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LYAPUNOV DESIGN OF A NEW MODEL REFERENCE ADAPTIVE CONTROL SYSTEM USING PARTIAL A PRIORI INFORMATION

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A new approach to adaptive model reference control, based on Lyapunov's direct method, is presented. A design procedure for single output systems has been developed and the results verified by computer simulation. The algorithm presented in this paper guarantees asymptotic stability, provided that the transfer function of the equivalent error system is strictly positive real. Since the direct Lyapunov's method is used, the stability conditions are sufficient but not necessary. Therefore, the assumptions are more stringent than they need be. Consequently, as verified by simulation, the algorithm performs very well even if those assumptions are violated. The implementation of the proposed algorithm requires a priori partial information on the plant.

1. INTRODUCTION

In a model reference control system the design specifications are represented by a reference model. A controller to be designed uses the model inputs, the model states, and the error between plant and model output to generate the appropriate control signals.

When the plant parameters are not well known, adaptive control is used to adjust the control law. In this paper, using the second or direct Lyapunov method a system is designed that adjusts the control law to minimize the error between the plant and the ideal target system states. However, the verification of the assumptions made, as well as effective implementation of the algorithm proposed, requires a priori partial information on the plant, i.e., bounds on the plant parameters have to be known.

One of the first researchers who used Lyapunov's second method to design a stable adaptive controller for single input single output systems (SISO) was Parks [13]. Also, the same technique was used by Grayson [6], and Winsor and Roy [19] for the design of multiple input multiple output model reference adaptive control systems. However, these algorithms required the satisfaction of Erzberger's perfect model following (PMF) conditions. In other words, these adaptive controllers function properly only if there exists a certain structural relationship between the plant and the model.

Another adaptive algorithm (whose stability is ensured by the hyperstability criterion of Popov) for multiple input multiple output continuous system subject to the PMF conditions was developed by Landau [8]. The papers by Broussard [5], and Mabius and Kaufman [9] were among the first ones that reported designs of adaptive controllers which do not require the satisfaction of PMF conditions.

In this paper, a new MRAC algorithm for SISO systems is presented. This algorithm does not require the satisfaction of PMF conditions. The design, based on Lyapunov's direct method, guarantees asymptotic stability provided that the transfer function of an equivalent error system is strictly positive real (SPR). In addition, it is shown that the adaptive algorithm guarantees that the error remains bounded under less restrictive positivity conditions.

Furthermore, the new MRAC system is quite simple, as compared to other MRAC algorithms. Despite its simplicity, the new MRAC algorithm is at least as effective as the more complex adaptive mechanisms. This may be demonstrated, by comparing for the same plant, the performance of the system implementing this algorithm with one based on adaptive model following control (AMFC) algorithm developed by Landau [8].

The paper is organized in the following manner. Section 2 presents the new design and formulates the stability theory which is proved in Appendix I. Section 3 presents a simple design example and outlines the design procedure. Also, in this section the simulation results are presented. These results give a comparison of the two systems mentioned above. Section 4 summarizes the results obtained and outlines a possible application of this algorithm to the adaptive control of the n-joint manipulator.

2. THE DESIGN PROBLEM

The design of a new MRAC system is based on a modification of the MRAC system proposed by Bialasiewicz and Proano [4]. Both systems have the same general structure, shown in Fig. 1, where the state estimator is described by the differential equation of a reference model with feedback. Due to this feedback, the estimator response is faster than that of the reference model, and therefore, the convergence rate of the adaptation algorithm is grately increased. Further improvement of the convergence rate is achieved by a modification of the algorithm used in [4]. This modification is based on the ideas presented by Sobel and Kaufman [17] and Bar-Kana [2].

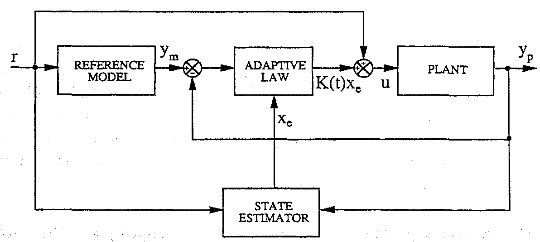

Fig. 1. MRAC system.

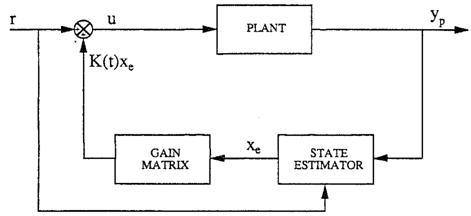

Fig. 2. Augmented system.

