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A TOTALLY SUBJECTIVE MULTIFACTORIAL CHOICE PROCEDURE

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This contribution deals *inter alia* with the fuzzy number crunching issue and with the subjective ranking of fuzzy numbers using the bounds of their peaks and supports.

1. INTRODUCTION

An operational form of a multifactorial subjective choice problem may be:

$$(\mathrm{P}_1) \qquad \mathrm{DR}\ [\mathrm{VDP}(x)] \qquad \text{subject to } x \in X$$

where X is a finite object set, DR a decision rule and VDP a variable descriptor pattern, i.e., a vector of variable descriptors used for the characterization of the various objects in X; and subjectively estimated by the decision maker for each object x considered. VDP(x) is, thus, a subjective input vector. Problem (P_1) reads "Apply the decision rule DR to choose the best-compromise object in the set X, given the judgemental input vectors VDP(x), for $x \in X$". It reduces, in many situations, to:

$$(\mathrm{P}_2) \qquad \mathrm{DR}'\ [\mathrm{AR}(\mathrm{VDP}(x))] \qquad \text{subject to } x \in X$$

where DR′ is a decision rule and AR is a rule for aggregating the different elements of vector VDP(x). The present contribution assumes that a decision maker (who is also the assessor) and a decision analyst (who is also the consultant and facilitator) compose a decision-making unit and cooperate in order to find a best-compromise solution x^* for problem P_2, i.e., an object $x^* \in X$ that satisfies the decision maker most [5]. It proposes a totally subjective multifactorial choice procedure which involves the decision maker in the selection process step by step, for we concede the point that a decision maker is more confident in a problem solution and feels more comfortable with it, if he/she is totally involved in the process generating it, and because it results from his/her own perception and judgements.

2. THE SUBJECTIVE INPUTS MODELING AND CRUNCHING

In this contribution, the subjective inputs are expressed in natural language and the decision maker is allowed to assign quantitative, semiquantitative and ordered or linguistic and ordered values (see [10] for the meanings of these concepts) to the different variable descriptors. Therefore, these judgemental inputs may be modeled conveniently by crisp numbers, triangular fuzzy numbers, crisp intervals or fuzzy numbers with trapezoidal membership functions which are all encompassed by fuzzy intervals.

In the sequel, a fuzzy interval $\underline{A}$ will be denoted by $\underline{A} = (m_1/m_2,\ m_3/m_4)$, which is a four-characteristic-value representation, also known as Buckley's notation [1,2,3]. The values m_1, m_2, m_3 and m_4 satisfy the condition $m_1 \leq m_2 \leq m_3 \leq m_4$. The interval $[m_2,\ m_3]$ is the peak of $\underline{A}$ and $[m_1,\ m_4]$ its support. See Figure 1.

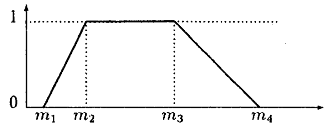

Fig. 1.

Let $\underline{A}_i$ for $i = 1, \ldots, p$ be a sequence of fuzzy intervals and let $T(\underline{A}_i)$ be the fuzzy interval image of $\underline{A}_i$ by a mapping T, then the $T(\underline{A}_i)$'s will be called well-conceived transformations of the $\underline{A}_i$'s if and only if:

1⁰) $\operatorname{Supp}(T(\underline{A}_i)) \subseteq [0,1]$, for $i = 1, \ldots, p$

2⁰) the transformation T favours a well specified fuzzy anchor value $\underline{A}_{i_0}$, among the $\underline{A}_i$'s, i.e., $T(\underline{A}_{i_0}) \geq T(A_i)$ for $i = 1, \ldots, p$.

Now let's assume that the object set X comprizes n distinct objects characterized by k different variable descriptors.

