KYBERNETIKA — VOLUME 28 (1992), NUMBER 5, PAGES 402–412

PARAMETRIZATION OF MULTI-OUTPUT AUTOREGRESSIVE–REGRESSIVE MODELS FOR SELF–TUNING CONTROL

MIROSLAV KÁRNÝ

Problem of parametrization of multi-output autoregressive regressive Gaussian model (ARX) is studied in the context of prior design of adaptive controllers.

The substantial role of prior distribution of unknown parameters on the parametrization is demonstrated. Among several parametrizations a nontraditional one is advocated which

- makes it possible to model the system output entrywise, thus it is very flexible;
- models relations among system outputs in a realistic way;
- is computationally cheap;
- adds an acceptable amount of redundant parameters comparing to the most general but computationally most demanding parametrization which organizes the unknown regression coefficients in column vector.

1. INTRODUCTION

Autoregressive-regressive model with exogenous inputs (ARX) is often used for modelling of controlled systems especially in self-tuning control [1]. Popularity of ARX models stems mainly from plausibility of least squares (LS) for estimating its parameters. If Bayesian setup is used, the statistics supplied by LS serve for a simple evalution of posterior probabilities on structures of ARX models which compete for the best description of the modelled system [2]. Thus, complete system identification can be performed within the LS framework.

The cited results proved to be reliable and quite complex identification tasks have been solved using them. In connection with preparation of theoretical tools for prior tuning of linear-quadratic-Gaussian selftuners [3] the problem of redundant parameters – which is of restricted importance in on line phase – has emerged.

This paper brings a sequence of simple propositions which summarize the relevant results on idenfitication of ARX model for multi-output (MO) systems and brings some arguments in favour of a nontraditional parametrization of ARX model called here separated parametrization.

2. PRELIMINARIES

2.1. Manipulations with arrays

An inspection of multivariate systems requires handling multi-index arrays. Readability of relations among them is much influenced by the notation. We hope the following one to be a lucky choice:

- the arrays are mostly column-oriented, a row-oriented array is gained by the transposition $'$ of the column-oriented one;
- ith column of a matrix with entries x_{jk} is denoted x_{*i}; x_{i*} means ith row, i.e. $x_{i*} = x'_{*i}$;
- the arrays assumed have generally non-rectangular shape (e.g. number of entries in x_{*i} varies with i): the quotation marks above should indicate this fact;
- the asterisk convention applies to tensors too: if a tensor S has the entries S_{ijk} then S_{ij*} means the vector gained after fixing the indices i, j and S_{i**} is a matrix selected from S_{ijk} when i is fixed.

3. BAYESIAN FORMALISM

Bayesian estimation adopted needs a probabilistic form of the model. For presenting it, we shall use the following notation and notions:

- $p(A|B)$ denotes the ***probability density function*** (abbr. p.d.f.) or the probability function (p.f.) of a random variable A conditioned on B (the random variable, its realization and the corresponding p.d.f. argument are not distinguished as usual; a distinction of the p.d.f. and p.f. will be clear from the context).
- $\mathcal{N}(y|\hat{y}, \sigma)$ denotes the *Gaussian p.d.f.* of a variable y determined by the expected ($\mathcal{E}$) value $\hat{y}$ of y and by the covariance σ.
- Data are measured on the system at discrete time moments labelled by $t = 1, 2, \ldots$ Those which can be directly manipulated are called the (system) *input* and denoted $u(t)$. The rest is called the (system) *output*, $y(t)$. The dimension of the output is denoted m.
- In conditioning, the following *data collections* are used

  $$t^i = \left\{\{y(\tau), u(\tau)\}_{\tau=1}^{t-1}, \{y_j(t)\}_{j=1}^{i}, u(t)\right\} \quad \text{for} \quad i = 0, 1, \ldots, m$$

