\left(\alpha_{T_j}(|(u-1)\,k+1|)\right)^{\gamma},\quad |u|\ge 2,$$

where C is a positive constant.

Proof of Theorem 5.

For the pure replacement model we get

$$m(\gamma,t)=E\,\psi(\underline{y}_1^{\gamma},t)$$

$$=P(v_1^{\gamma}=v_{1-j}^{\gamma}=0)\int\psi(\underline{x}_1,t)\,\mathrm{d}\mu_x+P(v_1^{\gamma}=v_{1-j}^{\gamma}=1)\int\psi(\underline{w}_1,t)\,\mathrm{d}\mu_w$$

$$+P(v_1^{\gamma}=0,v_{1-j}^{\gamma}=1)\int\psi(x_1,\ldots w_{1-j},t)\,\mathrm{d}\mu_{xw}$$

$$+P(v_1^{\gamma}=1,v_{1-j}^{\gamma}=0)\int\psi(w_1,x_0,\ldots x_{1-j},t)\,\mathrm{d}\mu_{xw},$$

where $\int\psi(\underline{x}_1,T_0)\,\mathrm{d}\mu_x=m(0,T_0)=0$. We also have

$$P(v_1^{\gamma}=v_{1-j}^{\gamma}=1)=\begin{cases}P(v_1^{\gamma}=1)\,P(v_{1-j}^{\gamma}=1) & \text{for case }(\mathbf{A})\\ p_1p_2 & \text{for case }(\mathbf{B}),\ \widetilde{k}\le j\\ p_3+p_4p_5p_6 & \text{for case }(\mathbf{B}),\ \widetilde{k}>j\end{cases}$$

where

$$p_1 = P\left(\bigcup_{l=0}^{\widetilde{k}-1}\{\widetilde{v}_{1-l}=1\}\right) \qquad p_2 = P\left(\bigcup_{l=0}^{\widetilde{k}-1}\{\widetilde{v}_{1-j-l}=1\}\right)$$

$$p_3 = P\left(\bigcup_{l=0}^{\widetilde{k}-j-1}\{\widetilde{v}_{1-j-l}=1\}\right) \qquad p_4 = P\left(\bigcup_{l=0}^{\widetilde{k}-j-1}\{\widetilde{v}_{1-j-l}=0\}\right)$$

$$p_5 = P\left(\bigcup_{l=0}^{j-1}\{\widetilde{v}_{1-l}=1\}\right) \qquad p_6 = P\left(\bigcup_{l=0}^{j-1}\{\widetilde{v}_{1-\widetilde{k}-l}=1\}\right).$$

Hence

$$P(v_1^\gamma = v_{1-j}^\gamma = 1) = \begin{cases} \gamma^2 + o(\gamma^2) & \text{for case (A)and case (B), } \widetilde{k} \le j \\ (1-j/\widetilde{k})\,\gamma + o(\gamma) & \text{for case (B), } \widetilde{k} > j. \end{cases}$$

In an analogous way we obtain

$$P(v_1^\gamma = 0, v_{1-j}^\gamma = 1) = \begin{cases} \gamma + o(\gamma) & \text{for case (A) and case (B), } \widetilde{k} \le j \\ j\gamma/\widetilde{k} + o(\gamma) & \text{for case (B), } \widetilde{k} > j. \end{cases}$$

Furthermore

$$\int \psi(x_1,\dots,w_{1-j},T_0)\,\mathrm{d}\mu_{xw} = \int \psi(w_1,x_0,\dots,x_{1-j},T_0)\,\mathrm{d}\mu_{xw} = -\frac{2}{\pi}\arcsin T_0.$$

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