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STABILITY OF THE FEEDBACK LINEARIZATION OF BILINEAR PLANTS WITH BOUNDED INPUTS[1]

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This paper discusses the effects of bounds on the input of bilinear systems when feedback linearization is used. It is shown that the stability can be analyzed as a linear multi-model problem, and sufficient conditions as well as testing approaches are given. The results are applied to a hydraulic control system.

1. INTRODUCTION

Many nonlinear plants can be better approximated and controlled using bilinear models instead of linear ones. For many bilinear models, a feedback linearization can be achieved, forcing the plant to behave like a linear system, with all known advantages. In practical applications, however, this means that the dynamic range of the applied control input is enlarged, increasing the danger of running into bounds, due, for example, to power saturation.

Running into bounds can cause problems even for linear systems (e.g. instability) but in general, if the open-loop system is stable, not too conservative conditions can be given to insure stability [1].

In bilinear systems the issue is more complex, as the control input and the state are connected multiplicatively. This makes the methods used in the linear case very conservative. However, they suggest a very simple analytical approach, as the saturated bilinear system actually becomes a linear system with different dynamics. These dynamics can be influenced by the choice of the saturation levels.

Starting from this key idea, the paper shows that the stability problem is actually a multi-model linear problem, and gives sufficient conditions for the global stability of the closed-loop system. An algorithmic approach is given based on the work presented in [2], and an alternative testing procedure is derived from the approach of [3]. The results are applied to a hydraulic system to show the practical aspects. A discussion of the implications of the results on the control system design closes the paper.

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2. PROBLEM STATEMENT

Consider a general smooth nonlinear system affine in the input

$$\dot{x} = f(x) + g(x)u \tag{1}$$

It is well known that such systems can be approximated by bilinear models, leading to a description of the form

$$\dot{x} = Ax + Nxu + bu \tag{2}$$

plus a residual $\mathcal{O}(x^2)$ [4, 5].

The special subclass of *dyadic* bilinear plants, on which the paper is focused, can be written as

$$\dot{x} = Ax + b(1 + r^{\mathrm{T}}x)u \tag{3}$$

As a real system cannot attain every state inside $\Re^n$, we shall assume the existence of an 'interesting' subset $\Omega_x \subset \Re^n$ to which we can limit all our considerations. We shall also assume that the plant inside Ω_x is completely controllable, *i.e.* that the Lie Algebraic Rank Condition [6] is fulfilled for all $x \in \Omega_x$. Of course this implies that the singularity-hyperplane defined by $1 + r^{\mathrm{T}}x = 0$ is not included in Ω_x such that we may assume that $1 + r^{\mathrm{T}}x > f_0 > 0$, $\forall x \in \Omega_x$. The control quantity u is assumed to be bounded by $[-\underline{u}, \bar{u}]$ with $\underline{u} > 0$ and $\bar{u} > 0$.

3. THE BASIC CONTROL

For a generic nonlinear smooth plant as in eq. (1) a linearizing feedback can be designed (see [7] for notation)

$$u_L = \alpha(x) + \beta(x)u_{ref} \tag{4}$$

where

$$\alpha(x) = \frac{L_f^n \lambda(x)}{L_g L_f^{n-1}\lambda(x)} \qquad \beta(x) = \frac{1}{L_g L_f^{n-1}\lambda(x)}$$

and λ is a (possibly fictive) output function which, together with the system (1), produces a relative degree equal to n. There are easily testable but quite restrictive necessary and sufficient conditions on the existence of such a λ [7].

The control (4) is known to transform the plant (1) (after a coordinate change $z = \Phi(x)$) into a linear system

$$z = Fz + gv \tag{5}$$

where the pair $\{F, g\}$ is in Brunovsky canonical form.

For the dyadic systems of eq. (3) satisfying the conditions stated above it is easy to show [8, 9] that the corresponding state transformation exists and is linear, *i.e.* $z = Dx$ where $D \in GL(\mathbb{R}, n)$. A possible output function is $\lambda = c^{\mathrm{T}}x$ where the pair $\{A, c\}$ must be observable.

By adding a classical state feedback *i.e.* by choosing

$$u_{ref} = v - \hat{k}^{\mathrm{T}} z = v - \hat{k}^{\mathrm{T}} \Phi(x) \tag{6}$$

the spectrum of the controlled system can be chosen arbitrarily.