  (by definition the set $\{\cdot\}_1^0$ is empty).
- The estimation task arises when a *system model* describing the output probability for a given past is *parametrized* by an *unknown (finite-dimensional) parameter* Θ, i.e. it is described by the p.d.f. $p(y(t)|t^0, \Theta)$. If the p.d.f. is viewed as function of Θ (data are fixed) then it is called *incremental likelihood function.*

- The estimation is performed assuming that the estimated parameter Θ is unknown to the input generator (it fulfils *natural conditions of control* [4]), i.e. assuming $u(t), \Theta$ to be independent under the condition t^0, i.e. $p(u(t)|t^0, \Theta) = p(u(t)|t^0)$.
- The posterior p.d.f. $p(\Theta|t^m)$ is *Bayesian parameter estimate*. Under natural conditions of control, it is related to the prior p.d.f. $p(\Theta)$ by the following version of *Bayes rule*: $p(\Theta|t^m) \propto M(\Theta; t^m)p(\Theta)$, where $\propto$ means proportionality by a Θ-independent factor. The *likelihood function* $M(\Theta; t^m)$ is the product of the incremental likelihood function $M(\Theta; t^m) = p(y(t)|t^0, \Theta)M(\Theta; (t-1)^m)$, $M(\Theta; 0^m) \equiv 1$.

4. MULTI-OUTPUT ARX MODEL

Two parametrizations of the multi-output autoregressive-regresive model of controlled systems (MO ARX) will be discussed.

Fundamental parametrization: The outputs are related to the past history by the equations

$$y_i(t) = \sum_{k=1}^{\bar{l}_i} \bar{\theta}_{ki}\bar{\psi}_{ki}(t) + \bar{e}_i(t) = \bar{\theta}_{i*}\bar{\psi}_{*i}(t) + \bar{e}_i(t), \quad i = 1, \ldots, m \tag{1}$$

where

$\bar{\theta}_{ki}$ are regression coefficients to be estimated;

$\bar{l}_i$ denotes the number of coefficients related to the ith output;

$\bar{\psi}_{*i}(t)$ is the regression vector available for predicting $y_i(t)$, the regressor is a known function of t^0;

$\bar{e}_i(t)$ are zero mean Gaussian random variables with the covariance structure

$$\mathcal{E}[\bar{e}_i(t)\bar{e}_j(\tau)|t^0, \bar{\Theta}] = \begin{cases} 0 & \text{for } t \neq \tau \\ \bar{\sigma}_{ij} & \text{for } t = \tau \end{cases}$$

given by an unknown *symmetric* positive definite covariance matrix $\bar{\sigma}$.

The unknown parameter $\bar{\Theta}$ of the fundamental parametrization is

$$\bar{\Theta} \equiv \left(\bar{\theta}_{ki},\ \bar{\sigma}_{ij},\ i = 1, \ldots, m,\ i \geq j,\ k = 1, \ldots, \bar{l}_i\right).$$

Separated parametrization: The outputs are related to the past history by the equations formally identical to (1)

$$y_i(t) = \sum_{k=1}^{l_i} \theta_{ki}\psi_{ki}(t) + e_i(t) = \theta_{i*}\psi_{*i}(t) + e_i(t), \quad i = 1, \ldots, m \tag{2}$$

with elements defined in a way which guarantees *independency of* $e(t)$ *entries*

θ_{ki} are regression coefficients to be estimated;

l_i denotes the number of coefficients related to the ith output;

$\psi_{*i}(t)$ is the regression vector available for predicting $y_i(t)$, the regressor is a known function of t^{i-1} (!);

$e_i(t)$ are zero mean Gaussian random variables having independent also entries (!)

$$\mathcal{E}[e_i(t)e_j(\tau)|t^j,\Theta] = \begin{cases} 0 & \text{for} \quad t \neq \tau \text{ or } i \neq j \\ \sigma_i & \text{for} \quad t = \tau \text{ and } i = j \end{cases}$$

given by an unknown *diagonal* positive definite covariance matrix σ.

