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A VIEW ON FILTERING OF CONTINUOUS DATA SIGNALS

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A spline approximation of continuous signals and its extension to the piecewise polynomial functions has been derived. Bayesian identification is used for determining the parameters of the approximation. The conditions on the smoothness of the approximation are introduced in the form of prior information about the parameters through so called fictitious data. The approximation can be used e.g. for spline modelling, filtering of data signals and it enables differentiation of filtered signals.

1. INTRODUCTION

Adaptive control based on discrete-time models of the controlled plant is widely spread, simple, theoretically well elaborated and easy to implement on digital computers. Often, especially for slow processes, the digital control with "reasonably" long period of sampling and a low order model of the plant is fully satisfactory. Sometimes, however, the information about the controlled system lost between two sampling instants is non-negligible. An extraction of this information by a continuous or high rate filtering can be of a significant help.

In control problems in which noise and system dynamics are well separated almost any of the vast amount of filters available can be used. Situation becomes more difficult when the dynamics are close each other. Then, filters based on local modelling of the filtered signal [4, 5] seem to be the only feasible way. Because of the difficulty of the filtering problem solved quality of the solution depends much on the prior information fed into the filter. This naturally supports use of Bayesian methodology for it and especially new methodology [2] of fictitious data which admits to incorporate the available information in a systematic way.

Technically, the paper deals with a filtering based on approximation of the measured noisy signal by a function defined in a piecewise manner. The partial functions used for creating the approximation are made mutually dependent. For each node of the approximating-function domain, linkage conditions are set which connect the partial functions. Each connection is specified with a weight determining the importance of its precise fulfilling.

Here, a piecewise polynomial approximation is considered with conditions on continuity of the approximating function and some of its derivatives. For the conditions precisely fulfilled, a spline approximation is obtained.

In this way, the piecewise linear filtering described in [4] which considered no relation of generated lines is generalized in two directions:

1) Instead of piecewise lines it generates piecewise polynomials of arbitrary order.

2) The accuracy of connecting adjoint polynomials can be controlled by the introduced weights.

The paper starts with preliminaries with which the explanation of main results gets rid of the technicalities. The application areas which motivated the reported research, namely,

- spline modelling
- filtering of data signals
- differentiation of data signals

are described in Section 3 in detail. The main results are given in Sections 4 (filtering by spline approximation) and 5 (filtering by piecewise-defined functions). In Section 6, the theory is illustrated by a simple analytical example. Simulation results are presented in Section 7.

2. PRELIMINARIES

2.1. Spline functions

Consider a finite time interval $[0, T]$ divided by N nodes N_i, $i = 1, \dots, N$, $N_1 = 0$, $N_N = T$ to $N-1$ subintervals. A function $x(t)$, $t \in [0, T]$ is called spline of the degree m and the defect d (cf. [1], [6]) iff

- $x(t)$ is a polynomial of the degree at most m on each subinterval of the interval $[0, T]$,
- $x(t)$ possesses continuous derivatives up to the order $m-d$ on the open interval $(0, T)$.

As polynomials are naturally continuous with all their derivatives, it is sufficient to ensure the continuity conditions only at inner nodes of the domain. Thus, by the definition, a spline $x(t)$

1) has the form

$$x(t) = x_i(t) = \sum_{j=0}^{m} a_{j,i}(t - N_i)^j, \quad t \in (N_i, N_{i+1}) \tag{1}$$

for $i = 1, 2, \dots, N-1$

2) with continuity conditions on $m - d$ derivatives

$$\begin{aligned} x_i(t) &= x_{i-1}(t)|_{t=N_i}, \\ x_i^{(1)}(t) &= x_{i-1}^{(1)}(t)|_{t=N_i}, \\ \dots & \quad \dots \\ x_i^{(m-d)}(t) &= x_{i-1}^{(m-d)}(t)|_{t=N_i}, \end{aligned} \tag{2}$$

for $i = 2, 3, \dots, N-1$.

