λ-MEASURES OF HYPOENTROPY AND COMPARISON OF EXPERIMENTS: BLACKWELL AND LEHMANN APPROACH

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Ferreri [5] introduced and studied a λ-measure of hypoentropy. Taneja [10] extended it towards applications point of view. Recently, authors [9] studied its applications to compare the experiments using Bayesian approach. In this paper, our aim is to study more applications of this measure in comparison of experiments applying Blackwell's as well as Lehmann's approach.

1. INTRODUCTION

Let $(\Theta, \beta_\Theta, \tau)$ be the parametric probability space where we suppose that τ is dominated either by a countable measure or by the Lebesgue measure; i.e., $\tau \ll \nu$. Let $p(\theta) = (\mathrm{d}\tau/\mathrm{d}\nu)(\theta)$ denote its Random-Nykodym derivative. The statistical space $(\mathscr{X}, \beta_{\mathscr{X}}, P_\theta)_{\theta\in\Theta}$ associated to the experiment $\mathscr{E}_X$ is such that $P_\theta \ll \mu\ \forall\theta \in \Theta$, where μ could be either a countable measure or the Lebesgue measure. Let $f(x/\theta) = \mathrm{d}P_\theta/\mathrm{d}\mu\,(x)$ be its probability function or its density function. Finally, we denote by $f(x)$ and $p(\theta/x)$, the predictive and posterior distributions respectively.

The expected λ-measure of information obtained before observing the experiment $\mathscr{E}_X$ is given by

$$I_\lambda(X; p(\cdot)) = \mathsf{E}_X[H_\lambda(p(\cdot)) - H_\lambda(p(\cdot \mid x))] = \\ = \frac{1}{\lambda}\left\{\int_{\mathscr{X}}\int_\Theta (1 + \lambda p(\theta \mid x)) \log\left(\frac{1 + \lambda p(\theta \mid x)}{1 + \lambda p(\theta)}\right) f(x)\,\mathrm{d}\nu(\theta)\,\mathrm{d}\mu(x)\right\}, \quad \lambda > 0 \tag{1}$$

where

$$H_\lambda(p(\cdot)) = \left(1 + \frac{1}{\lambda}\right)\log\left(1 + \frac{1}{\lambda}\right) - \frac{1}{\lambda}\int_\Theta (1 + \lambda p(\theta)) \log(1 + \lambda p(\theta))\,\mathrm{d}\nu(\theta)$$

is the well known λ-hypoentropy (cf. [5]).

Bayesian's approach adopted by the authors [9] in comparison of experiments

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based on λ-measure of hypoentropy is as follows:

Definition 1. Let $\mathscr{E}_X = \{\mathscr{X}, \beta_{\mathscr{X}}, P_\theta;\ \theta \in \Theta\}$ and $\mathscr{E}_Y = \{\mathscr{Y}, \beta_{\mathscr{Y}}, Q_\theta;\ \theta \in \Theta\}$ be two experiments. We say that the experiment $\mathscr{E}_X$ is preferred to $\mathscr{E}_Y$, for given prior $p(\cdot)$, denoted by, $\mathscr{E}_X \geqq^\lambda \mathscr{E}_Y$, iff

$$I_\lambda(X; p(\cdot)) \geqq I_\lambda(Y; p(\cdot)) \tag{2}$$

In this, paper we analyse the relation between the criterion (2) given in Definition 1 and the classical criterion of Lehmann. We also study the relation between (2) and the two well known Blackwell's criteria based on the sufficiency property and on the decision theory.

2. RELATION WITH THE CLASSICAL CRITERIA

The following section studies the relation between the criterion given in (2) and Lehmann's criterion.

2.1 Relation with the Lehmann's criterion

Lehmann's [7] definition of comparing two experiments is stated as follows:

Definition 2. The experiment $\mathscr{E}_X$ is preferred to experiment $\mathscr{E}_Y$, denoted by $\mathscr{E}_X \geqq^L$ $\geqq^L \mathscr{E}_Y$ iff there exists an experiment $\mathscr{E}_U$ with known distribution independent of $\mathscr{E}_X$, and also there exists a measurable function $h(x, u)$ such that the random variable $H = h(X, U)$ is equally distributed to Y for every $\theta \in \Theta$.