If $\underline{A}_{ij} = (a^1_{ij}/a^2_{ij},\ a^3_{ij}/a^4_{ij})$, where $0 < a^1_{ij} \leq a^2_{ij} \leq a^3_{ij} \leq a^4_{ij}$ for $j = 1, \ldots, k$, denotes the k positive fuzzy intervals used for the modeling of the various elements of the subjective input vector $\mathrm{VDP}(x_i)$ relative to the ith object x_i, for $i = 1, \ldots, n$ (at this point, one should notice that different scales may be employed), then $\underline{A}_{ij}$ for $i = 1, \ldots, n$, will be the sequence of estimates of the jth variable descriptor over the n objects.

Let $a^+_j = \max_{1 \leq i \leq n} a^4_{ij}$ and $a^-_j = \min_{1 \leq i \leq n} a^1_{ij}$, then the following well-conceived transformations may be considered:

$$\mathrm{T}_1: \quad \underline{A}_{ij} \to \underline{d}_{ij} = \left(\frac{a^1_{ij}}{a^+_j} \Big/ \frac{a^2_{ij}}{a^+_j},\ \frac{a^3_{ij}}{a^+_j} \Big/ \frac{a^4_{ij}}{a^+_j} \right), \qquad j = 1, \ldots, k$$

$$\mathrm{T}_2: \quad \underline{A}_{ij} \to \underline{d}_{ij} = \left(\frac{a^1_{ij} - a^-_j}{a^+_j - a^-_j} \Big/ \frac{a^2_{ij} - a^-_j}{a^+_j - a^-_j},\ \frac{a^3_{ij} - a^-_j}{a^+_j - a^-_j} \Big/ \frac{a^4_{ij} - a^-_j}{a^+_j - a^-_j} \right), \qquad j = 1, \ldots, k$$

$$\mathrm{T}_3: \quad \underline{A}_{ij} \to \underline{d}_{ij} = \left(\frac{a^-_j}{a^4_{ij}} \Big/ \frac{a^-_j}{a^3_{ij}},\ \frac{a^-_j}{a^2_{ij}} \Big/ \frac{a^-_j}{a^1_{ij}} \right), \qquad j = 1, \ldots, k$$

$$\mathrm{T}_4: \quad \underline{A}_{ij} \to \underline{d}_{ij} = \left(\frac{a_j^+ - a_{ij}^4}{a_j^+ - a_j^-} \Big/ \frac{a_j^+ - a_{ij}^3}{a_j^+ - a_j^-}, \frac{a_{ij}^+ - a_{ij}^2}{a_j^+ - a_j^-} \Big/ \frac{a_j^+ - a_{ij}^1}{a_j^+ - a_j^-} \right), \qquad j = 1, \ldots, k$$

$$\mathrm{T}_5: \quad \underline{A}_{ij} \to \underline{d}_{ij} = \left(d_{ij}^1 \,/\, d_{ij}^2,\, d_{ij}^3 \,/\, d_{ij}^4\right), \qquad j = 1, \ldots, k$$

where $d_{ij}^s = \frac{a_{ij}^s}{a_{ij}^4} \cdot \left[\frac{1}{2}\left(\frac{a_{ij}^4}{r_j^4} + \frac{r_j^4}{a_{ij}^4}\right)\right]^{-1}$ for $s = 1, 2, 3, 4$ and $\underline{r}_j = \left(r_j^1 \,/\, r_j^2,\, r_j^3 \,/\, r_j^4\right)$ the fuzzy anchor value of the jth variable descriptor. It is to be noted that:

1^0) The transformations T_1 and T_2 preserve the natural ordering already existing among the various estimates of any variable descriptor and favour the largest value assigned to it over the various objects considered, whereas the transformations T_3 and T_4 reverse it and favour the smallest value and the transformation T_5 violates it and favour any given preferred value other than the largest or the smallest.

2^0) Iff all the $\underline{A}_{ij}$'s are crisp numbers then the $T_m(\underline{A}_{ij})$'s for $m = 1, \ldots, 5$ are nothing but the degrees of closeness proposed by Zeleny [13].