The smoothness conditions can be expressed in terms of the spline parameters $a_{j,i}$. They read

$$\begin{aligned} a_{0,i} &= \sum_{j=0}^{m} a_{j,i-1}(N_i - N_{i-1})^j, \\ a_{1,i} &= \sum_{j=1}^{m} j a_{j,i-1}(N_i - N_{i-1})^{j-1}, \\ \dots & \quad \dots \\ a_{m-d,i} &= \sum_{j=m-d}^{m} \binom{j}{j-m+d} a_{j,i-1}(N_i - N_{i-1})^{j-m+d}. \end{aligned} \tag{3}$$

The formulas (3) imply that $m - d + 1$ parameters of the polynomial at each interval (N_i, N_{i+1}) are deterministic functions of parameters from the previous interval (N_{i-1}, N_i).

As splines of given order and defect form a linear space, any spline $x(t)$ can be expressed in terms of a space basis

$$x(t) = \sum_{i=1}^{N} x_i q_{x,i}(t) = x^{\mathrm{T}} q_x(t), \tag{4}$$

where $q_{x,i}(t)$ are the base functions of the spline space, $^{\mathrm{T}}$ denotes transposition, x_i are coefficients of the spline approximation and

$$x^{\mathrm{T}} = [x_1, x_2, \dots, x_N], \qquad q_x^{\mathrm{T}}(t) = [q_{x,1}(t), q_{x,2}(t), \dots, q_{x,N}(t)].$$

As the basis we shall use the fundamental splines which are defined by the following property

$$q_{x,i}(N_j) = \delta_{i,j} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

for $i, j = 1, \dots, N$. For the base considered, the coefficients x_i are values of the spline $x(t)$ at its nodes N_i

$$x_i = x(N_i), \quad i = 1, 2, \dots, N.$$

2.2. Bayesian estimation of regression model

Consider a linear stochastic regression model

$$y(t) = P^{\mathrm{T}} z(t) + \varepsilon(t) \tag{5}$$

where

y is modelled variable (regressand),

$P^{\mathrm{T}} = (P_0, P_1, \dots, P_m)$ is vector of regression coefficients,

$z^{\mathrm{T}} = (z_0, z_1, \dots, z_m)$ is data regressor and

$\varepsilon(t)$ is noise term of the model which is supposed to be white and Gaussian with zero mean and variance r.

The unknown parameters of the model to be estimated are $\Theta = (P, r)$.

Suppose that at a time instant t we have at disposal the measured data $y(t), z(t)$ and the conditional probability density function (p.d.f.) $p(\Theta|D_{t-1})$ from the previous time instant $t-1$. D_{t-1} denotes the data $y(\tau), z(\tau)$, $\tau = 1, 2, \ldots, t-1$. Then, the posterior p.d.f. $p(\Theta|D_t)$ in which the piece of information from $y(t)$ and $z(t)$ is included can be computed as follows

$$p(\Theta|D_t) \propto p(y(t)|\Theta, z(t))p(\Theta|D_{t-1})$$

where the p.d.f. $p(y(t)|\Theta, z(t))$ is determined by the system model (5) as Gaussian p.d.f. with mean $P^{\mathrm{T}}z(t)$ and variance r.

It can be shown that the prior p.d.f. $p(\Theta)$ determining a proper Gauss-inverse-Wishart distribution reproduces for the model assumed. Thus, the conditional p.d.f. $p(\Theta|D_t)$ has a fixed functional form determined by statistics $\hat{P}, C, \nu, \hat{r}$ which evolve according to the following formulas

$$\begin{aligned} e_p &= y - \hat{P}^{\mathrm{T}}z, \quad \hat{P}_n = \hat{P} + \frac{Cz}{1+\zeta}e_p, \\ C_n &= C - \frac{Czz^{\mathrm{T}}C}{1+\zeta}, \\ \nu_n &= \nu + 1, \quad \hat{r}_n = \frac{1}{\nu_n}\left[\frac{e_p^2}{1+\zeta} - \hat{r}\right] \end{aligned} \tag{6}$$

where index n denotes new (updated) statistics, no index means old one,
y is the value of the approximated signal (regressand) measured at time instant t,
z denotes data vector (regressor) at the same time instant t,
$\hat{P}$ coincides with the least square estimate of the identified parameters,
C is proportional to the covariance matrix of P,
ν denotes a positive scalar counting the number of measurements,
$\hat{r}$ coincides with a point estimate of the noise variance r,
$\zeta = z^{\mathrm{T}}Cz$.