Theorem 1. Let $\{\mathscr{X}, \beta_{\mathscr{X}}, P_\theta; \theta \in \Theta\}$ and $\{\mathscr{Y}, \beta_{\mathscr{Y}}, Q_\theta; \theta \in \Theta\}$ be two statistical experiments with the same parameter space. If $\mathscr{E}_X \geqq^L \mathscr{E}_Y$, then for every prior $p(\cdot)$ $\mathscr{E}_X \geqq^\lambda \mathscr{E}_Y$.

Proof. As $\mathscr{E}_X \geqq^L \mathscr{E}_Y$, there exist a random variable U independent of X and θ with completely known distribution and a measurable function h such that $H = h(X, U)$ is equally distributed to Y for every $\theta \in \Theta$. Since $f(u \mid \theta) = f(u)$ and $f(x, u \mid \theta) = f(x \mid \theta) f(u \mid \theta)$, then we have

$$f(x, u \mid \theta_0, x_0) = f(u) \quad \text{and} \quad f(h \mid \theta, x) = f(h \mid x)$$

We know that [see [9], property 2]

$$I_\lambda(X; p(\cdot)) = I_\lambda(X, H; p(\cdot)) \geqq I_\lambda(H; p(\cdot)) \tag{3}$$

Also,

$$f(h \mid \theta) = f(y \mid \theta) \quad \forall p(\cdot)$$

This gives

$$I_\lambda(H; p(\cdot)) = I_\lambda(Y; p(\cdot)) \tag{4}$$

Expressions (3) and (4) completes the required result, i.e.,

$$I_\lambda(X; p(\cdot)) \geqq I_\lambda(Y; p(\cdot))$$ □

2.2 Relations with Blackwell's criteria

First we study the relation between the criterion given in (2) and the Blackwell's sufficient criterion.

Definition 3. Let $\{\mathscr{X}, \beta_{\mathscr{X}}, P_\theta; \theta \in \Theta\}$ and $\{\mathscr{Y}, \beta_{\mathscr{Y}}, Q_\theta;\ \theta \in \Theta\}$ be two statistical experiments with the same parameter space. Blackwell's method for comparing two experiments states that the experiment $\mathscr{E}_X$ is sufficient for the experiment $\mathscr{E}_Y$, denoted by $\mathscr{E}_X \geqq^B \mathscr{E}_Y$, if there exists a measurable transformation $h\colon \mathscr{X} \times \mathscr{Y} \to \mathbb{R}$ satisfying:
(a) $f(y \mid \theta) = \int_{\mathscr{X}} h(x, y) f(x \mid \theta)\, \mathrm{d}\mu(x) \quad \forall \sigma \in \Theta \quad \forall y \in \mathscr{Y}$
(b) For every fixed $x \in \mathscr{X}$, $h(x, y)$ is a probability density function on $(\mathscr{Y}, \beta_{\mathscr{Y}})$, and
(c) $\int_{\mathscr{X}} h(x, y)\, \mathrm{d}\mu(x) < \infty \quad \forall y \in \mathscr{Y}$

To compare the maximizing criterion of λ-measure of hypoentropy with Blackwell's criterion, it is necessary to distinguish whether the parameter space is finite or not. In the latter case, we need a hypothesis of completeness over the family of distribution functions associated to the experiment $\mathscr{E}_X$.

The following theorems cover both the situations.

Theorem 2. Let $\{\mathscr{X}, \beta_{\mathscr{X}}, f(x \mid \theta); \theta \in \Theta\}$ and $\{\mathscr{Y}, \beta_{\mathscr{Y}}, f(y \mid \theta); \theta \in \Theta\}$ be two statistical experiments defined over $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ and $p(\theta) = (p(\theta_1), p(\theta_2), \ldots p(\theta_n))$ be a probability distribution on Θ. If $\mathscr{E}_X \geqq^B \mathscr{E}_Y$, then $\mathscr{E}_X \geqq^\lambda \mathscr{E}_Y$.