The unknown parameter Θ of the separated parametrization is

$$\Theta \equiv (\theta_{ki};\ \sigma_i,\ i = 1,\ldots,m,\ k = 1,\ldots,l_i).$$

Let us stress the difference of the above definitions:

Fundamental parametrization has symmetrical noise covariance and unit matrix at the predicted output;

Separated parametrization has diagonal noise covariance and a triangular matrix at the predicted output.

5. ESTIMATION OF MO ARX

We shall write explicitly likelihood function as the key item needed in estimation.

Proposition 1 [*Incremental likelihood of MO ARX model.*] The incremental likelihoods of Gaussian MO ARX model take the forms:
Fundamental parametrization

$$\begin{aligned} & p(y(t)|t^0,\bar{\Theta}) = \qquad (3) \\ = \;& |2\pi\bar{\sigma}|^{-0.5}\exp\left\{-0.5\sum_{i,j=1}^{m}[y_i(t)-\bar{\theta}_{i*}\bar{\psi}_{*i}(t)](\bar{\sigma}^{-1})_{ij}[y_j(t)-\bar{\theta}_{*j}\bar{\psi}_{*j}(t)]\right\} = \\ = \;& |2\pi\bar{\sigma}|^{-0.5}\exp\left\{-0.5\sum_{i,j=1}^{m}(\bar{\sigma}^{-1})_{ij}\begin{bmatrix}-1\\ \bar{\theta}_{*i}\end{bmatrix}'\bar{\Psi}_{*i}(t)\bar{\Psi}_{j*}(t)\begin{bmatrix}-1\\ \bar{\theta}_{*j}\end{bmatrix}\right\} \end{aligned}$$

where the (regression) data vectors $\bar{\Psi}_{*i}(t)$ related to the time moment t and the ith output are introduced

$$\bar{\Psi}_{i*}(t) = [y_i(t), \bar{\psi}_{i*}(t)].$$

The dimension of the ith data vector is $\bar{L}_i = \bar{l}_i + 1$.
Separated parametrization

$$p(y(t)|t^0,\Theta) = \prod_{i=1}^{m}(2\pi\sigma_i)^{-0.5}\exp\left\{-\frac{0.5}{\sigma_i}[y_i(t)-\theta_{i*}\psi_{*i}(t)]^2\right\} \tag{4}$$
$$= \prod_{i=1}^{m}(2\pi\sigma_i)^{-0.5}\exp\left\{-\frac{0.5}{\sigma_i}\begin{bmatrix}-1\\ \theta_{*i}\end{bmatrix}'\Psi_{*i}(t)\Psi_{i*}(t)\begin{bmatrix}-1\\ \theta_{*i}\end{bmatrix}\right\}$$

where the (regression) data vectors $\Psi_{*i}(t)$, $i = 1,\dots,m$, related to the time moment t and the ith output are introduced

$$\Psi_{i*}(t) = [y_i(t),\dots,y_1(t),\psi_{i*}(t)]. \tag{5}$$

The dimension of the ith data vector is $L_i = l_i + i$.

Proof. By straightforward manipulations. □

Proposition 2 [*Likelihood of MO ARX model.*] The likelihoods of a Gaussian MO ARX model take the forms:
Fundamental parametrization

$$M(\bar{\Theta};t^m) = |2\pi\bar{\sigma}|^{-0.5\nu(t)}\exp\left\{-0.5\sum_{i,j=1}^{m}(\bar{\sigma}^{-1})_{ij}\begin{bmatrix}-1\\ \bar{\theta}_{*i}\end{bmatrix}'\bar{V}^{ij}(t)\begin{bmatrix}-1\\ \bar{\theta}_{*j}\end{bmatrix}\right\} \tag{6}$$

where the scalar $\nu(t)$ and $(\bar{L}_i,\bar{L}_j)$ matrices $\bar{V}^{ij}(t)$, $i,j = 1,\dots,m$, are sufficient statistics for estimating the parameter $\bar{\Theta}$. They are defined by the formulae

$$\bar{\nu}(t) = \bar{\nu}(t-1)+1, \quad \bar{\nu}(0) = 0$$
$$\bar{V}^{ij}(t) = \bar{V}^{ij}(t-1)+\bar{\Psi}_{*i}(t)\bar{\Psi}_{j*}(t), \quad \bar{V}^{ij}(0) = 0, \quad i,j = 1,\dots,m.$$