For dyadic bilinear systems after some simple transformation the control

$$u_L = \frac{v - k^{\mathrm{T}} x}{1 + r^{\mathrm{T}} x} \tag{7}$$

is found where $k = D\hat{k}$. Note that this solution can be obtained by quite different approaches, either as in [10] or simply by using the "computed-torque" approach well-known in the robotics literature.

Due to the bounds on the input of the bilinear part the actual control applied to the plant is given by

$$u \;=\; u_L - \delta(x) \tag{8}$$

where

$$\delta(x) \;=\; \begin{cases} u_L - \bar{u} & u_L > \bar{u} \\ 0 & u_L \in [\underline{u}, \bar{u}] \\ -\underline{u} + u_L & u_L < \underline{u} \end{cases} \tag{9}$$

We shall assume the external reference v to be varying slowly enough to be considered as fixed for any single settle process, or, conversely, that the dynamic of the local loop is quick enough to allow this assumption. In a digital framework, which would be the only practical way to implement a feedback linearizing control, this condition either is automatically fulfilled or can be easily enforced.

4. EFFECTS OF THE BOUNDS

Under normal operating conditions the controlled system behaves like a linear system

$$\dot{x} = (A - b\,k^{\mathrm{T}})x + bv. \tag{10}$$

Clearly the choice of k is important for the actual value of the control (7) and therefore for the reaching of the bounds.

Reaching the bounds means that the system is fed by a constant input $\bar{u}$ or $-\underline{u}$, depending on which bound is active. But this implies that the motion of the controlled system is described by a *linear* state equation

$$\dot{x} = (A + u_c b\, r^{\mathrm{T}})x + b\, u_c \tag{11}$$

where u_c is either $\bar{u}$ or $-\underline{u}$.

In the case of dyadic systems, it is easy to show that there are always combinations of upper and lower bounds for which the system is stable, provided that the open-loop system is stable. To this purpose, we use the fact that $b\,r^{\mathrm{T}}$ has rank one, and compute the characteristic equation of the system at the bounds

$$\begin{aligned} P(s) &= \det(sI - A + u_c b\, r^{\mathrm{T}}) \\ &= \det(sI - A)(I + u_c r^{\mathrm{T}}(sI - A)^{-1} b) \\ &= \det(sI - A) + u_c r^{\mathrm{T}} Adj(sI - A) b \end{aligned} \tag{12}$$

If we consider a linear system $\{A, b, r^{\mathrm{T}}\}$, then eq. (12) is the formulation of its root locus with a feedback gain u_c. Obviously, if the system is open-loop stable, choosing $\bar{u}$ and $\underline{u}$ sufficiently small the closed-loop system will remain stable. For unstable plants this may not hold.

Of course even if all three linear systems (10) and (11) are stable, this does not guarantee the stability of the system (3) controlled by (7).

4.1. Conditions for stability

In this section we analyze the stability of the controlled system and shall assume without loss of generality a reference signal $v = 0$. A way to prove the stability of the controlled system is the existence of a common Lyapunov function for all three subsystems. Sufficient stability conditions are stated in the following theorem.

Theorem. Consider a system defined by

$$\dot{x} = \begin{cases} (A + \bar{u} b r^{\mathrm{T}})x + b\bar{u} & \text{if} \quad x \in \Omega_+ := \{x \in I\!R^n \ : \ (k + \bar{u} r)^{\mathrm{T}} x + \bar{u} < 0\} \\ (A - b k^{\mathrm{T}})x & \text{if} \quad x \in \Omega_0 := \{x \in I\!R^n \ : \ -\underline{u} \le u_L \le \bar{u}\} \\ (A - \underline{u} b r^{\mathrm{T}})x - b\underline{u} & \text{if} \quad x \in \Omega_- := \{x \in I\!R^n \ : \ (k - \underline{u} r)^{\mathrm{T}} x - \underline{u} > 0\} \end{cases} \tag{13}$$

with $\bar{u} > 0$, $\underline{u} > 0$, $(k + \bar{u} r) \neq 0$ and $(k - \underline{u} r) \neq 0$. If the conditions