Proof. As Y is sufficient for X, then by the Blackwell's criterion, there exists a measurable function $h\colon \mathscr{X} \times \mathscr{Y} \to \mathbb{R}^+$ satisfying the following:
(a) $f(y \mid \theta) = \int_{\mathscr{X}} h(x, y) f(x \mid \theta)\, \mathrm{d}\mu(x) \quad \forall \theta \in \Theta \quad \forall y \in \mathscr{Y}$
(b) For every fixed $x \in \mathscr{X}$, $h(x, y)$ is a probability density function on $(\mathscr{Y}, \beta_{\mathscr{Y}})$, and
(c) $\int_{\mathscr{X}} h(x, y)\, \mathrm{d}\mu(x) < \infty \quad \forall y \in \mathscr{Y}$

By considering y fixed, we define

$$g_y(\theta_i) = \frac{\int_{\mathscr{X}} h(x, y) f(x \mid \theta_i)\, \mathrm{d}\mu(x)}{\int_{\mathscr{X}} h(x, y)\, \mathrm{d}\mu(x)}$$

Then

$$p(\theta_i|y) = \frac{f(y|\theta_i)p(\theta_i)}{f(y)} = \frac{\int_{\mathscr{X}} h(x, y) f(x \mid \theta_i)\, \mathrm{d}\mu(x)}{f(y)}\, p(\theta_i) =$$

$$= \frac{p(\theta_i)\,[g_y(\theta_i) \int_{\mathscr{X}} h(x, y)\, \mathrm{d}\mu(x)]}{\int_{\mathscr{X}} h(x, y) f(x)\, \mathrm{d}\mu(x)}$$

Let us consider an N-dimensional random variable $Z = (Z_{\theta_1} = (f(x \mid \theta_1)), \ldots, Z_{\theta_N} = f(x \mid \theta_N))$, where $\forall i\ Z_{\theta_i}$ have the density

$$f(x \mid y) = \frac{h(x, y)}{\int_{\mathscr{X}} h(x, y)\, \mathrm{d}\mu(x)}$$

Moreover, we suppose that the variables Z_{θ_i} $(i = 1, \ldots, n)$ are independent. Let us consider the following convex function:

$$H(x_1, \ldots, x_N) = -\left(\sum_{i=1}^{N} x_i p_i\right)\left[\sum_{i=1}^{N}\left(1 + \lambda \frac{x_i p_i}{\sum_{i=1}^{N} x_i p_i}\right)\log\left(1 + \lambda \frac{x_i p_i}{\sum_{i=1}^{N} x_i p_i}\right)\right]$$

where p_i is $p(\theta_i)$.

As

$$\mathsf{E}[Z_{\theta_i}] = \int_{\mathscr{X}} f(x \mid \theta_i) \frac{h(x, y)}{\int_{\mathscr{X}} h(x, y)\, d\mu(x)}\, d\mu(x) = g_y(\theta_i)$$

it follows that

$$H(\mathsf{E}[Z_{\theta_1}], \ldots, \mathsf{E}[Z_{\theta_N}]) = H(g_y(\theta_1), \ldots, g_y(\theta_N)) =$$

$$= -\left(\sum_{i=1}^{N} g_y(\theta_i)\, p(\theta_i)\right)\left[\sum_{i=1}^{N}\left(1 + \lambda \frac{g_y(\theta_i)\, p(\theta_i)}{\sum_{i=1}^{N} g_y(\theta_i)\, p(\theta_i)}\right)\log\left(1 + \lambda \frac{g_y(\theta_i)\, p(\theta_i)}{\sum_{i=1}^{N} g_y(\theta_i)\, p(\theta_i)}\right)\right] =$$

$$= -\frac{f(y)}{\int_{\mathscr{X}} h(x, y)\, d\mu(x)}\left[\sum_{i=1}^{N}(1 + \lambda p(\theta_i \mid y))\log(1 + \lambda p(\theta_i \mid y))\right]$$

Applying Jensen's inequality, we have

$$H(\mathsf{E}[Z_{\theta_1}], \ldots, \mathsf{E}[Z_{\theta_N}]) \leqq \mathsf{E}[H(Z_{\theta_1}, \ldots, Z_{\theta_N})]$$

i.e.,

$$-\frac{f(y)}{\int_{\mathscr{X}} h(x, y)\, d\mu(x)}\left[\sum_{i=1}^{N}(1 + \lambda p(\theta_i \mid y))\log(1 + \lambda p(\theta_i \mid y))\right] \leqq$$

$$\leqq \frac{-1}{\int_{\mathscr{X}} h(x, y)\, d\mu(x)} \int_{\mathscr{X}} \sum_{i=1}^{N}(1 + \lambda p(\theta_i \mid x))\log(1 + \lambda p(\theta_i \mid x))\, h(x, y) f(x)\, d\mu(x)$$

