

PIECE-WISE OPTIMAL CONTROL IN TRANSPORT ALLOCATION PROBLEM

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An optimal control in real time operating system is examined. A special case of control process is presented according to the available old state information and unknown state description for the future system behavior. Planning and adjusting rules are implemented satisfying the optimal control policy in a restricted time base. A piece-wise optimal strategy is proposed and a suitable real time-algorithm for optimal control is examined. The control system is related to the real time automatic vehicle location. An example of the real time operating rules is presented.

1. INTRODUCTION

To implement an optimal control policy it is necessary to define an appropriate optimization problem and to resolve it. This paper deals with the case when the system behavior is not analytically defined. Only few values of the system state for the past moments are available. Hence the insufficient and inadequate state information prevents from analytical definition of the optimization control problem. Here an identification procedure which analyzes the old system states in real time and performs a prediction of the object state in a near future time interval is proposed. Hence using all available knowledge of the past system behaviour, a formalization is performed to achieve analytical description of the optimal control problem. This procedure is introduced for a piece-wise optimal control of an automatic vehicle location system (AVLS). The main topic of the AVLS is to estimate and to trace in real time the vehicle location changes in a central control (CCU). The real time

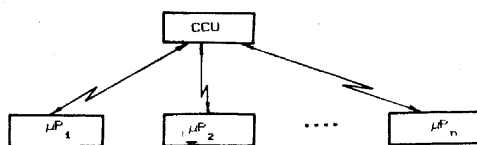


Fig. 1.

tracing of the vehicle motion is useful to implement for solving different kinds of control problems: real time request satisfactions, real time effective decisions, minimizing fuel and time consumptions [1-5].

The functional and technical structure of the AVLS is a hierarchical one with two control levels, see Fig. 1. The upper level is represented by the CCU and the lower level is constituted by the microprocessor units — μP_i in the vehicles — vh_i , $i = 1, \dots, n$. n — the number of vehicles. The system normally operates in a consequent manner [3]. At first the CCU calls the i th vehicle for data transfer. After recognizing the call, the vehicle sends its locational data $x_i(T_i)$ [4, 5] or transmits a measure signal to determine its position [6]. After receiving the data, the CCU performs the new data transfer with the next vehicle by increasing the index $i = i + 1$, $i \leq n$ or starts from $i = 1$, if $i = n$. It is obvious that the call order and data transmissions will define the system operation rules, its real time capabilities of the vehicle locations and the location accuracy. Hence the optimal sequence for data transmission between the CCU and the vehicles will implement the real time optimal management of the AVLS.

2. SYSTEM OPERATION MODEL

The formal presentation of the system operation could be introduced using the optimization model [5, 7]

$$\min \sum_{i=1}^n |y_i(t_i)| \quad (1)$$

$$y_i(t_i) = x_i(t_i) - x_i(T_i) - Q_{ad}, \quad \forall i = 1, \dots, n \quad (1a)$$

$$|t_i - t_j| \geq t_c, \quad \forall i, j = 1, \dots, n, \quad i \neq j, \quad (1b)$$

$$t_i \geq t_c, \quad \forall i = 1, \dots, n, \quad (1c)$$

where $x_i(t_i)$ — the current position of the vh_i ,

$x_i(T_i)$ — the latest transmitted data to the CCU by the i th vehicle, $t_i \geq T_i$,

t_c — the time delay of the system,

Q_{ad} — admissible space interval which could be passed by the vehicle without tracing in the CCU.

The expression (1a) evaluates the locational error between the current vehicle position $x_i(t_i)$ and that presented in the CCU — $x_i(T_i)$. It is obvious that in the ideal case when $t_i = T_i$, $\forall t_i = [0, \infty]$ every vehicle location changes will be presented in the CCU. But such a rapidity normally could not be achieved. Moreover the vehicle motion is traced in scale representation of the observable region in the CCU. Hence there exists a minimal space interval Q_{ad} which has to be passed by the vehicle in order to modify the vehicle locational presentation in the CCU. The value Q_{ad} refers to the minimal change noticeable by the operator in the CCU. The difference $y_i(t_i)$ is an inaccuracy of the system which determines its real time operational

features. According to the optimal system processing the global goal is to minimize the sum of the inaccuracies $\sum_{i=1}^n |y_i(t_i)|$ for all vehicles $\mathbf{v}h_i$, $i = 1, \dots, n$. The constraints (1b) refer to the inoperability conditions of the system in the time interval t_c due to the fact that the CCU cannot operate with more than one vehicle in the same time. The time delay t_c includes the time for data call from the CCU to $\mathbf{v}h_i$, the time for data transmissions, the time for information processing and data preparation for the next call.