3. AGGREGATION SCHEMES

The crunching of subjective inputs results for each object considered into a mathematical object which could be respectively a k-tuple $(\underline{A}_1, \underline{A}_2, \ldots, \underline{A}_k)$, such that $\mathrm{Supp}\,(\underline{A}_j) \subseteq [0,1]$, for $j = 1, \ldots, k$ a level 2 fuzzy set $((\underline{A}_1, p_1), (\underline{A}_2, p_2), \ldots, (\underline{A}_k, p_k))$, such that $\mathrm{Supp}\,(\underline{A}_j) \subseteq [0,1]$ and $p_j \in [0,1]$ for $j = 1, \ldots, k$ or a mixed fuzzy set $((\underline{A}_1, \underline{\pi}_1), (\underline{A}_2, \underline{\pi}_2), \ldots, (\underline{A}_k, \underline{\pi}_k))$, such that $\mathrm{Supp}\,(\underline{A}_j)$ and $\mathrm{Supp}\,(\underline{\pi}_j) \subseteq [0,1]$ for $j = 1, \ldots, k$, this will depend on whether the decision maker assigns equal crisp weights, unequal crisp weights or fuzzy weights to the different variable descriptors. If the assigned weights are crisp (equal or unequal), it is possible to use appropriate aggregation connectives as follows [11]:

if $\underline{A}_1 = (a_1 \,/\, b_1,\, c_1 \,/\, d_1)$ and $\underline{A}_2 = (a_2 \,/\, b_2,\, c_2 \,/\, d_2)$ are two fuzzy intervals having their supports in $[0,1]$, then $\underline{A}_1 \blacklozenge \underline{A}_2$ will be defined by:

$$\underline{A}_1 \blacklozenge \underline{A}_2 = (a_1 \Diamond a_2 \,/\, b_1 \Diamond b_2,\, c_1 \Diamond c_2 \,/\, d_1 \Diamond d_2)$$

where $\blacklozenge$ is an aggregation connective defined on the collection of fuzzy intervals having their supports in $[0,1]$ associated with the aggregation connective $\Diamond$ defined on the unit interval $[0,1]$. The operator $\Diamond$ may be chosen on the basis of relationships of compensation and competitiveness involved between the different variable descriptors [12]. If the assigned weights of importance are fuzzy, it is possible to employ a linear aggregation scheme [6, 8, 9] or generalized fuzzy sets reductions [4].

4. TOTALLY SUBJECTIVE MULTIFACTORIAL CHOICE PROCEDURE

4.1. The subjective rank-ordering of fuzzy numbers

The decision maker assigns subjectively probabilistic weights of importance w_i ($i = 1, 2, 3, 4$) respectively to the four characteristic values of the fuzzy interval. It can be easily shown that the pessimism-optimism indices associated with the support's level and the peak's level are respectively $\lambda = \frac{w_1}{w_1+w_4}$ and $\lambda' = \frac{w_2}{w_2+w_3}$ therefore a coherence condition could be: $\lambda = \lambda'$ that is, λ and λ' are independent of the level considered. Now, let Eval $(x) = (e_1(x) \,/\, e_2(x),\ e_3(x) \,/\, e_4(x))$ be the fuzzy-interval-valued evaluation of object x, the decision analyst starts by computing WACV (Eval (x)) $= \sum_{r=1}^{r=4} w_r \times e_r(x)$, for $x \in X$, i. e., a weighted average of the characteristic values and then uses the obtained values for the ordering purpose, using the following rules:

$$x_i > x_j \Longleftrightarrow \text{WACV}(\text{Eval}(x_i)) > \text{WACV}(\text{Eval}(x_j))$$

i. e., x_i is preferred to x_j

$$x_i \sim x_j \Longleftrightarrow \text{WACV}(\text{Eval}(x_i)) = \text{WACV}(\text{Eval}(x_j))$$

i. e., x_i is indifferent to x_j.

If the assigned weights are possibilistic, it is possible to transform these weights into probabilistic ones by means of the formula suggested by Dubois and Prade in [7].

The ordering functions WACV satisfies the following properties:

1^0) If $\underline{A} = (a \,/\, a,\ a \,/\, a)$, where a is a crisp number, then WACV $(\underline{A}) = a$. It follows that WACV preserves the natural ordering among crisp numbers.