Generally, it is needed $\bar{st} = 1 + \sum_{i=1}^{m}\left[0.5\bar{L}_i(\bar{L}_i+1)+\sum_{j=1}^{i-1}\bar{L}_i\bar{L}_j\right]$ storage elements for keeping the values of these statistics.
Separated parametrization

$$M(\Theta;t^m) = \prod_{i=1}^{m}(2\pi\sigma_i)^{-0.5\nu(t)}\exp\left\{-\frac{0.5}{\sigma_i}\begin{bmatrix}-1\\ \theta_{*i}\end{bmatrix}'V^i(t)\begin{bmatrix}-1\\ \theta_{*i}\end{bmatrix}\right\} \tag{7}$$

where the scalar $\nu(t)$ and (L_i,L_i) matrices $V^i(t)$, $i = 1,\dots,m$, are sufficient statistics for estimating the parameter Θ. They are defined by the formulae

$$\nu(t) = \nu(t-1)+1, \quad \nu(0) = 0$$
$$V^i(t) = V^i(t-1)+\Psi_{*i}(t)\Psi_{i*}(t), \quad V^i(0) = 0, \quad i = 1,\dots,m.$$

Generally, it is needed $st = 1 + \sum_{i=1}^{m} 0.5L_i(L_i + 1) \approx \bar{st}/m$ storage elements for keeping the values of these statistics.

Proof. Implied by the Bayes rule and previous proposition. □

The exceptional position of Gaussian ARX model among practically used system descriptions stems from the fact that it possesses finite dimensional statistics and thus admits self-reproducing prior p.d.f.

Proposition 3 [*Estimation of MO ARX model with self-reproducing prior.*] Let us change the zero initial conditions in the recursions for sufficient statistics to:
Fundamental parametrization

$$\bar{\nu}(0) = \bar{\nu}_0, \text{ with a positive } \bar{\nu}_0$$

$$\bar{V}^{ij}(0) = \bar{V}_0^{ij} \text{ giving a positive definite } \begin{bmatrix} \bar{V}_0^{11} & \dots & \bar{V}_0^{1m} \\ \dots & \dots & \dots \\ \bar{V}_0^{m1} & \dots & \bar{V}_0^{mm} \end{bmatrix}.$$

Separated parametrization

$$\nu(0) = \nu_0, \text{ with a positive } \nu_0$$

$$V^i(0) = V_0^i \text{ with a positive definite } V_0^i.$$

Then the likelihood functions modified in this way are respectively proportional to $p(\bar{\Theta}|t^m)$ and $p(\Theta|t^m)$ with finite proportionality factors.

Proof. See [4]. □

Remark. It can be shown [4] that collection of the sufficient statistics is algebraically equivalent to least squares. The prior p.d.f. adds non-trivial initial conditions to recursive version of LS and regularize them.

6. COMPUTATION AND REDUNDANCY ASPECTS

6.1. Relations of the parametrizations

The parametrizations assumed are generically equivalent.

Proposition 4 [*Relations of the fundamental and separated parametrizations.*] Let $\bar{\sigma}$ be a positive definite matrix with the (necessarily unique) factorization $\bar{\sigma} = \mathcal{L}^{-1}S(\mathcal{L}^{-1})'$ where $\mathcal{L}$ is the lower triangular matrix with unit diagonal and S is the diagonal matrix.