In order to develop the new adaptive algorithm we first formulate the design problem. This formulation is based on the idea of the augmented system shown in Fig. 2. We assume for simplicity that both the plant and the reference model are of order n. The elements of the augmented system may be described by the following equations:

PLANT:

$$\dot{x}_p = A_p\, x_p + B_p\, u \qquad x_p \in R^n \tag{1}$$

$$y_p = C_p\, x_p \qquad y_p \in R \tag{2}$$

STATE ESTIMATOR:

$$\dot{x}_e = A_m\, x_e + B_m\, r + L(C_1\, x_p - C_e\, x_e) \qquad x_e \in R^n \tag{3}$$

$$y_e = C_e\, x_e \qquad y_e \in R \tag{4}$$

In particular, $C_1 = C_p$. In this case for the second order system $L = [\ell_1\ \ell_2]^{\mathrm{T}}$. However, if x_p is available or if x_p is replaced by an estimate that for the second order system can be assumed to be $[y_p\ \dot{y}_p]^{\mathrm{T}}$, then one can choose $C_1 = C_e = I$ and $L = \mathrm{diag}(\ell_1, \ell_2)$. This means that in the latter case the feedback term in (3) becomes $[\ell_1 y_p\ \ell_2 \dot{y}_p]^{\mathrm{T}}$.

PLANT INPUT:

$$u = K(t)\, x_e + r. \tag{5}$$

Then, the state space representation of the augmented system is given by

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} A_p & B_p K(t) \\ LC_1 & A_m - L\,C_e \end{bmatrix} \begin{bmatrix} x_p \\ x_e \end{bmatrix} + \begin{bmatrix} B_p \\ B_m \end{bmatrix} r \tag{6}$$

$$y = \begin{bmatrix} C_p & 0 \end{bmatrix} \begin{bmatrix} x_p \\ x_e \end{bmatrix} \tag{7}$$

or in a compact form

$$\dot{x} = Ax + Br \tag{8}$$

$$y = Cx. \tag{9}$$

The output of the augmented system (selected as the plant output) is required to track the output of a nth order reference model

$$\dot{x}_m = A_m x_m + B_m r \tag{10}$$
$$y_m = C_m x_m \tag{11}$$

that is, it is required that

$$y = y_m . \tag{12}$$

Therefore, the purpose of the gain matrix $K(t)$ is to permit an adjustment of the dynamics of the augmented system so that it performs as a stable reference model.

Assume that there exists an ideal target system

$$\dot{x}^* = A^* x^* + Br \tag{13}$$
$$y^* = Cx^* \tag{14}$$

that satisfies the equation

$$y^* = y_m \tag{15}$$

with

$$A^* = \begin{bmatrix} A_p & B_p\tilde{K} \\ LC_1 & A_m - LC_e \end{bmatrix} \tag{16}$$

where $\tilde{K}$ is an unknown constant gain. We define the generalized state error as

$$e_x = x^* - x \tag{17}$$

and the output error as

$$e_y = y_m - y = y^* - y = Cx^* - Cx = Ce_x . \tag{18}$$

The augmented system state error equation then becomes

$$\begin{aligned} \dot{e}_x &= \dot{x}^* - \dot{x} = \dot{x}^* - A^*x + A^*x - \dot{x} = A^*x^* + Br - A^*x + A^*x - Ax - Br \\ &= A^*e_x + (A^* - A)\,x = A^*e_x - B^*(K(t) - \tilde{K})\,x_e \end{aligned} \tag{19}$$

with

$$B^* = \begin{bmatrix} B_p \\ 0 \end{bmatrix} .$$

Therefore, an equivalent error system is

$$\begin{aligned} \dot{e}_x &= A^*e_x - B^*(K(t) - \tilde{K})\,x_e \\ e_y &= Ce_x \end{aligned} \tag{20}$$

which has the transfer function

$$Z(s) = C(sI - A^*)^{-1}\,B^* . \tag{21}$$

$K(t)$ should be defined by an adaptation law such that e_x approaches zero as t tends to infinity. It is proposed that this adaptation law has the following form:

$$u_c = K(t)\, x_e \tag{22}$$

$$K(t) = K_P(t) + K_I(t) \tag{23}$$

$$K_P(t) = e_y\, x_e^{\mathrm{T}}\, \overline{S} \tag{24}$$

$$\dot{K}_I(t) = e_y\, x_e^{\mathrm{T}}\, S \tag{25}$$

where $\overline{S}$ and S are properly selected, positive-definite symmetric adaptive coefficient matrices.

As discussed below, the proportional term (24) facilitates the direct control of the output error e_y. Because of this, as indicated in [3], the error can be ultimately reduced to zero under the assumption of a disturbance free environment. It is worthwhile to note, that the adaptive controller, defined by the equations (22) through (25), is extremely simple.