The above formulas coincide formally with famous recursive least squares [7] but they have fruitful (at least to the studied case) Bayesian interpretation which admits to built in prior information available (see the next paragraph).

For detailed discussion of Bayesian view point see e. g. [8].

2.3. Additional conditions in Bayesian estimation

Suppose that at a time instant t we have the p.d.f. $p(\Theta|D_{t-1})$. Instead of building in a new piece of information from the measured data $y(t)$ and $z(t)$ we are to build in some additional condition. The condition carries a piece of information (denoted by I) concerning Θ, for instance, that some of the parameters is more or less known: i. e. its mean is known and its variance is zero or small. An example of this type is given in Section 6.

Suppose that the condition can be expressed in the following form

$$\mathsf{E}\,[P^{\mathrm{T}}\bar{z} - \bar{y}|D_{t-1}, I] \;=\; 0, \quad \mathsf{E}\,[(P^{\mathrm{T}}\bar{z} - \bar{y})^2|D_{t-1}, I] = \bar{r} \tag{7}$$

where $\bar{y}, \bar{z}$ are so called fictitious data.

Another form of (7) is

$$\bar{y} = P^{\mathrm{T}}\bar{z} + \bar{e}$$

$\bar{e}$ is a fictitious noise with the conditional mean $\mathsf{E}\,[\bar{e}|D_{t-1}, I] = 0$ and the given variance $\mathsf{E}\,[(\bar{e})^2|D_{t-1}, I] = \bar{r}$.

Thus, the fictitious data express the information I in the form similar to that of the system model (5).

It has been shown recently in [2] that the additional condition can be built in to the p.d.f. $p(\Theta|\cdot)$ in the way of regular recursion (6). The data y, and z used in the recursion are computed from the fictitious data $\bar{y}$, and $\bar{z}$ mentioned above simply by multiplying and shifting them

$$y = \alpha\bar{y} + \beta, \qquad z = \alpha\bar{z} \tag{8}$$

where constant coefficients α and β depend on the values of the statistics (6) resulting from the previous identification and the variance $\bar{r}$ of the fictitious noise $\bar{e}$.

3. MOTIVATION FOR PIECEWISE FILTRATION

3.1. Spline-based modelling

Spline models (see [3]) belong to the class of so called hybrid models. They describe continuous reality in the sense that the modelled continuous variable can be predicted in an arbitrary (continuous) time instant but they have the form suitable for digital (discrete) treating. They have been developed mainly to improve discrete control with high sampling rate.

The starting point of spline-based modelling is the continuous convolution model

$$\int_0^t g(t-\tau)y(\tau)\mathrm{d}\tau = \int_0^t h(t-\tau)u(\tau)\mathrm{d}\tau + \varepsilon(t), \tag{9}$$

where the finite-support kernels g, h and the signals y (output) and u (input) are considered to be splines, $\varepsilon(t)$ is a noise term of the model. The finite supports of the kernels determine the length of the history of signals y and u necessary for the model to remember. They assure that the integrations in the model (9) are performed over a finite path even if the time t goes to infinity.

For the description of the splines g, h, y, u in the model, the form (4) has been chosen. After substituting into the convolution model (9) and integrating over the functions depending on time (the base functions) we obtain the discrete form of the model

$$g^{\mathrm{T}}P(t)y = h^{\mathrm{T}}Q(t)u + \varepsilon(t), \tag{10}$$

where
$g,\ h$ are vectors of model coefficients (samples of kernels $g(t),\ h(t)$ at their nodes),
$y,\ u$ are output and input data vectors (for finite support kernels they are finite vectors of samples of the signals $y(t)$ and $u(t)$ at their nodes).
P, Q are matrices of integrals of base functions products

$$P(t) = \int_0^t q_g^{\mathrm{T}}(t-\tau)q_y(\tau)\mathrm{d}\tau\ , \quad Q(t) = \int_0^t q_h^{\mathrm{T}}(t-\tau)q_u(\tau)\mathrm{d}\tau. \tag{11}$$

For given base functions they can be computed off-line and used as fixed filter matrices.