1. there exists a positive definite matrix $P \in I\!R^{n \times n}$ such that

$$\begin{aligned} (A + \bar{u} b r^{\mathrm{T}})^{\mathrm{T}} P + P(A + \bar{u} b r^{\mathrm{T}}) &=: -Q_+ < 0 \\ (A - b k^{\mathrm{T}})^{\mathrm{T}} P + P(A - b k^{\mathrm{T}}) &=: -Q_0 < 0 \\ (A - \underline{u} b r^{\mathrm{T}})^{\mathrm{T}} P + P(A - \underline{u} b r^{\mathrm{T}}) &=: -Q_- < 0 \end{aligned} \tag{14}$$

2. P and Q_+ satisfy the inequality

$$2|Pb||k + \bar{u} r| < \lambda_{\min}(Q_+) \tag{15}$$

3. P and Q_- satisfy the inequality

$$2|Pb||k - \underline{u} r| < \lambda_{\min}(Q_-) \tag{16}$$

are fulfilled then the system (13) is globally asymptotically stable.

Proof. The function

$$v(x) = x^{\mathrm{T}} P x \tag{17}$$

is by assumption a candidate Lyapunov function. The time derivative of (17) along the trajectories of (13) is given by three different expressions corresponding to the three subsets $\Omega_i, i = +, 0, -$.

If $x \in \Omega_0$ then

$$\frac{d}{dt}v = -x^{\mathrm{T}} Q_0 x < 0 \tag{18}$$

and as long as x stays in this region the magnitude of v has to decrease.

If $x \in \Omega_+$ then

$$\frac{d}{dt}v = -x^{\mathrm{T}} Q_+ x + 2\bar{u} b^{\mathrm{T}} P x =: \dot{v}_+ \tag{19}$$

The first summand in the last equation is negative definite, the second one is in general indefinite and we are looking for (conservative) bounds which guarantee that $\dot{v}_+ < 0$ for all possible $x \in \Omega_+$. Using the relation

$$\lambda_{\mathsf{min}}(Q_+)|x|^2 \leq x^{\mathrm{T}} Q_+ x \leq \lambda_{\mathsf{max}}(Q_+)|x|^2 \tag{20}$$

($\lambda_{\mathsf{min}\,/\,\mathsf{max}}(Q)$ denoting the minimal respectively maximal eigenvalue of Q) a sufficient condition for $\dot{v}_+ < 0$ is found to be

$$\frac{2\bar{u}|Pb|}{|x|} < \lambda_{\mathsf{min}}(Q_+) \tag{21}$$

If the last expression is true for the smallest possible $|x|$ for $x \in \Omega_+$ then it will be true for all $x \in \Omega_+$.

In the general case, the set Ω_+ consists of all points which satisfy the condition

$$v - \bar{u} - (k + \bar{u}r)^{\mathrm{T}} x > 0 \tag{22}$$

In our case ($v = 0$), the state x withy smallest length which satisfies this condition is

$$x = \frac{-(k + \bar{u}r)\bar{u}}{|(k + \bar{u}r)|^2}$$

and its length is

$$\rho_+ = \frac{\bar{u}}{|k + \bar{u}r|}. \tag{23}$$

Inserting this expression into eq. (21) condition 2 of the theorem follows immediately.

In a completely similar way condition 3 can be derived which guarantees that also if $x \in \Omega_-$ the time derivative of $v(x)$ is negative. But, if this is true, we have shown that in all three possible regions the common positive definite function $v(x)$ has to decrease and therefore the origin has to be globally asymptotically stable. □

The problem of how to find a suitable matrix P will be tackled in the next section. Once such a P has been found the evaluation of the remaining conditions 2 and 3 is straightforward. Of course we can not draw any conclusions if these tests fail since they represent only sufficient but not necessary conditions.

4.2. Computational verification

An explicit solution can be given only for extremely specific systems. However, as we limit ourselves to equations of the kind $V = x^T Px$, the search can be strongly simplified using the fact that the solutions P of the Lyapunov equation

$$A^T P + PA = -Q \tag{24}$$

with A stable, Q a positive definite matrix, can be expressed in form of a linear mapping of the Q matrix

$$\begin{aligned} \mathrm{col}(A^T P + PA) &= -(A^T \otimes I + I \otimes A^T)\mathrm{col}(P) \\ &= -L\ \mathrm{col}(P) \end{aligned} \tag{25}$$

where $\otimes$ means the Kronecker tensor product [2]. Note that L depends only on the system matrix. Let $Q_2 > 0$. Then if Q_i, defined as

$$\mathrm{col}(Q_i) = L_i L_2^{-1} \mathrm{col}(Q_i) = LL_i \mathrm{col}(Q_i) \tag{26}$$

is positive definite, the corresponding P define a common Lyapunov function. For low order systems a search can be done relatively easily (for order 2 even by hand), for more complex systems the approach proposed in [2] in a different framework can be very useful.