The following theorem formulates the stability result of the proposed design for a new MRAC system:

Theorem. Consider the system described by equations (6), (7), (10) and (11), with the adaptation law defined by equations (22) through (25). This system is asymptotically stable or, in other words, e_x and e_y (defined by (17) and (18)) approach zero as t tends to infinity if

(a) S is a positive definite symmetric matrix,
(b) $\overline{S}$ is a positive semidefinite symmetric matrix, and
(c) $Z(S)$, defined by (21), is strictly positive real (SPR).

Proof. This theorem is proven in Appendix 1. □

Since the plant is not fully known it is not possible to verify the SPR condition. Instead, one can assume the existence of a gain matrix $\tilde{K}$ such that A^* is a stable matrix that satisfies the Lyapunov equation

$$A^{*\mathrm{T}} P + P\, A^* = -Q \tag{26}$$

where Q and P are positive-definite symmetric matrices. Then, the total derivative $\dot{V}(t)$ of the Lyapunov function candidate, specified by (A1) in the Appendix, has the following form:

$$\dot{V}(t) = -e_x^{\mathrm{T}}\, Q\, e_x - 2e_x^{\mathrm{T}}(PB^* - C^{\mathrm{T}})\,(K(t) - \tilde{K})\, x_e - 2e_y^2\, x_e^{\mathrm{T}}\, \overline{S}\, x_e. \tag{27}$$

For a stable system $\dot{V}(t)$ is negative definite because $K(t)$ is bounded and a positive definite symmetric matrix $\overline{S}$ may be chosen to be sufficiently large. The third term in (27) is

$$-2e_y^2\, x_e^{\mathrm{T}}\, \overline{S}\, x_e = -2e_y\, K_P\, x_e$$

which is a negative definite quadratic in the output error e_y, and is proportional to the gain K_P. Due to the proper selection of $\overline{S}$, it can remain large even if the error e_y is arbitrarily reduced. As a result, the set $\{e_x, K_I | \dot{V}(t) = 0\}$ may be arbitrarily reduced.

Comment. The simulation results, presented below and obtained for a system which does not meet the SPR condition, show that the designer does not necessarily need to be concerned with the positivity condition. This is a consequence of the fact that Lyapunov's direct method sometimes provides very conservative stability conditions which are sufficient, but not necessary except in very special cases [10]. In other words, the assumptions made are usually more stringent than required. Similar conclusions can be found in the papers by Bar-Kana [2] and Seraji [15], in which the design based on Lyapunov's direct method is presented.

3. AN EXAMPLE

A. Plant and reference model

The plant to be controlled is a single-link arm that consists of a rigid link coupled through a gear train to DC-motor [18], which is shown in Fig. 3. Its linearized state space equation is

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -G/E & -F/E \end{bmatrix} x + \begin{bmatrix} 0 \\ H/E \end{bmatrix} V_s$$

where $x = [\theta_1 \ \dot{\theta}_1]^{\mathrm{T}}$ and the constants G, E, F, H are defined by the physical parameters of the plant, and V_s is the input voltage to the motor. Since the constants E and G depend on M, the total mass of the link, the dynamics of the plant are directly related to the playload and an adaptive controller is needed to make the system performance independent of the mass of the link.

Figure 4 shows that the link dynamics changes drastically with the total mass M. The required dynamics, represented by the reference model, are also shown in Fig. 4 in the form of the unit step response.

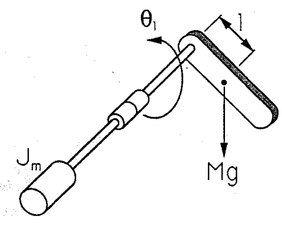

Fig. 3. The plant.

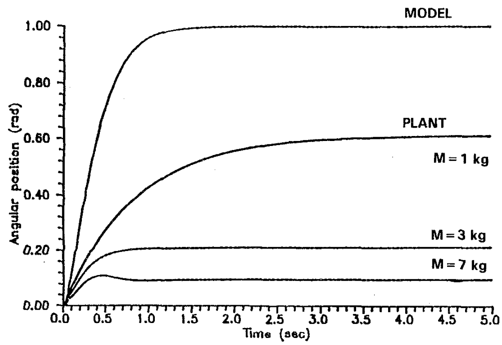

Fig. 4. Plant and model dynamics.