Integrating with respect to Y, we have

$$-\int_{\mathscr{Y}}\left[\sum_{i=1}^{N}(1 + \lambda p(\theta_i \mid y))\log(1 + \lambda p(\theta_i \mid y))\right] f(y)\, d\mu(y) \leqq$$

$$\leqq -\int_{\mathscr{X}}\left[\sum_{i=1}^{N}(1 + \lambda p(\theta_i \mid x))\log(1 + \lambda p(\theta_i \mid x))\right]\left(\int_{\mathscr{Y}} h(x, y)\, d\mu(y)\right) f(x)\,.$$

$$.\, d\mu(x)$$

i.e.,

$$I(X; p(\cdot)) \geqq I(Y; p(\cdot))\,. \qquad \square$$

Theorem 3. Let $\{\mathscr{X}, \beta_{\mathscr{X}}, f(x \mid \theta);\ \theta \in \Theta\}$ and $\{\mathscr{Y}, \beta_{\mathscr{Y}}, g(y \mid \theta); \theta \in \Theta\}$ be two experiments defined over Θ such that $(P_\theta)_{\theta\in\Theta}$ is a complete family of distributions. Then, if $\mathscr{E}_X$ is preferred to the experiment $\mathscr{E}_Y$ according to Blackwell's criterion, then $\mathscr{E}_X \geqq^{\lambda} \mathscr{E}_Y$.

Proof. As $\mathscr{E}_X$ is sufficient for $\mathscr{E}_Y$, we have

$$g(y \mid \theta) = \int_{\mathscr{X}} h(x, y) f(x \mid \theta)\, d\mu(x)\,.$$

On the other side, we have

$$g(y \mid \theta) = \int_{\mathscr{X}} g(y \mid x, \theta) f(x \mid \theta) \, d\mu(x) .$$

This gives

$$\int_{\mathscr{X}} (h(x, y) - g(y \mid x, \theta)) f(x \mid \theta) \, d\mu(x) = 0$$

i.e.,

$$\mathsf{E}_\theta[h(x, y) - g(y \mid x, \theta)] = 0 .$$

Since $\{f(x \mid \theta))\}_{\theta \in \Theta}$ is a complete family, then

$$h(x, y) - g(y \mid x, \theta) = 0 \quad \text{a.e.} \quad \mu(x) , \quad \forall y, \quad \forall \theta .$$

i.e., $g(y \mid x, \theta)$ is independent of θ a.e. $\mu(x)$. Therefore

$$I_\lambda(X, Y; p(\cdot)) = I_\lambda(Y; p(\cdot)) .$$

Also, we know that (see authors [9])

$$I_\lambda((X, Y); p(\cdot)) \geqq I_\lambda(Y; p(\cdot)) .$$

This gives

$$I_\lambda(X; p(\cdot)) \geqq I_\lambda(Y; p(\cdot)) .$$

□

Now we shall study the relation between the Blackwell's criterion based on the Decision Theory and the λ-measure of hypoentropy criterion, when the parameter space is finite and the prior distribution is uniform.

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$ be a finite parameter space. Let $\mathscr{E}_X = \{\mathscr{X}, \beta_{\mathscr{X}}, P_{\theta_i}\}_{i=1,2,\ldots N}$ be an experiment for each i. Let us consider a pair $(\mathscr{E}_X, A)$, where A is a closed bounded convex subset of $\mathbb{R}^N$ whose elements are terminal action points $a = (a_1, a_2, \ldots, a_N)$, i.e., $a_j = L(\theta_j, d)$ $(j = 1, 2, \ldots, N)$ is the loss from action a, and d is an arbitrary decision function. When the state of nature is θ_j, the risk is

$$R_j = R(\theta_j, d) = \int_{\mathscr{X}} L(\theta_j, d) \, dP_{\theta_j}(x) \quad (j = 1, 2, \ldots, N) .$$

As d varies over all possible decision functions, for the risk problem we have

$$B(\mathscr{E}_X, A) = \{R = (R_1, \ldots, R_N) \mid d \in D\} .$$

According to Blackwell's definition, we say that the experiment $\mathscr{E}_X$ is more informative than $\mathscr{E}_Y$, written by $\mathscr{E}_X \supset \mathscr{E}_Y$, if for every $A \subset \mathbb{R}^N$, closed, bounded and convex set, we have $B(\mathscr{E}_X, A) \supset B(\mathscr{E}_Y, A)$.

Reduction to standard experiment gives a condition equivalent to $\mathscr{E}_X \subset \mathscr{E}_Y$.