For identification of the parameters g, h , the model can be written as a regression one with filtered data $g^{\mathrm{T}}\tilde{y} = h^{\mathrm{T}}\tilde{u} + \varepsilon$, where $\tilde{y} = Py, \quad \tilde{u} = Qu$.

For control design, the model with filtered parameters $\tilde{g}^{\mathrm{T}}y = \tilde{h}^{\mathrm{T}}u + \varepsilon$, is used with $\tilde{g}^{\mathrm{T}} = g^{\mathrm{T}}P, \quad \tilde{h}^{\mathrm{T}} = h^{\mathrm{T}}Q$.

Further details can be found in [3].

3.2. Filtering of data signals

Let us list other cases where filtering of measured noisy data signals is indispensable.

- Smoothing, which removes high frequency noise from the data signals, is known to be necessary in almost all practical control problems.
- Equidistant sampling of data signals is supposed in the digital control. If the data are measured irregularly the filtering helps to recover continuous approximation of the measured data which can be sampled with a fixed desired period.
- Discrete control with low model order requires relatively long period of sampling. In order to utilize the information between sampling instants it is reasonable to use continuous or high rate measuring of data for the filtering. The low rate discrete data can be either simply sampled on the filtered signal or generated in some other more sophisticated way (e. g. as in the spline-based control).

3.3. Differentiation of data signals

The filtered data signal in the form of a piecewise-defined function which is composed of known smooth functions (e. g. polynomials) can also be differentiated. If, for instance, the approximation (4) is considered

$$x(t) = \sum_{i=1}^{N} x_i q_{x,i}(t)$$

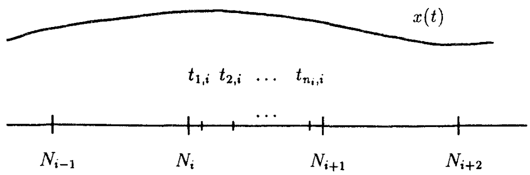
then its kth derivative $x^{(k)}(t)$ is

$$x^{(k)}(t) = \sum_{i=1}^{N} x_i q_{x,i}^{(k)}(t) = \sum_{i=1}^{N} x_i p_{x,i}(t)$$

where $p_{x,i}(t) = q_{x,i}^{(k)}(t),\ i = 1, 2, \ldots, N$ are fixed known functions. Thus, knowing the samples x_i of $x(t)$ signal values and derivatives can be computed at any selected point.

4. SPLINE APPROXIMATION OF DATA SIGNALS

For spline approximation of measured data signals, we shall consider the following arrangement



where
x is the signal to be approximated,
N_i are nodes of the approximating spline and
$t_{j,i}$ are time instants at which the continuous signal is measured and the spline approximation is fitted ($t_{j,i}$ denotes jth point in the ith interval (N_i, N_{i+1})). The distance between $t_{j,i}$ and $t_{j+1,i}$ is arbitrary.

Note, that the domain $[0, T]$ of the filtered signals is usually potentially infinite. Then, N increases with T.

The data measured on the ith interval are

$$\{y(t_{1,i}), y(t_{2,i}), \ldots, y(t_{n_i,i})\} \tag{12}$$

where n_i is the number of measurements on the ith interval.