The constraints (1c) include the initial restrictions after the system starts. The solution of the optimization problem (1) would be the row matrix $J = \{t_i^*\}$, $i = 1, \dots, n$ where t_i^* are the time moments which are searched for when the CCU has to accomplish next data transmission with the i th vehicle. As $J = \{t_i^*\}$, $i = 1, \dots, n$ is determined according to (1), hence the implementation of J will manage the AVLS in an optimal manner.

3. GLOBAL OPTIMIZATION PROBLEM

The optimization problem (1) in such respect is nonanalytically defined, due to the fact that $x_i(t_i)$ is an unknown stochastic function. In this case analytical relations of the system behavior $x_i(t_i)$, $i = 1, \dots, n$, are not available. The states are known as exact values only in few last moments $x_i(T_i)$, $x_i(T_i^{-1})$, \dots , $x_i(T_i^{-m})$, $m = 1, \dots, \infty$, $T_i > T_i^{-1} > \dots > T_i^{-m} > \dots$, $i = 1, \dots, n$.

In this paper an approach for real time simulation of the current system state $x_i(t_i)$, $i = 1, \dots, n$, is proposed using the old state data, which have been transmitted to the CCU

$$x_i(t_i) = x[t_i, x_i(T_i), x_i(T_i^{-1}), \dots, x_i(T_i^{-k}), \dots, T_i, T_i^{-1}, \dots, T_i^{-m}, \dots]$$

where $T_i > T_i^{-1} > T_i^{-2} \dots$ and prediction of the space positions $x_i(t_i)$, $i = 1, \dots, n$ in the near future after the latest report $x_i(T_i)$, $t_i > T_i$ is performed.

Here the Newton polynomial formula is implemented [5, 7] for the formal presentation of the vehicle motion

$$x_i(t_i) = x_i(T_i) + \frac{x_i(T_i) - x_i(T_i^{-1})}{T_i - T_i^{-1}} (t_i - T_i) + \dots, T_i > T_i^{-1}. \quad (2)$$

Using the substitution $t_i > T_i$, the vehicle location prediction is achieved from (2). Due to the stochastic feature of the vehicle motion it is sufficient to use only the first two terms in (2)

$$x_i(t_i) = x_i(T_i) + v_{avi}(T_i, T_i^{-1}) \cdot (t_i - T_i),$$

where

$$v_{avi}(T_i, T_i^{-1}) = \frac{x_i(T_i) - x_i(T_i^{-1})}{T_i - T_i^{-1}}. \quad (2)$$

Hence the expression (1a) becomes

$$y_i(t_i) = v_{avi}(T_i, T_i^{-1}) \cdot t_i - S_i(T_i, T_i^{-1}),$$

where

$$S_i(T_i, T_i^{-1}) = v_{avi}T_i + Q_{ad}.$$

To obtain a suitable analytical form of the problem, here a substitution from $|y_i|$ to y_i^2 is performed, which does not change the physical meaning of the control process. Hence the global optimization problem becomes

$$Z(t^k, T_i, T_i^{+1}) = \min \sum_{i=1}^n y_i^2(t_i), \quad (3)$$

$$y_i(t_i) = v_{avi}(T_i, T_i^{-1}) \cdot t_i - S_i(T_i, T_i^{-1}), \quad \forall i = 1, \dots, n$$