B. Design procedure

Since for a manipulator the full state x_p is available one can choose $C_e = C_1 = I$ and $L = \mathrm{diag}(\ell_1, \ell_2)$. Then, (3) becomes

$$\dot{x}_e = (A_m - L)\,x_e + L\,x_p + B_m r$$

and the matrix A^* can be written as

$$A^* = \begin{bmatrix} A_p & B_p \tilde{K} \\ L & A_m - L \end{bmatrix}$$

The following steps are involved:

1. Choose L such that the submatrix $A^*_{22} = A_m - L$ is a stable matrix.
2. Using an a priori known range of plant parameters, check that there exists a $\tilde{K}$ such that A^* is a stable matrix. In the case of the system discussed, this condition should be checked for all values of M. Recall that $\tilde{K}$ is not used in the adaptation mechanism. If it cannot be found return to step 1.
3. Next, assume a positive-definite symmetric matrix Q and make sure that for all values of M there exists a positive-definite symmetric matrix P that satisfies (26). Return to step 1 if such a matrix P cannot be found.
4. Simulate the system, and varying $\overline{S}$ and S, try to obtain the best tracking error response.

The following gain matrices have been experimentally found and used for the simulation:

$$L = \mathrm{diag}(10,\,2), \qquad \overline{S} = \mathrm{diag}(200,\,600), \qquad S = \mathrm{diag}(600,\,600).$$

Simulation results

To illustrate the effectiveness of the new MRAC system, the design of a classical AMFC system (developed by Landau [8]) for a single link manipulator is considered, and the simulations for both systems performed. The results using three values of M are shown in Fig. 5 through Fig. 7.

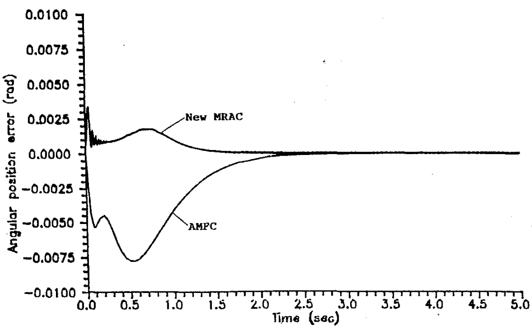

Fig. 5. Angular position error ($M = 1$kg) for tracking the unit step response of a reference model.

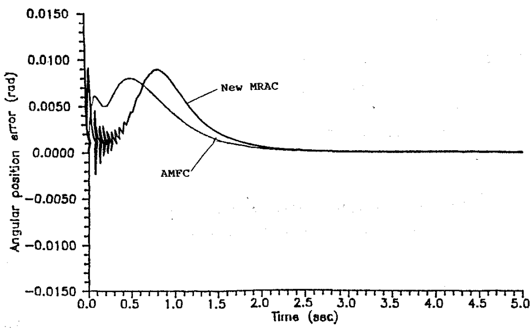

Fig. 6. Angular position error ($M = 3$kg) for tracking the unit step response of a reference model.

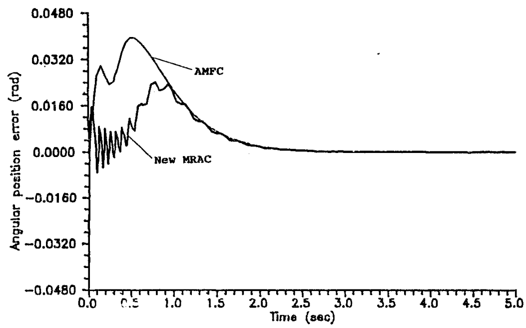

Fig. 7. Angular position error ($M = 7$kg) for tracking the unit step response of a reference model.

The simulation results show that the new MRAC algorithm is at least as effective as the AMFC algorithm. Also, it was found that the new MRAC is much simpler to implement. That is, it requires a smaller number of gains and integrators.

4. SUMMARY

In this paper the development and simulation results for a new model reference adaptive control algorithm have been presented. This algorithm does not require that the PMF conditions be satisfied. Therefore, the plant does not need to be structurally similar to the reference model.

The design of the new control algorithm is based on Lyapunov techniques. Asymptotic stability is assured, provided that the transfer function of the equivalent error system is strictly positive real. A discussion is given of what happens when the SPR requirement is replaced by the condition of positivity. Since the positivity conditions are imposed on the equivalent error system and not on the control plant, the new adaptive controller is really quite versatile and can be used with many different types of plant configurations.

Despite its simplicity the new MRAC system is robust to the changes in plant dynamics. Therefore, it would be worthwhile to consider the application of this algorithm to adaptive control of manipulators. This implementation could be done following the Seraji approach to decentralized control of manipulators [15]. In this application, the control law for each joint of an n-joint manipulator has the same form and would involve only four gains and two integrators. The required dynamics of each joint would be specified by a reference model.