Then – with probability 1 specified by $p(\bar{\Theta})$ – there is one-to-one mapping between both parametrizations which is given by the equalities

$$\begin{aligned} \sigma &= S, \quad l_i = \bar{l}_i + i - 1 \\ \psi_{ki}(t) &= \begin{cases} y_k(t) & \text{for } k < i \\ \bar{\psi}_{(k-i+1)i} & \text{for } k \geq i \end{cases}, \quad \theta_{ki} = \begin{cases} -\mathcal{L}_{ik} & \text{for } k < i \\ \sum_{j=1}^{i} \bar{\theta}_{(k-i+1)j}\mathcal{L}_{ij} & \text{for } k \geq i \end{cases} \end{aligned} \tag{8}$$

Proof. By straightforward algebraic manipulations. □

Proposition 4 is seemingly in a contradiction with Propositions 2 as the sufficient statistics of both models differ. The real difference, however, enters through the self-reproducing priors.

Proposition 5 [*Richness of self-reproducing priors.*] Let $\bar{\mathcal{P}}$, $\mathcal{P}$ be sets of all proper self-reproducing priors related to fundamental and separated models, respectively. Let us denote $\mathcal{T}$ mapping of $\bar{\mathcal{P}}$ described by (8). Then $\mathcal{P}$ is proper subset $\mathcal{T}\bar{\mathcal{P}}$.

Proof. The inclusion $\mathcal{P} \subseteq \mathcal{T}\bar{\mathcal{P}}$ is implied directly by the definition of $\mathcal{T}$. The strict inclusion is seen from the following example which will be used in the furher discussion too.

Let $m = 2$ and $p(\bar{\Theta}) = \bar{p}$ assign nonzero probability to the following $\bar{\Theta}$

$$\begin{aligned} \bar{\theta}_{1\bullet} &= [\bar{f}', \bar{g}', 0'], \quad \text{number of zeros } = dim(\bar{k}) \neq 0 \\ \bar{\theta}_{2\bullet} &= [0', \bar{h}', \bar{k}'], \quad \text{number of zeros } = \dim(\bar{f}) \neq 0, \ \dim(\bar{h}) = \dim(\bar{g}) \\ \mathcal{L}_{21} &= \alpha \end{aligned}$$

Then, there is no $p \in \mathcal{P}$ such that the corresponding image is $\bar{p}$ because the mapping

$$\begin{aligned} \theta_{1\bullet} &= [\bar{f}', \bar{g}', 0'] = \bar{\theta}_{1\bullet} \\ \theta_{2\bullet} &= [\alpha\bar{f}', \alpha\bar{g}' + \bar{h}', \bar{k}'] \end{aligned}$$

is non-invertible. □

Remarks.

1. Proposition 5 exemplifies that the prize we paid for smaller dimension of sufficient statistics is the loss of modelling flexibility: we are not able to assign fixed (zero) values to arbitrary entries of the estimated regression coefficients.

2. The superfluous parameters introduced are called *redundant*. With this notion, we can formulate the above statement in another way: redundancy is the price paid for handling MO model as independent single output models.

6.2. Special cases

Various special cases of the above parametrizations have been published. This paragraph discusses mutual relations of the most often met cases.

- *Matrix coefficients:*
  This is the case for which the fundamental parametrization fulfils $\bar{l}_i = \bar{l}$ and $\bar{\psi}_{*i}(t) = \bar{\psi}(t)$ with some fixed $\bar{l}$ and $\bar{\psi}(t)$. If we look on $\bar{\psi}_{*i}$ as selections from a "maximal" $\bar{\psi}$, the definition is equivalent to the requirement: if indices k, i exist such that $\bar{\theta}_{ki} \equiv 0$ then $\bar{\theta}_{k*} \equiv 0$.

  Separated-parametrization counterpart is gained through the mapping (8).

- *Independent parametrization:*
  This case is specified by diagonal noise covariance $\bar{\sigma}$ in the fundamental parametrization.