$$|t_i - t_j| \geq t_c, \quad \forall i, j = 1, \dots, n \quad i \neq j,$$

$$t_i \geq t_c, \quad \forall i = 1, \dots, n.$$

In every time period $[t^k, t^{k+1}]$, where t^k is the initial moment of this period, according to the system modeling and prediction, the problem (3) is well defined and $v_{avi}(T_i, T_i^{-1})$ and $S_i(T_i, T_i^{-1})$ are computed. The solution of (3) consists of the time sequences $J = \{t_i^* = T_i^{+1}\}$, $i = 1, \dots, n$. The minimal (the first) element r of the row $\{T_i^{+1}\}$, $i = 1, \dots, n$ determines that at the time moment T_r^{+1} data $x_r(T_r^{+1})$ of the vh_r have to be transmitted. Hence in the time period $[t^k, t^{k+1}]$, where $t^{k+1} = T_r^{+1}$, the global optimization problem $Z(t^k, T_i, T_i^{+1})$ is constant and defines the optimal operation rules of the system. At time T_r^{+1} a data transfer is performed and $x_r(T_r^{+1})$ is obtained in the CCU. These new data are used for actualization and modification of v_{avi} and S_i , $i = 1, \dots, n$ and the problem (3) is settled according to the latest received data to the new optimization problem $Z(t^{k+1}, T_i, T_i^{+1})$, where t^{k+1} is the new initial

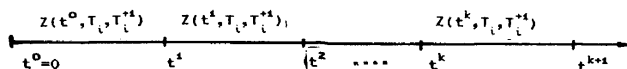


Fig. 2.

moment for the new time period, see Fig. 2. Hence the form of the global optimization problem (3) is modified and adapted according to the recent received data $x_r(T_r^{+1})$. The system operates in optimal manner with constant optimal problem (3) only in the time interval $[t^k, t^{k+1}]$, $k = 0, 1, 2, \dots$, see Fig. 2. Since the optimization problem (3) is modified and adapted in every time interval $[t^k, t^{k+1}]$, $k = 0, 1, 2, \dots$, piece-wise optimal solutions are achieved for the overall period $t_i = [0, \infty]$, $i = 1, \dots, n$. Hence optimality cannot be claimed for the overall period $[0, \infty]$, but this will be a compromise settlement between the requirement of the real time control strategy and the possibilities to achieve it.

4. GLOBAL OPTIMIZATION PROBLEM SOLUTION

To reduce the complexity of the investigations, here i is restricted to $i = 1, 2$. Hence (3) becomes

$$\begin{aligned} \min \{F = y_1^2(t_1) + y_2^2(t_2)\} \\ y_1(t_1) = v_1 \cdot t_1 - S_1 \\ y_2(t_2) = v_2 \cdot t_2 - S_2 \\ |t_1 - t_2| \geq t_c \\ t_1 \geq t_c, \quad t_2 \geq t_c. \end{aligned} \quad (4)$$

The condition $|t_1 - t_2| \geq t_c$ could be introduced as

$$\text{A) } -t_1 + t_2 + t_c \leq 0 \quad \text{and} \quad \text{B) } t_1 - t_2 + t_c \leq 0. \quad (5)$$

As the constraints (5) define a non compact admissible areas, the optimization problem (4) has to be solved both for the cases A) and B). The local solutions $[t_1^A, t_2^A]$ and $[t_1^B, t_2^B]$ after substitution in F will determine the global optimal solution according to which is the smallest value: $F(t^A)$ or $F(t^B)$.

The condition (5) prevents the global optimization problem (4) from decomposition. To achieve it, the hierarchical control theory is used. By forming the right Lagrange problem, case A) is obtained as:

$$\begin{aligned} L^A(t_1, t_2) = (v_1 \cdot t_1 - S_1)^2 + (v_2 \cdot t_2 - S_2)^2 + \lambda(-t_1 + t_2 + t_c), \\ t_1, t_2 \geq t_c. \end{aligned}$$

By solving the right Lagrange problem $\min_{t_1, t_2} L^A(t_1, t_2)$ two subproblems arise in the form

$$\min_{t_1 \geq t_c} (v_1 \cdot t_1 - S_1)^2 - \lambda \cdot t_1 \Rightarrow \arg: t_1^A(\lambda) = \max \left[t_c, \frac{S_1}{v_1} + \frac{\lambda}{2v_1^2} \right] \quad (6a)$$

$$\min_{t_2 \geq t_c} (v_2 \cdot t_2 - S_2)^2 + \lambda \cdot t_2 \Rightarrow \arg: t_2^A(\lambda) = \max \left[t_c, \frac{S_2}{v_2} - \frac{\lambda}{2v_2^2} \right] \quad (6b)$$