The observed data are assumed to be related to the approximated signal $x(t)$ by

$$y(t_{j,i}) = x(t_{j,i}) + e(t_{j,i}), \quad j = 1, 2, \ldots, n_i \tag{13}$$

where
$e(t_{j,i})$ represents a combination of random and approximation errors and it is supposed to be white with normal distribution, zero mean and variance r,
$x(t)$ is mth order spline i.e.

$$x(t) = x_i(t) = \sum_{j=0}^{k} a_{j,i}(t - N_i)^j \quad \text{for} \quad t \in (N_i, N_{i+1})$$

and $i = 1, 2, \ldots, N$, i.e. on each interval (N_i, N_{i+1}), $x(t)$ is an mth order polynomial. We require $x(t)$ to be a spline of a defect d. Thus, each pair of the adjoin polynomials $x_{i-1}(t)$ and $x_i(t)$ is coupled by the conditions (3). The coupling restrict the $m - d + 1$ parameters related to the interval (N_i, N_{i+1}) $a_{j,i}$, $j = 0, 1, \ldots, m - d$ to be deterministic functions of the parameters from the previous interval.

Our task is to identify the parametèrs $\{a_{0,i}, a_{1,i}, \ldots, a_{m,i}\}$ of the polynomial $x_i(t)$ on the ith interval (N_i, N_{i+1}) using the data (12).

In order to reach recursive filtering we shall restrict ourselves by identifying the parameters a separately in separate intervals. Just smoothness conditions (3) will be used as additional information when starting the identification on a new interval.

We formulated the knowledge of a parameter subset on the given interval as additional conditions of Section 2. Clearly, we shall proceed more generally than necessary for the task solved. But it offers the direct hint for the task generalization treated in the next section.

For the interval (N_i, N_{i+1}) the model relating the measured data and unknown parameters has the form

$$y_i(t) = x_i(t) + e_i(t) = \sum_{j=0}^{m} a_{j,i}(t - N_i)^j + e_i(t) = P_i^{\mathrm{T}} z_i(t) + e_i(t)$$

where

$$\begin{aligned} P_i^{\mathrm{T}} &= (a_{0,i},\ a_{1,i},\ \ldots,\ a_{m,i}) \\ z_i^{\mathrm{T}}(t) &= (1,\ (t - N_i),\ \ldots,\ (t - N_i)^m). \end{aligned}$$

Thus, the gained model has the form (5) and the recursive least squares algorithm (6) is directly applicable.

The additional conditions for $x(t)$ to be a spline in the node N_i are (3). They can be written as the sequence of conditions having the form (7) with

$$\bar{z}_{li} = [\underbrace{0, \ldots, 1}_{l}, \ldots, 0] \tag{14}$$

$$\begin{aligned} \bar{y}_{li} &= \mathsf{E}\left[\sum_{j=l}^{m} \binom{j}{j-l} a_{l,i-1}\,(N_i - N_{i-1})^{j-l} | \mathcal{D}_{i-1}, I\right] \\ &= \mathsf{E}\left[\sum_{j=l}^{m} \binom{j}{j-l} a_{l,i-1}\,(N_i - N_{i-1})^{j-l} | \mathcal{D}_{i-1}\right] \end{aligned} \tag{15}$$

for $l = 0, 1, \ldots, m - d$. In the equation (15) $\mathcal{D}_{i-1}$ denotes the information about the approximated signal extracted from the data measured on the previous $(i-1)$th interval and the additional (continuity) information concerning the interval. I denotes the additional information concerning the ith interval. The second equality in (15) is a formal expression of our assumption that the information I from the ith interval does not influence the parameter estimates from the interval $(i - 1)$ which are used for computing the continuity conditions on the ith interval.

The exact meeting of the conditions (3) will be reached when

$$\bar{r} = 0.$$

The conditions are built in to the identified parameters according to (8).

5. PIECEWISE APPROXIMATION OF DATA SIGNALS

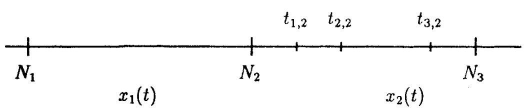
In the previous section, the conditions (3) determining the identified signal $x(t)$ to be a spline have been considered in a deterministic way, i.e. the variances of their fictitious models have been set to zero. However, the experiments with the designed algorithm show that even splines (especially those with defect 1) are not flexible enough to follow suitably the approximated signal. Especially, at the very beginning of the experiment, the initial value $x(0)$ and the derivatives $x^{(l)}(0)$, $l = 1, \dots, m-d$ have to be chosen very close to those of the approximated signal x. If not, the approximation error damps rather slowly.