For any experiment $\mathscr{E}_X = \{\mathscr{X}, \beta_{\mathscr{X}}, P_{\theta_i}\}_{i=1,2,\ldots,N}$, let $p_{\theta_i}(x)$ be the density of P_{θ_i} with respect to $NP_0 = P_{\theta_1} + \ldots + P_{\theta_N}$. Let $\mathscr{Z}$ be the set of N-tuples $z = (z_1, z_2, \ldots, z_n)$, $z_i \geqq 0$, $\sum_{i=1}^{N} z_i = 1$. For any Borel subset A of $\mathscr{Z}$, let us define

$$m_i(A) = P_{\theta_i}(\{x \in X \mid p(x) = (p_{\theta_1}(x), \ldots, p_{\theta_N}(x)) \in A\})$$

so that m_i $(i = 1, 2, \ldots, N)$ is the distribution of z, where x has distribution P_{θ_i}.

We now have a new experiment

$$\mathscr{E}_X^* = \{\mathscr{X}, \beta_{\mathscr{X}}, m_i\}_{i=1,2,\dots,N}$$

called standard experiment and the measure

$$m_X = \frac{1}{N} \sum_{i=1}^{N} m_i$$

defined over $(\mathscr{X}, \beta_{\mathscr{X}})$ is called the standard measure.

The following result (cf. [1]) is a valuable tool in the comparison of experiments:

Let $\mathscr{E}_X$ and $\mathscr{E}_Y$ be two experiments with standard measures m_X and m_Y respectively. $\mathscr{E}_X \supset \mathscr{E}_Y$ if and only if for every continuous convex function $g(p)$,

$$\int_{\mathscr{X}} g(p)\, \mathrm{d}m_X \geqq \int g(p)\, \mathrm{d}m_Y$$

Theorem 4. Let $\Theta = \{\theta_1, \theta_2, \dots, \theta_N\}$ be a finite parameter space with the uniform distribution and let $\mathscr{E}_X$ and $\mathscr{E}_Y$ be two experiments. If $\mathscr{E}_X \supset \mathscr{E}_Y$, then $\mathscr{E}_X \geqq^{\lambda} \mathscr{E}_Y$.

Proof. We have to prove that

$$\mathsf{E}_X[H_\lambda(p(\cdot \mid x))] \leqq \mathsf{E}_Y[H_\lambda(p(\cdot \mid y))]$$

for all $p(\theta_i) = 1/N$, $i = 1, 2, \dots, N$ i.e.,

$$\int_{\mathscr{X}} [\int_\Theta (1 + \lambda p(\theta \mid x)) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)]\, f(x)\, \mathrm{d}\mu(x) \geqq$$
$$\geqq \int_{\mathscr{Y}} [\int_\Theta (1 + \lambda p(\theta \mid y)) \log (1 + \lambda p(\theta \mid y))\, \mathrm{d}\lambda(\theta)]\, f(y)\, \mathrm{d}\mu(y)$$

Considering the left hand side of the above inequality, we have

$$\int_{\mathscr{X}} [\int_\Theta (1 + \lambda p(\theta \mid x)) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)]\, f(x)\, \mathrm{d}\mu(x) =$$
$$= \int_{\mathscr{X}} \int_\Theta (f(x) + \lambda f(x \mid \theta)\, p(\theta)) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)\, \mathrm{d}\mu(x) =$$
$$= \int_{\mathscr{X}} \int_\Theta f(x) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)\, \mathrm{d}\mu(x) +$$
$$+ \lambda \int_{\mathscr{X}} \int_\Theta f(x \mid \theta)\, p(\theta) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)\, \mathrm{d}\mu(x) =$$
$$= \int_{\mathscr{X}} \int_\Theta [\int_\Theta f(x \mid \theta)\, p(\theta)\, \mathrm{d}\lambda(\theta)] \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)\, \mathrm{d}\mu(x) +$$
$$+ \lambda \int_{\mathscr{X}} \int_\Theta f(x \mid \theta)\, p(\theta) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)\, \mathrm{d}\mu(x) =$$
$$= \int_\Theta [\int_\Theta [\int_{\mathscr{X}} f(x \mid \theta) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\mu(x)]\, p(\theta)\, \mathrm{d}\lambda(\theta)]\, \mathrm{d}\lambda(\theta) +$$
$$+ \lambda \int_{\mathscr{X}} \int_\Theta f(x \mid \theta)\, p(\theta) \log (1 + \lambda p(\theta \mid x))\, \mathrm{d}\lambda(\theta)\, \mathrm{d}\mu(x) =$$
$$= \sum_{j=1}^{N} \sum_{i=1}^{N} \int_{\mathscr{X}} f(x \mid \theta_j) \log (1 + \lambda p(\theta_i \mid x)) \frac{1}{N}\, \mathrm{d}\mu(x) +$$
$$+ \lambda \sum_{i=1}^{N} \int_{\mathscr{X}} 1 f(x \mid \theta_i) \log (1 + \lambda p(\theta_i \mid x)) \frac{1}{N}\, \mathrm{d}\mu(x) =$$
$$= \lambda \sum_{i=1}^{N} \frac{1}{N}\, \mathsf{E}_{X \mid \theta_i}[\log (1 + \lambda p(\theta_i \mid x))] +$$