A different approach is offered by the formulation of the circle criterion given in [3]. The authors study the problem of the stability of a perturbed system

$$\dot{x} = \{A_0 + DF(t)E\}\, x \tag{27}$$

with $||F(t)|| \leq 1$. They show that the system is quadratically stable if and only if it satisfies the following conditions:

1. A_0 is a stability matrix
2. $||E(sI - A_0)^{-1} D||_\infty < 1$

We can now restate our problem as the problem of the stability of a basic linear system plus a disturbance which can assume the following values

$$\dot{x} = \{A - bk^T\}\, x + \left\{ \begin{array}{c} bk^T + \bar{u}br^T \\ 0 \\ bk^T - \underline{u}br^T \end{array} \right\} x \tag{28}$$

In order to use the form of eq. (27) we can rewrite the set of systems in the form

$$\dot{x} = A_0 x + \underbrace{b}_{D} \underbrace{\rho \left\{ \tilde{k}^T - \delta \Delta r^T \right\}}_{F} \underbrace{\gamma I}_{E}\, x \tag{29}$$

where $\tilde{k} = k + \bar{u}r$, δ can be either 0 or 1, $\Delta = \bar{u} - \underline{u}$, $\gamma = \max_{\delta=0,1} ||F||^{-1}$ and ρ can be either 0 or $\frac{1}{\gamma}$. From the above mentioned stability theorem we obtain that the stability is preserved if

$$||(sI - A_0)^{-1} b||_\infty < \frac{1}{\gamma} = ||F(t)|| \tag{30}$$

As $A_0 = A - bk^{\mathrm{T}}$, both sides depend on k. The norm $\|F(t)\|$ is given by $\|\rho(\tilde{k} - \delta\Delta r)\|$). The smallest value for the maximum parameter variation (*i.e.* $\rho = \frac{1}{\gamma}$), is

$$\|\tilde{k} - \delta\Delta r\| = \sqrt{\left(\tilde{k}^{\mathrm{T}} - \delta\Delta r^{\mathrm{T}}\right)\left(\tilde{k} - \delta\Delta r\right)} = \sqrt{\tilde{k}^{\mathrm{T}}\tilde{k} - 2\delta\Delta\tilde{k}^{\mathrm{T}}r + \delta^2\Delta^2 r^{\mathrm{T}}r} \qquad (31)$$

so that the smallest maximum is for $\tilde{k}^{\mathrm{T}}r > 0$, *i.e.* for k parallel to r.

The stability can be verified by testing the following condition

$$\max_{\omega} \bar{\sigma}\left[(j\omega I - A_0)^{-1}b\right] < \frac{1}{\gamma} \qquad (32)$$

which is equivalent to asking

$$\|(sI - A_0)^{-1}b\|_{\infty} < \frac{1}{\gamma} \qquad (33)$$

This condition is easily tested by evaluating the eigenvalues of an Hamiltonian matrix (see *e.g.* [11], p.27). Testing is therefore much easier than using the other approach presented. However, it tests the common stability of all possible intermediate system representations as well, and is therefore, in general, much more conservative.

5. AN APPLICATION TO HYDRAULIC DRIVES

In a previous work [12], a complex hydraulic system has been controlled by using, basically, a linear representation of the system. The first part of the plant, a high speed linear drive, is reproduced schematically in Figure 1.

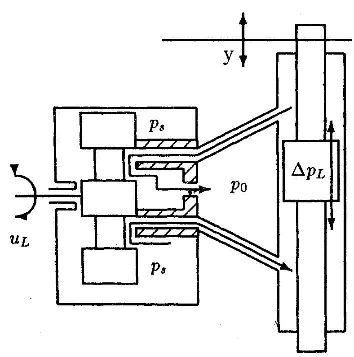

Fig. 1. Scheme of the linear actuator.