The simple and very efficient way how to increase flexibility of the signal $x(t)$ is to release deterministic (precisely zero error variance) conditions on smoothness and to consider them in the probabilistic way in the form (15) with nonzero variances $\bar{r}$.

The approximation $x(t)$ we obtain in this way is not a spline but only a piecewise polynomial function. It has not the smoothness required for splines. Even, it need not be continuous if the variance of the condition requiring continuity of $x(t)$ is greater than zero. On the other hand such curve is very flexible and approximates acceptably not only values of $x(t)$ but gives us also some information about its derivatives which can be computed almost everywhere (not in the nodes).

6. EXAMPLE

To exemplify the results derived we shall consider the signal $x(t)$ to be approximated on two intervals with nodes N_1, N_2, N_3 as it is indicated in the following diagram.



The model of the signal $x(t)$ (2) is

$$\begin{aligned} x(t) &= x_1(t) + e(t) \quad t \in (N_1, N_2), \\ x(t) &= x_2(t) + e(t) \quad t \in (N_2, N_3). \end{aligned}$$

The spline approximation is of the order $m = 2$ with defect $d = 1$. For the 2nd order the polynomials x_1 and x_2 are

$$\begin{aligned} x_1(t) &= a_{0,1} + a_{1,1}(t - N_1) + a_{2,1}(t - N_1)^2, \\ x_2(t) &= a_{0,2} + a_{1,2}(t - N_2) + a_{2,2}(t - N_2)^2. \end{aligned}$$

We suppose the parameters on the first interval have already been estimated and we are to compute the smoothness conditions and to perform identification for the second interval using the data measured at time instants $\{t_{1,2}, t_{2,2}, t_{3,2}\}$ and respecting the smoothness conditions. For $m = 2$ and $d = 1$ we demand $2 - 1 = 1$ continuous derivative of the approximating spline. With the condition on the continuity of the spline itself we have two conditions on smoothness

$$x_2(N_2) = x_1(N_2) \text{ and } \quad x_2'(N_2) = x_1'(N_2),$$

which according to (3) gives

$$\begin{aligned} a_{0,2} &= a_{0,1} + a_{1,1}(N_2 - N_1) + a_{2,1}(N_2 - N_1)^2 = \bar{a}_{0,2} \\ a_{1,2} &= a_{1,1} + 2a_{2,1}(N_2 - N_1) = \bar{a}_{1,2} \end{aligned} \tag{16}$$

The regression model (5) for the second interval has the form

$$y(t) = P_2^{\mathrm{T}} z_2(t) + e \qquad t \in (N_2, N_3)$$

where

$$P_2^{\mathrm{T}} z_2(t) \; = \; x_2(t), \quad P_2^{\mathrm{T}} = [a_{0,2}, a_{1,2}, a_{2,2}], \quad z_2^{\mathrm{T}} = [1, (t - N_2), (t - N_2)^2]$$

for all measured t, i.e. $t \in \{t_{1,2}, t_{2,2}, t_{3,2}\}$.

The fictitious data for introducing the conditions (16) are

$$\begin{aligned} &\bar{y} = \bar{a}_{0,2}, \quad \bar{z} = [1, 0, 0] \quad \text{for the first condition, and} \\ &\bar{y} = \bar{a}_{1,2}, \quad \bar{z} = [0, 1, 0] \quad \text{for the second one.} \end{aligned}$$

7. EXPERIMENTAL RESULTS

For illustration of experimental results, the second order piecewise polynomial filtering has been chosen. The typical results are shown on the previous figures. Here the dashed lines represent the noisy signal to be filtered, the solid lines are the deterministic signals (derivatives of the signals) i.e. the signals without noise, the dotted lines are the filtered signals. Figure 1 shows the filtering by spline with defect $d = 1$ i.e. both the filtered signal and its derivative are continuous. It can be clearly seen that the spline approximation is not able to follow the signal. The improvement of approximation, attained in Figure 2, is caused by increasing the defect of the spline used for approximation to $d = 3$ (to discontinuity). It means that, now, both the approximation and its derivative are not forced to be continuous at nodes. The approximation is performed by a piecewise polynomial function.