APPENDIX I

Proof of the stability theorem

The following function is chosen as a Lyapunov function candidate:

$$V(t) = e_x^{\mathrm{T}} P\, e_x + \mathrm{tr}[(K_I - \tilde{K})\, S^{-1}(K_I - \tilde{K})^{\mathrm{T}}] \tag{A1}$$

where $\tilde{K}$ is an unspecified matrix. $\tilde{K}$ appears only in the function $V(t)$ and not in the control algorithm. Then

$$\dot{V}(t) = \dot{e}_x^{\mathrm{T}} P\, e_x + e_x^{\mathrm{T}} P\, \dot{e}_x + 2\mathrm{tr}[(K_I - \tilde{K})\, S^{-1}\, \dot{K}_I^{\mathrm{T}}] \tag{A2}$$

which is calculated along the system trajectory (20) and has the following form:

$$\dot{V}(t) = e_x^{\mathrm{T}}(A^{*\mathrm{T}}\, P + P\, A^*)\, e_x - 2e_x^{\mathrm{T}}\, P\, B^*(K(t) - \tilde{K})\, x_e + 2\mathrm{tr}[e_y(K_I - \tilde{K})\, x_e]. \tag{A3}$$

Combining (A3), (23) and (24), one obtains

$$\dot{V}(t) = e_x^{\mathrm{T}}(A^{*\mathrm{T}}\, P + P\, A^*)\, e_x - 2e_x^{\mathrm{T}}\,(P\, B^* - C^{\mathrm{T}})\,(K(t) - \tilde{K})\, x_e - 2e_y^2\, x_e^{\mathrm{T}}\, \overline{S}\, x_e. \tag{A4}$$

Due to condition (b) of the theorem, the third term of (A4) is negative semidefinite. In addition, we require the existence of a gain matrix $\tilde{K}$ such that

$$A^{*\mathrm{T}} P + P\, A^* = -Q \qquad \text{and} \qquad P\, B^* = C^{\mathrm{T}}$$

where P and Q are positive-definite symmetric matrices. This requirement, according to the Kalman–Yakubovitch Lemma [11], [16], is satisfied if, and only if, the transfer function $Z(s)$, defined by (21), is strictly positive real. This is guaranteed by the condition (c) of the theorem. Then,

$$\dot{V}(t) = -e_x^{\mathrm{T}}\, Q\, e_x - 2e_y^2\, x_e^{\mathrm{T}}\, \overline{S}\, x_e. \tag{A5}$$

Therefore, $\dot{V}(t)$ is negative definite in $e_x(t)$, and is negative semidefinite in the augmented state $[e_x,\, K_I(t)]$. Since $V(t)$ is positive the new MRAC system is stable. The asymptotic stability can be seen using the Lemma of Barbalat [11,16].

Note that $V(t)$ is bounded from below because it is positive definite, and is a nonincreasing function since $\dot{V}(t) \leq 0$. Therefore, it converges to a finite value V_∞ as t tends to infinity. Then,

$$\lim_{t\to\infty} \int_0^t \dot{V}(t)\, \mathrm{d}t = \lim_{t\to\infty} V(t)\Big|_0^t = V_\infty - V(0) \tag{A6}$$

exists and is finite. To use the Lemma of Barbalat, one still has to show that $\dot{V}(t)$ is uniformly continuous or that $\ddot{V}(t)$ is bounded. This can be seen from equation (A5) since $\dot{e}_x$, x_e, and $\dot{x}_e$ are bounded. Then

$$\lim_{t\to\infty} \dot{V}(t) = \lim_{t\to\infty} \left[-e_x^{\mathrm{T}}\, Q\, e_x - 2e_y^2\, x_e^{\mathrm{T}}\, \overline{S}\, x_e\right] = 0 \tag{A7}$$

or

$$\lim_{t \to \infty} e_x = 0 \qquad \text{and} \qquad \lim_{t \to \infty} e_y = 0. \tag{A8}$$

Also, since x_e is bounded, one obtains by (24), (25), and (A8) that the

$$\lim_{t \to \infty} K_P(t) = 0 \qquad \text{and} \qquad \lim_{t \to \infty} \dot{K}_I(t) = 0. \tag{A9}$$

Also, $K_I(t)$ is square integrable. Therefore, $K(t)$, given by (23), is a bounded gain matrix. Summarizing, it is shown that under the assumptions of the Theorem the new MRAC system is asymptotically stable and that the output error e_y tends to zero as t approaches infinity. In other words, the output of the augmented system approaches the output of the model asymptotically.

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