To estimate the optimal solutions it is necessary to resolve the inverse Lagrange problem and to determine λ . The possible cases using the local solutions of the subproblems (6) are:

$$\begin{aligned} \text{Case A 1: } \begin{cases} t_1^A(\lambda) = \frac{S_1}{v_1} + \frac{\lambda}{2v_1^2} \\ t_2^A(\lambda) = \frac{S_2}{v_2} - \frac{\lambda}{2v_2^2} \end{cases}, \quad \text{Case A 2: } \begin{cases} t_1^A(\lambda) = t_c \\ t_2^A(\lambda) = \frac{S_2}{v_2} - \frac{\lambda}{2v_2^2} \end{cases} \\ \text{Case A 3: } \begin{cases} t_1^A(\lambda) = \frac{S_1}{v_1} + \frac{\lambda}{2v_1^2} \\ t_2^A(\lambda) = t_c. \end{cases} \end{aligned}$$

After substitution the cases A 1, A 2, A 3 into the Lagrange function $L^\Lambda(t)$, the inverse Lagrange function $H^\Lambda(\lambda)$ is determined. By solving the inverse Lagrange problem $\max_{\lambda} H^\Lambda(\lambda)$ the optimal values λ^{opt} are defined:

For case A 1: $\lambda^{\text{opt}} = 2v_1^2v_2^2[(S_2/v_2) - (S_1/v_1) + t_c]/(v_1^2 + v_2^2)$,

For case A 2: $\lambda^{\text{opt}} = 2v_2S_2$,

For case A 3: $\lambda^{\text{opt}} = 2v_1^2(2t_c - S_1/v_1)$.

From the subproblems (6) it follows that for $\lambda = 0$ the equations $t_1(\lambda = 0) = t_1(0) = S_1/v_1$, $t_2(\lambda = 0) = t_2(0) = S_2/v_2$ takes place. These relations after substitution into $t^\Lambda(\lambda)$ define the optimal solutions as:

$$\text{Case A: } \begin{cases} t_1^\Lambda(\lambda) = \max \left\{ t_c; t_1(0) + \frac{v_2^2[t_2(0) - t_1(0) + t_c]}{v_1^2 + v_2^2} \right\} \\ t_2^\Lambda(\lambda) = \max \left\{ t_c; t_2(0) - \frac{v_1^2[t_2(0) - t_1(0) + t_c]}{v_1^2 + v_2^2} \right\} \\ \text{if } t_2^\Lambda(\lambda) = t_c, \quad t_1^\Lambda(\lambda) = 2t_c. \end{cases} \quad (7)$$

By the same manner it is proved that in case B the optimal solutions are:

$$\text{Case B: } \begin{cases} t_1^\text{B}(\lambda) = \max \left\{ t_c; t_1(0) - \frac{v_2^2[t_1(0) - t_2(0) + t_c]}{v_1^2 + v_2^2} \right\} \\ t_2^\text{B}(\lambda) = \max \left\{ t_c; t_2(0) + \frac{v_1^2[t_1(0) - t_2(0) + t_c]}{v_1^2 + v_2^2} \right\} \\ \text{if } t_1^\text{B}(\lambda) = t_c, \quad t_2^\text{B}(\lambda) = 2t_c. \end{cases} \quad (8)$$

5. REAL TIME OPERATION ALGORITHM

The relations (7) and (8) determine the operation algorithm of the system. Every vehicle vh_i , $i = 1 \dots n$ solves the local optimization problem (6) assuming the Lagrange multiplier $\lambda = 0$. The solution is achieved by accomplishing some algebraic and logic operations. This preserves the real time capabilities of the algorithm. After

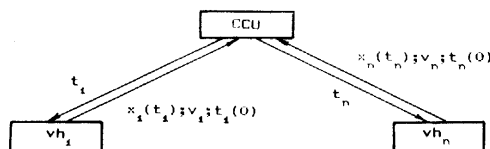


Fig. 3.

receiving a data call from the CCU, the i th vehicle sends its locational data $x_i(T_i)$, computes $v_{avi}(T_i, T_i^{-1})$ and $S_i(T_i, T_i^{-1})$ with these new data and according to (6) predicts the preposition $t_i(0) = T_i^{+1}$ for the next future data transfer. At this step, model modification of (2) is performed using the new data $x_i(T_i)$, see Fig. 3. After receiving the