Such systems can be easily represented by 'almost bilinear' models [10] or combinations of such models and linear filters [13].

With some simplifications, the state of the servocylinder can be described by three state quantities, the pressure drop on the cylinder Δp_L, the speed $\dot{y}$ and the position y of the cylinder. We obtain the following model:

$$\dot{\Delta p_L} = \frac{B}{V}[k_v \,\mathrm{u}\sqrt{1-\frac{\Delta p_L}{p_s}} - K_L \Delta p_L - A\dot{y}] \tag{34}$$

$$\ddot{y} = \frac{A}{m}\Delta p_l - \frac{R}{m}\dot{y} \tag{35}$$

where B is the compression module of oil, V the compression volume, k_v the specific flow of the valve, p_s the supply pressure, K_L the leakage coefficient, A the equivalent surface of the cylinder, m the mass and R the friction coefficient. In order to keep the example clear, we shall concentrate us on the second order problem of keeping the servocylinder speed constant. Choosing Δp_L and $\dot{y}$ as the state variables x_1, x_2, we can write the whole system as

$$\dot{x} = \begin{bmatrix} -a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} x + \begin{bmatrix} b_1\sqrt{1-a_{14}x_1} \\ 0 \end{bmatrix} u. \tag{36}$$

The design of the linear state feedback is made on the basis of the behavior of the linear part of the model - the desired poles are chosen in order to reduce rise time and to increase the damping of the system. Figure 2 shows the behavior of the linear part of the plant without the control, while Figure 3 shows the one after implementing the control.

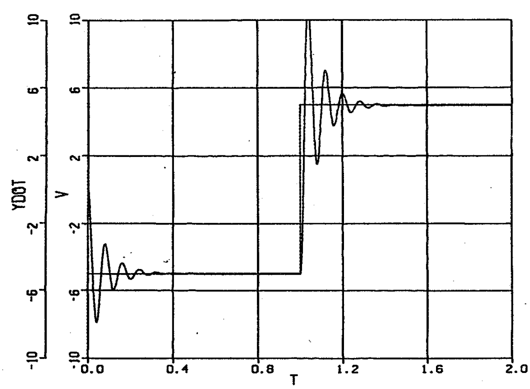

Fig. 2. Behavior of the linear part of the system.

By applying this state feedback together with the feedback linearization, we obtain the behavior shown in Figure 4 for three different bounds: ± 0.25, ± 1.0 and

$\pm 10V$. The reference value was always the same, $\pm 5V$. In the first case, the system does not reach the desired value, and the dynamic behavior is clearly asymmetric - less damped on the negative side. In the second case, the system runs again into bounds, but the negative side shows a strong oscillation. In the third case, again, the system behaves correctly form both sides, with a small residual oscillation on the negative side.

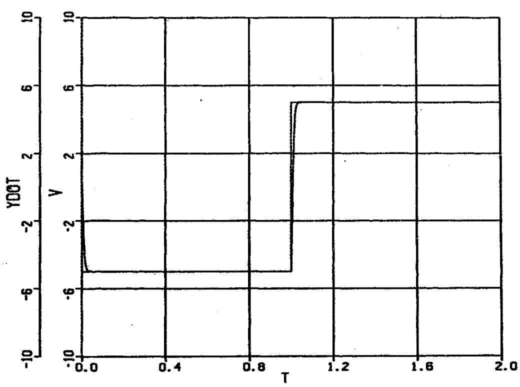

Fig. 3. Theoretical behavior expected by the feedback linearization plus pole placement.

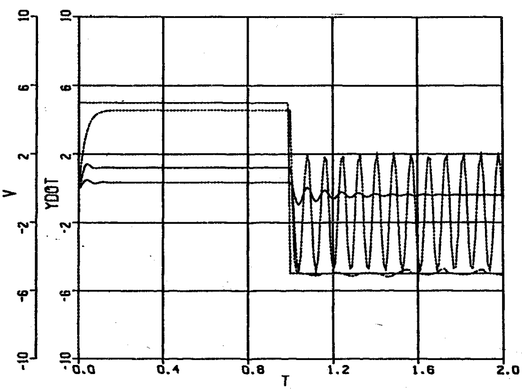

Fig. 4. Effect of the bounds on the speed control.