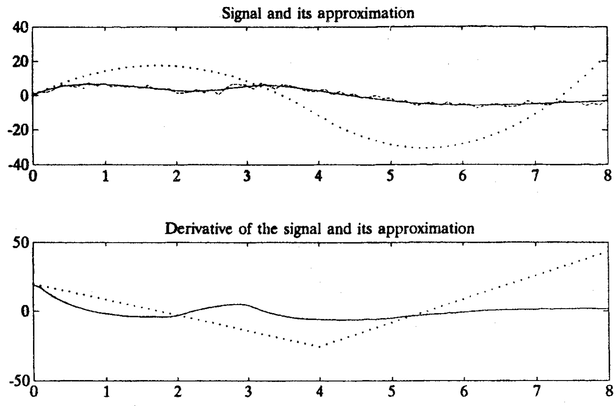


Fig. 1. The second order spline approximation.

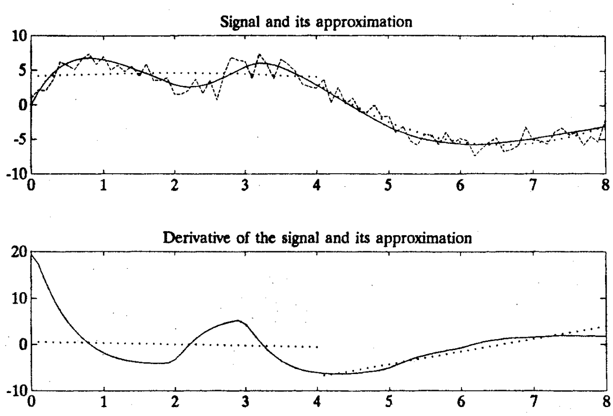


Fig. 2. The second order piecewise approximation.

8. CONCLUSIONS

A spline approximation of continuous signals and its extension to the piecewise polynomial functions has been derived. The extension of spline approximation consists in releasing strict conditions on spline smoothness in the nodes (i. e. continuity of some derivatives). Instead, only approximate conditions admitting some errors in fulfilling the required conditions are introduced. As a result, an approximation in the form of a piecewise (polynomial) function is obtained. This function need not even be continuous but it is more flexible than splines. It can better approximate both the function itself and also some of its derivatives (with the nodes avoided).

The approximation is a starting point for so called spline models, but, it can also be considered a suitable filtering for noisy measured data.

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REFERENCES

[1] C. De Boor: A Practical Guide to Splines. Springer-Verlag, Berlin – Heidelberg – New York 1978.

[2] M. Kárný: A note on feeding uncertain knowledge into recursive least squares. In: Proc. 30th CDC, Brighton, U.K., 1991.

[3] M. Kárný, I. Nagy, J. Böhm and A. Halousková: Design of spline-based self-tuners. Kybernetika *26* (1990), 1, 17–30.

[4] M. Kárný, A. Halousková and I.Nagy: Modelling, identification and adaptive control of cross-direction basis weight of paper sheets. Internat. Conf. CONTROL 88, Oxford 1988, pp. 159–164.

[5] R. Kohn and G. Ansley: Equivalence between Bayesian smoothness priors and optimal smoothning for function estimation. In: Bayesian Analysis of Time Series and Dynamic Models (J. C. Spall, ed.), Marcel Dekker, New York 1988, pp. 393–430.

[6] N. P. Kornejčuk: Splajny v teorii aproximacii. Nauka, Moskva 1984.

[7] L. Ljung and T. Söderström: Theory and Practice of Recursive Identification. MIT Press, Mass. 1983.

[8] V. Peterka: Bayesian approach to system identification. In: Trends and Progress in System Identification (P. Eykhoff, ed.), Pergamon Press, Oxford 1981, pp. 239–304.

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