preposition $t_i(0) = T_i^{+1}$, the CCU checks for feasibility the new solution T_i^{+1} . If T_i^{+1} satisfies the conditions (5), the CCU assumes $t_i(0) = T_i^{+1}$ as a global optimal solution and at time T_i^{+1} will perform next data transfer with the i th vehicle. If conditions (5) are not satisfied, for example there exists index p such that $|t_p(0) - t_i(0)| < t_c$, the CCU corrects the sequence $t_i(\lambda)$ and $t_p(\lambda)$ in right order according to the relations (7) and (8). It is important that in both cases (7) and (8) the CCU obtains solutions $t_i(\lambda)$ and $t_p(\lambda)$ using the preposition $t_i(0)$ and $t_p(0)$ respectively. In the general case when $i = 1 \dots n$, $n > 2$ the expressions (7) and (8) have to be performed for every couple of indices, which do not satisfy constraints (5). Using the minimal element from the solution $J = \{t_i(0)\}$, $i = 1, \dots, n$, the CCU performs next data call. As the problem (3) is actualized with every new data $x_i(T_i)$, the control process possesses adaptive features.

6. EXAMPLE

Two vehicles are observed with initial defined models $y_1(t_1) = v_1 t_1 - Q_{ad} = 10t_1 - 300$ and $y_2(t_2) = v_2 t_2 - Q_{ad} = 30t_2 - 300$, where $Q_{ad} = 300$ m, $t_c = 7$ s, $v_1 = 10$, $v_2 = 30$.

STEP 1. The global optimization problem will be

$$Z(t^0 = 0, T_i, T_i^{+1}) \equiv \min [y_1^2(t_1) + y_2^2(t_2)]$$

$$y_1(t_1) = 10t_1 - 300$$

$$y_2(t_2) = 30t_2 - 300$$

$$|t_1 - t_2| \geq 7$$

$$t_1, t_2 \geq 7.$$

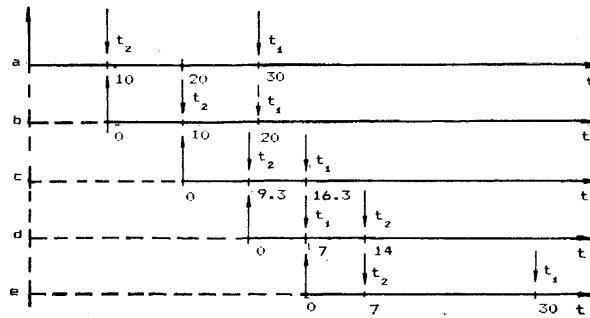


Fig. 4.

The solutions are $t_1^1(0) = 30$ s, $t_2^1(0) = 10$ s. Hence in the 10th second the second vehicle will transmit data, see Fig. 4a.

STEP 2. At the 10th second the data transfer between \mathbf{vh}_2 and the CCU is performed. Assuming that the motion aspects of \mathbf{vh}_2 are not changed, the velocity $v_2 = 30$ will preserve its value. Giving into account that $t^{k+1} = 10$ s, the global optimization problem will be

$$\begin{aligned} Z[t^1 = t_2^1(0) = 10 \text{ s}, T_i, T_i^{+1}] &\equiv \min \{y_1^2 + y_2^2\} \\ y_1(t_1) &= v_1 t_1 - Q_{ad} + S_1 = 10t_1 - 200, \quad S_1 = v_1 t_2(0) \\ y_2(t_2) &= v_2 t_2 - Q_{ad} = 30t_2 - 300, \\ |t_1 - t_2| &\geq 7, \quad t_1, t_2 \geq 7. \end{aligned}$$

The solution is $t_1^2(0) = 10$ s, $t_1^2(0) = 20$ s, see Fig. 4b. It is obvious that $t_1^2(0)$ is obtained from $t_1^1(0)$ only by a simple coordinate translation using the value $t_2^1(0) = 10$ s. Hence it is not necessary to resolve the global optimization problem (5) again but only this subproblem (6), where new data have been obtained. The new optimal solutions of (5) consist of all old solutions of (5) obtained up to now modified by the coordinate translation. The main new part of the solution of (5), during the current step, is constituted by the solution of subproblem (6) where the new data $x_i(T_i)$ are involved for v_{avi} and S_i computation. The coordinate translation ensures that the time scale is the same for all vehicles and the sequence of data transmission takes place in "common" time for the overall system.