What actually happens can be understood by looking at the corresponding root locus diagram. For negative values of $u_c$below $-.4459$ the system is unstable, for positive values always stable. The correct behavior of the third case, which should also lead to instability, is due to the fact that the bounds are reached only very shortly, so that the behavior of the linearized structure is determining.

From the root locus one would expect the behavior of the second case, for $u_c = 1.0V$, to be also quite unstable, and not only a limit cycle as shown in Figure 4. By observing the control quantity, however, it can be easily seen that a mixed case occurs, in which *both* bounds are reached during the negative phase. In this case, the combination of the three regions has a positive effect, as no exponential state evolution happens, but the effect could as well be negative. This stresses again the importance of considering mixed movements.

In order to perform the second test we start by constructing the three matrices L_i, as defined earlier, corresponding to the three cases of eq. (17). We check the existence of common Lyapunov functions for two neighbor regions at a time. Taking as boundary values +10 (the normal physical bound) and -0.3 (still acceptable, although with a poor damping: the poles lie at $-4.288 \pm 76.392i$), we find that no solution with $V = x^{\mathrm{T}}Px$ can exist, and we are forced to increase the lower bound up to -0.1. No problem arises for the positive bound.

6. CONCLUSION AND OUTLOOK

The theoretical considerations and the application presented above show that a bilinear system under saturation can be represented as a multi-model problem, in which the bordering subsystems must be jointly stabilized. Sufficient conditions for stability have been given, whereas the most critical point seems to be the distance of the stationary solutions of the saturated systems from the corresponding border to the unsaturated region.

From a practical point of view, the options for the designer are basically two: either to exploit his freedom in setting smaller bound values, or to use some prefiltering technique, to keep the system away from a bound that would make the system unstable. A possible scheme to increase the tracking region respecting boundary conditions under feedback linearization is given in [14], but other schemes should be considered as well.

The results have been derived for a specific class of nonlinear plants, and many specific conclusions will not hold for other classes. The basic idea, to consider the stability problem as a multi-model problem, can be easily extended to consider much more general classes. Furthermore, the solution of the problem for bilinear plants may prove very useful as a first degree approximation of more complex plants, especially in view of the approximation properties of bilinear systems [4].

Clearly, there are still many open questions worth studying. So, if a pole placement approach is taken, the designer has mostly the possibility to pick its k feedback vector from a set $\mathcal{K}$ of ks [15]. He can try to reduce the danger of running into bounds by choosing a k

$$\hat{u} = \min_{k \in \mathcal{K}} \max_{x \in \Omega_x} \left(\frac{v - k^{\mathrm{T}}x}{1 + r^{\mathrm{T}}x} \right) \tag{37}$$

It can be easily shown that no maxima or minima can lie inside the region $\mathcal{S}$, so that the search can be concentrated on the border. This approach, however, is extremely application depending, and closed results can be given mainly for very simple $\mathcal{S}$ structures, like hyperellipsoides. More general results would be of interest.

APPENDIX: NUMERICAL VALUES

The full system matrix (measuring in bar, for numerical reasons) is given by

$$\begin{aligned} A &= \begin{bmatrix} -2.2500 & -1.9080e4 \\ 3.2615e-1 & -2.3965e1 \end{bmatrix} \\ b &= \begin{bmatrix} 2.8485e4 \\ 0 \end{bmatrix} \\ c &= \begin{bmatrix} 0 & 1 \end{bmatrix} \\ r &= \begin{bmatrix} -2.0641e-3 & 0 \end{bmatrix} \\ k &= \begin{bmatrix} 1.4284e-2 & 3.4868 \end{bmatrix} \end{aligned}$$

The transformation matrix for the $2 \rightarrow 3$-case is (for a bound at .3)

$$LL = \begin{bmatrix} 5.54e1 & 6.61e-2 & 6.61e-2 & 8.99e-4 \\ 4.43e3 & 3.19e1 & -1.8577e1 & 8.40e-2 \\ 4.43e3 & -1.86e1 & 3.19e1 & 8.40e-2 \\ -9.05e5 & -7.30e2 & -7.30e2 & -8.93 \end{bmatrix}$$

Clearly, there is no real row vector $q = \mathrm{col}(Q)$ such that $q_1 > 0$, $q_1 q_4 - q_2^2 > 0$ that is projected into another vector $\hat{q}$ with the same properties.

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