Proposition 6 [*Relations of special parametrizations.*]

- *Matrix parametrization*

$$M(\bar{\Theta}; t^m) \propto |\bar{\sigma}|^{-0.5\bar{\nu}(t)} \exp\left\{-0.5\mathrm{tr}\begin{bmatrix} -I \\ \bar{\theta} \end{bmatrix}' \bar{V}^M(t) \begin{bmatrix} -I \\ \bar{\theta} \end{bmatrix}\right\} \tag{9}$$

  where I is unit matrix of an appropriate dimension, the matrix $\bar{\theta}$ consists of columns $\bar{\theta}_{*i}$ and

$$\bar{V}^M(t) = \bar{V}^M(t-1) + \bar{\Psi}(t)\bar{\Psi}'(t), \quad \bar{\Psi}'(t) = [y'(t), \bar{\psi}'(t)], \quad \bar{V}^M(0) = 0.$$

  For separated parametrization, the likelihood $M(\theta; t^m)$ coincides with (7) for $V^i(t) = \bar{V}^M(t)$, $i = 1, \ldots, m$.

  If $\bar{\mathcal{P}}^M$ is the set of prior p.d.f. in $\bar{\mathcal{P}}$ restricted to matrix models then the mapping $\mathcal{T}$ restricted to it is always invertible.

- *Independent parametrization*
  The separated parametrization coincides with the fundamental one, thus likelihoods are given by (7): models related to the respective outputs are estimated independently.

Proof. By straightforward algebra. □

Proposition 7 [*Redundancy ordering of special parametrizations.*] Let n_F, n_S, n_M be minimal numbers of parameters of fundamental, separated and matrix parametrizations of the ARX model for describing of the same data, respectively. Then,

$$n_F \leq n_S \leq n_M$$

Proof. The minimality of n_F is implied by the construction. The second inequality can be proved by induction (over m) starting from the case given in the proof of Proposition 5. □

Remarks.

1. The matrix version is rather often used in MO system modelling. When some entry of $\bar{\psi}$ influences some output then this entry has to be assumed in the remaining channels too. This is the key drawback of this model.

   Separated model brings no advantages in this case.

2. Independent parametrization is plausible as it requires no redundant parameters. As the noise models all unmeasurable influences, the independency of the noise entries might be quite poor model of reality in MO cases.

6.3. Selection matrices in off-line estimation

The collection of statistics may be quite demanding task. In off-line identification, especially in connection with structure estimation, the collection of maximal statistics combined with use of "selection" matrices may be advantageous.

The *maximal data vectors* $\Phi(t)$ are defined as known functions of t^0 which:
- have a fixed length L;
- the data vectors $\bar{\Psi}_{*i}$ ($\Psi_{*i}(t)$) can be constructed from them by fixed selection ($\bar{L}_i, L$) ((L_i, L)) matrices $\bar{S}_{i**}$ (S_{i**})

$$\bar{\Psi}_{*i}(t) = \bar{S}_{i**}\Phi(t), \quad \Psi_{*i}(t) = S_{i**}\Phi(t), \quad t = 1, 2, \ldots.$$

Typically, $\bar{S}_{i**}$ and S_{i**} consist of zero and units and they are of a row-like shape. Some entries of the maximal data vector $\Phi(t)$ have to coincide necessarily with $y(t)$.

Proposition 8 [*Likelihood functions with selection matrices.*] Using selection matrices, the statistics $\bar{V}^M(t)$ (see Proposition 6) with $\bar{\Psi}(t) = \Phi(t)$ have to be stored for determining likelihoods for both parametrization assumed:

Fundamental model

$$M(\bar{\Theta}; t^m) = |2\pi\bar{\sigma}|^{-0.5\nu(t)} \exp\left\{-0.5 \sum_{i,j=1}^{m} (\bar{\sigma}^{-1})_{ij} \left[\begin{array}{c} -1 \\ \bar{\theta}_{*i} \end{array}\right]' \bar{S}_{i**}\bar{V}^M(t)\bar{S}_{**j} \left[\begin{array}{c} -1 \\ \bar{\theta}_{*j} \end{array}\right]\right\}.$$