$$+\frac{1}{N}\sum_{j=1}^{N}\sum_{i=1}^{N}\mathsf{E}_{X|\theta_j}[\log(1+\lambda p(\theta_i\mid x))] =$$

$$=\frac{1}{N}\left(1+\frac{\lambda}{N}\right)\sum_{j=1}^{N}\sum_{i=1}^{N}\mathsf{E}_{X|\theta_j}[\log(1+\lambda p(\theta_i\mid x))] =$$

$$=\frac{1}{N}\left(1+\frac{\lambda}{N}\right)\sum_{j=1}^{N}\sum_{i=1}^{N}\mathsf{E}_{X|\theta_j}\left[\log\left(1+\lambda\frac{f(x\mid\theta_i)}{N f(x)}\right)\right].$$

Furthermore, we have

$$\frac{f(x\mid\theta_i)}{f(x)}=\frac{\dfrac{\mathrm{d}P_{\theta_i}(x)}{\mathrm{d}\mu(x)}}{\dfrac{\mathrm{d}P(x)}{\mathrm{d}\mu(x)}}=\frac{\dfrac{\mathrm{d}P_{\theta_i}(x)}{\mathrm{d}\sum_{i=1}^{N}P_{\theta_i}(x)}}{\dfrac{\frac{1}{N}\mathrm{d}\sum_{i=1}^{N}P_{\theta_i}(x)}{\mathrm{d}\sum_{i=1}^{N}P_{\theta_i}(x)}}=N\frac{\mathrm{d}P_{\theta_i}(x)}{\mathrm{d}\sum_{i=1}^{N}P_{\theta_i}(x)}=Nf_{\theta_i}(x).$$

Considering the standard experiment associated to $\mathscr{E}_X$:

$$\mathscr{E}_X^*=\{\mathscr{X},\beta_{\mathscr{X}},m_i\}_{i=1,2,\ldots,N}$$

As $\mathrm{d}m_i/\mathrm{d}m_X = Nf_{\theta_i}$, we have

$$\int_{\mathscr{X}}\left[\int_{\Theta}(1+\lambda p(\theta\mid x))\log(1+\lambda p(\theta\mid x)\,\mathrm{d}\lambda(\theta)\right]f(x)\,\mathrm{d}\mu(x) =$$

$$=\frac{1}{N}\left(1+\frac{\lambda}{N}\right)\sum_{j=1}^{N}\sum_{i=1}^{N}\mathsf{E}_{X/\theta_j}\left[\log\left(1+\lambda\frac{f(x\mid\theta_i)}{N f(x)}\right)\right]=$$

$$=\frac{1}{N}\left(1+\frac{\lambda}{N}\right)\sum_{j=1}^{N}\sum_{i=1}^{N}\mathsf{E}_{m_j}[\log(1+\lambda f_{\theta_i})] =$$

$$=\left(1+\frac{\lambda}{N}\right)\sum_{i=1}^{N}\mathsf{E}_{m_X}[f_{\theta_i}\log(1+\lambda f_{\theta_i})].$$

We now define the following convex function

$$g(f_{\theta_1},\ldots,f_{\theta_N})=\sum_{i=1}^{N}f_{\theta_i}\log(1+\lambda f_{\theta_i}).$$

Using the standard experiment's condition for $\mathscr{E}_X \supset \mathscr{E}_Y$, we obtain

$$\mathsf{E}_{m_X}[f_{\theta_i}\log(1+\lambda f_{\theta_i})] \geqq \mathsf{E}_{m_Y}[f_{\theta_i}\log(1+\lambda f_{\theta_i})].$$

i.e., $\mathscr{E}_X \geqq \mathscr{E}_Y$. □

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