2^0) $(\forall \underline{A} / \text{Supp}(\underline{A}) \subseteq [0,1])$: WACV(1) $\geq$ WACV$(\underline{A})$, that is, the maximum is attained for $(1/1,\ 1/1) = 1$ and WACW $(\underline{A}) \geq$ WACV(0), that is the minimum is attained for $(0/0,\ 0/0) = 0$.

3^0) WACV is continuous in the usual sense.

4.2. The steps of the selection process

The different steps of the selection process are the following:

STEP 1: The decision analyst obtains from the decision maker the subjective input vector relative to each object $x \in X$;

STEP 2: The decision analyst models the different subjective inputs by fuzzy intervals using Buckley's notation.

STEP 3: The decision analyst asks the decision maker to specify an anchor value among the estimates fo each variable descriptor over the objects considered.

STEP 4: Using well-conceived transformations, the decision analyst transforms the fuzzy intervals obtained in Step 2, in accordance with fuzzy anchor values specified in Step 3.

STEP 5: The decision analyst identifies a suitable aggregation operator compatible with the relationships of compensation and competitiveness involved by the decision maker between the different variable descriptors.

STEP 6: The decision analyst obtains from the decision maker the probabilistic weights of importance assigned to the four characteristic values and, then computes the fuzzy-interval-valued overall evaluations of the various objects.

STEP 7: The decision analyst applies the subjective rank-ordering procedure described in this contribution to the various fuzzy-interval-valued overall evaluations obtained in Step 5, and identifies the best-compromise solution(s).

5. CONCLUSION

The subject matter of this contribution was the making of a global subjective judgement based on an aggregation of crunched context-dependent formal representations (here fuzzy intervals) of judgemental inputs for a best-compromise alternative selection purpose. The procedure suggested takes, therefore, into account the human subjectivity and the fuzziness of the natural language in a practical way. A rather simple and easy to employ subjective approach for the rank-ordering of the fuzzy-interval-valued overall evaluations of the different objects was considered.

REFERENCES

[1] J. J. Buckley: Fuzzy hierarchical analysis. Fuzzy Sets and Systems *17* (1985), 233–247.

[2] J. J. Buckley: Ranking alternatives using fuzzy numbers. Fuzzy Sets and Systems *15* (1985), 21–31.

[3] J. J. Buckley: The fuzzy mathematics at finance. Fuzzy Sets and Systems *21* (1987), 257–273.

[4] J. J. Buckley: Generalized and extended fuzzy sets with applications. Fuzzy Sets and Systems *25* (1988), 159–174.

[5] V. Chankong and Y. Y. Haimes: Multiobjective Decision Making Theory and Methodology. North-Holland, Amsterdam 1983.

[6] D. Dubois and H. Prade: Decision-making under fuzziness. In: Advances in Fuzzy Set Theory and Applications (M. M. Gupta, R. K. Ragade and R. R. Yager eds.), North-Holland Amsterdam 1979, pp. 279–302.

[7] D. Dubois and H. Prade: Unfair coins and necessity measures: Towards a possibilistic interpretation of histograms. Fuzzy Sets and Systems *10* (1983), 15–20.

[8] D. Dubois and H. Prade: Fuzzy Sets and Systems: Theory and Applications. (Mathematics in Science and Engineering, Volume 144.) Academic Press, Orlando 1980.

[9] D. Dubois and H. Prade: The use of fuzzy numbers in decision analysis. In: Fuzzy Information and Decision Processes (M. M. Gupta and E. Sanchez, eds.), North-Holland, Amsterdam – Oxford – New York 1982, pp. 309–321.

[10] A. Pfeilsticker: The system approach and fuzzy set theory bridging the gap between mathematical and language-oriented economists. Fuzzy Sets and Systems *6* (1981), 209–233.

[11] A. M. Rebai: The aggregation of subjective inputs modeled by fuzzy intervals. Unpublished manuscript.

[12] R. R. Yager: Competetiveness and compensation in decision making: A fuzzy set based interpretation. Iona College Tech. Rep. RRY 78-14, New Rochelle, N. Y. 1978.

[13] M. Zeleny: Multiple Criteria Decison Making. McGraw-Hill, New York 1985.

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