STEP 3. At time $t_2^2(0)$ the second vehicle will operate with the CCU. If the motion aspect of the vehicles is preserved, hence $v_2(t_2^2(0), t_2^1(0)) = \text{const}$. The global optimization problem will be

$$\begin{aligned} Z[t^3 = t_2(0) = 10, T_i, T_i^{+1}] &\equiv \min \{y_1^2(t_1) + y_2^2(t_2)\} \\ y_1(t_1) &= v_1 t_1 - Q_{ad} + v_1 t_2^2(0) = 10t_1 - 100 \\ y_2(t_2) &= v_2 t_2 - Q_{ad} = 30t_2 - 300 \\ |t_1 - t_2| &\geq 7, \quad t_1, t_2 \geq 7. \end{aligned}$$

The solution is $t_1^3(0) = t_2^3(0) = 10$ s. Hence a competition arises between the vehicles which intend to operate at the same time with the CCU. Using (7) and (8) the optimal solutions could be $[t_1^A(\lambda) = 16.3 \text{ s}, t_2^A(\lambda) = 9.3 \text{ s}]$ and $[t_1^B(\lambda) = 7 \text{ s}, t_2^B(\lambda) = 14 \text{ s}]$. As the value of the performance index $F^A = 882 < F^B = 15\,300$, the global optimal solution is $t_1^3(\lambda) = 16.3 \text{ s}, t_2^3(\lambda) = 9.3 \text{ s}$, see Fig. 4c.

STEP 4. At time moment $t_2^3(\lambda) = 9.3 \text{ s}$ the second vehicle will transmit data. Assuming the preserved aspects of the motion, the global optimization problem will be

$$\begin{aligned} Z[t^4 = t_2^4(\lambda) = 9.3 \text{ s}, T_i, T_i^{+1}] &\equiv \min \{y_1^2(t_1) + y_2^2(t_2)\} \\ y_1(t_1) &= 10t_1 - 7 \\ y_2(t_2) &= 30t_2 - 300 \\ |t_1 - t_2| &\geq 7, \quad t_1, t_2 \geq 7. \end{aligned}$$

The solution of subproblems (6) $t_1(0) = 7$ s, $t_2(0) = 10$ s defines that the constraint (1b) is not satisfied. According to (7) and (8) the right solutions are $t_1^B(\lambda) = 7$ s, $t_2^B(\lambda) = 14$ s, see Fig. 4d.

STEP 5. With the same aspects of the motion after performing data transfer with vh_1 the solution of the problem $Z[t^S = t_1^S(\lambda) = 7, T_i, T_i^{+1}]$ is $t_1(0) = 30$ s, $t_2(0) = 7$ s, see Fig. 4e.

7. CONCLUSIONS

As a result, instead of the classical regular and consequent calling procedure, the system performs an irregular data transfer. This gives the most precise and optimal tracing of the vehicle motion in the sense of the optimization problem (3). Hence using real time modeling and adaptation of the global optimization problem (3), a piece-wise real time optimal strategy of the AVLS rules is achieved.

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REFERENCES

- [1] Д. И. Младенов, К. П. Стоилова, Т. А. Стоилов: Вычислительная система для управления автомобильным парком в движении. Автоматика и вычислительная техника (Рига) (1983), No. 6, 18—23.
- [2] Д. И. Младенов, К. П. Стоилова, Т. А. Стоилов: Някои проблеми при реализиране на автоматизиран оперативен контрол на клас подвижни обекти. Комплексни системи за управление (София) (1984), No. 1, 74—78.
- [3] Т. А. Стоилов, К. П. Стоилова, Г. Ф. Тодоров, В. Г. Петков, Г. В. Шейтанов, А. П. Кемалов: Система за управление на обекти с пространствено разпределени параметри. Труды международной конференции „Проблемы комплексных систем управления“, Т. 1, 1985, Варна, 333—340.
- [4] Т. А. Стоилов: Информационни системи за оперативен контрол и тяхното изграждане като йерархични. Анализ. Техническа мисъл (1987), No. 5, 25—28.
- [5] Т. А. Стоилов: Информационни системи за оперативен контрол и тяхното изграждане като йерархични. Синтез. Техническа мисъл (1987), No. 6, 19—24.
- [6] R. W. Gibson: Pinpoint — radio system for locating and monitoring vehicles. Philips Tech. Rev. 35 (1975), 1, 15—22.
- [7] T. A. Stoilov, K. P. Stoilova: An hierarchical adaptive algorithm for data transmission in distributed microprocessor network. In: Proc. IFIP TC6/TC8 International Symposium, Network Information Processing Systems, Sofia, May 1988, pp. 75—87.

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