Separated parametrization

$$M(\Theta; t^m) = \prod_{i=1}^{m} (2\pi\sigma_i)^{-0.5\nu(t)} \exp\left\{ -\frac{0.5}{\sigma_i} \begin{bmatrix} -1 \\ \theta_{*i} \end{bmatrix}' S_{i**} \bar{V}^M(t) S_{**i} \begin{bmatrix} -1 \\ \theta_{*i} \end{bmatrix} \right\}$$

The posterior p.d.f.'s are gained from the above likelihoods by choosing a positive definite $\bar{V}^M(0)$ iff for

Fundamental parametrization

$$\bar{V}^{ij}(0) = \bar{S}_{i**} \bar{V}^M(0) \bar{S}_{**j}, \; i, j = 1, \dots, m.$$

Separated parametrization

$$V^i(0) = S_{i**} \bar{V}^M(0) S_{**i}, \; i = 1, \dots, m.$$

Proof. A simple consequence of the definitions of the elements involved. □

Remark. The second part of Proposition 7 stresses again the influence of the prior p.d.f. on the parametrization: the possibility to update the statistics $\bar{V}^M(t)$ only depends on properties of $\bar{V}^M(0)$.

7. CONCLUSIONS

Parametrizations of multi-output ARX model have been discussed. They are characterized as follows:

Fundamental parametrization has symmetrical noise covariance and unit matrix at the predicted output;

Separated parametrization has diagonal noise covariance and a triangular matrix at the predicted output.

Matrix parametrization coincides with the fundamental one when no regressor entry can be omitted in predicting any output entry.

Independent parametrization coincides with the separated one with triangular matrix reduced to unit matrix.

Methodological gain of the paper lies in the recognition of the key role of prior probability density functions in defining a model structure.

From the practical view point, it has been shown that

- the fundamental parametrization is the most flexible in exploiting a priori known values of regression coefficients; the flexibility is, however, paid by substantial increase of computational demands;

- the separated parametrization retains a lot of flexibility of the fundamental parametrization; it introduces some redundant parameters but it spares a lot of computations;
- the matrix parametrization used up to now is uniformly worse that the separated one and should be avoided;
- the independent parametrization often recommended and used is computationally close to separated parametrization but rather often it can be poor model of reality and should be avoided, too.

(Received August 6, 1991.)

REFERENCES

[1] M. Kárný, A. Halousková, J. Böhm, R. Kulhavý and P. Nedoma: Design of linear quadratic adaptive control: theory and algorithms for practice. Kybernetika *21* (1985), Supplement to numbers 3, 4, 5 and 6.

[2] M. Kárný and R. Kulhavý: Structure determination of regression-type models for adaptive prediction and control. In: Bayesian Analysis of Time Series and Dynamic Models (J. C. Spall, ed.), Marcel Dekker, New York 1988.

[3] M. Kárný and A. Halousková: DESIGNER – tool for preliminary tuning of LQG self-tuners. In: Advanced Methods in Adaptive Control for Industrial Applications (Lecture Notes in Control and Information Sciences 158 (K. Warwick, M. Kárný and A. Halousková, eds.), Springer-Verlag, Berlin – Heidelberg – New York – London – Paris – Tokyo 1991, pp. 305–322.

[4] V. Peterka: Bayesian system identification. In: Trends and Progress in System Identification (P. Eykhoff, ed.), Pergamon Press, Oxford 1981, pp. 239–304.

Ing. Miroslav Kárný, DrSc., Ústav teorie informace a automatizace ČSAV (Institute of Information Theory and Automation – Czechoslovak Academy of Sciences), Pod vodárenskou věží 4, 182 08 Praha 8. Czechoslovakia.