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DECENTRALIZED ADAPTIVE STABILIZATION WITH STATE REGULATORS

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In the paper, two variants of a new approach to the decentralized control design of linear dynamic systems based on the large-scale system decomposition into subsystems are given. At the subsystem level, the state regulators are applied. The adaptation of local state regulators is effectuated according to the state variables deviation from the equilibrium. The theoretical base used is represented by the notion of the stability respecting the part of variables and the second Lyapunov method. An example is presented.

1. INTRODUCTION

The mathematical description of a dynamic system is not a trivial task. Especially in the case of large scale system, the exact modelling can never be achieved [1]. A great number of real applications deal with interconnected subsystems with poorly known or varying structure and parameters. Hence, the adaptive control schemes have been extended to the decentralized systems (cf. [2]–[9]).

Direct schemes are more frequently used (cf. [2]–[5]). Usually, the model reference adaptive control is applied to the subsystems, and when the interconnections among them are sufficiently small, the overall system stability is not disturbed.

The indirect schemes [6]–[9] are modified self-tuning single-input, single-output regulators operating at the subsystem level. In this case, the problem of identification loop robustness to the unmodelled dynamics (interconnections) has to be solved.

In this paper, two adaptive decentralized control algorithms using local subsystem state regulators are designed. The feedback gains of subsystems state regulators are adapted to achieve the stability in the presence of arbitrary interconnections.

2. DECENTRALIZED ADAPTIVE CONTROL

Consider the linear large scale system consisting of N subsystems

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{B}_i \boldsymbol{u}_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} \boldsymbol{A}_{ij} \boldsymbol{x}_j \,, \quad i \in \mathcal{N} \,. \tag{1}$$

where

$$\boldsymbol{x}_i \in \mathbb{R}^{n_i} \,, \quad \boldsymbol{u}_i \in \mathbb{R}^{m_i} \,, \quad \mathcal{N} = \{1, 2, \ldots, N\}$$

and $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{A}_{ij}$ are real matrices of appropriate sizes. We have to design the adaptive local control laws such that the state of global system (1)

$$\boldsymbol{x} = [\boldsymbol{x}_1^{\mathrm{T}}, \boldsymbol{x}_2^{\mathrm{T}}, \ldots, \boldsymbol{x}_N^{\mathrm{T}}]^{\mathrm{T}} \,, \quad \boldsymbol{x} \in \mathbb{R}^n \,, \quad n = \sum_{i=1}^{N} n_i \tag{2}$$

is regulated to zero.

We will distinguish two decentralized control types acting upon the system (1) via local regulators. In the first case, we consider the following control law

$$\boldsymbol{u}_i = -(1 + \alpha_i)\, \boldsymbol{K}_{0i} \boldsymbol{x}_i \,, \quad i \in \mathcal{N} \,, \tag{3}$$

where $\boldsymbol{K}_{0i} \in \mathbb{R}^{m_i \times n_i}$ are determined according to the arbitrary prespecified requirements on the system so that the isolated subsystems are stable and $\alpha_i \in \mathbb{R}^1$ is the ith subsystem adaptation parameter. We use the adaptation law

$$\dot{\alpha}_i = \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{C}_i \boldsymbol{x}_i \,, \quad i \in \mathcal{N} \,, \tag{4}$$

where $\boldsymbol{C}_i \in \mathbb{R}^{n_i \times n_i}$, $i \in \mathcal{N}$, are symmetric positive definite matrices. The initial conditions $\alpha_i(0)$, $i \in \mathcal{N}$, are arbitrary small positive numbers.

In the second case, n_i parameters are adapted in the subsystem control law

$$\boldsymbol{u}_i = -(\boldsymbol{K}_{0i} + \boldsymbol{K}_{0i} \boldsymbol{\varkappa}_i)\, \boldsymbol{x}_i \,, \quad i \in \mathcal{N} \,, \tag{5}$$

where $\boldsymbol{K}_{0i}$ has the same sense as in (3) and $\boldsymbol{\varkappa}_i = (\varkappa_{ilk})$, $\boldsymbol{\varkappa}_i \in \mathbb{R}^{n_i \times n_i}$, $i \in \mathcal{N}$, are diagonal matrices of adapted parameters. The adaptation law is

$$\dot{\varkappa}_{ilk} = \begin{cases} (\boldsymbol{D}_i \boldsymbol{x}_i \boldsymbol{x}_i^{\mathrm{T}})_{lk} \,, & \text{if} \quad l = k \,, \\ \varkappa_{ilk} = 0 \,, & \text{if} \quad l \neq k \,, \end{cases} \quad i \in \mathcal{N}, \tag{6}$$

where $\boldsymbol{D}_i \in \mathbb{R}^{n_i \times n_i}$ are diagonal positive definite matrices and $\boldsymbol{\varkappa}_i(0) > 0$. Hence, we have to prove the stability of systems (1)–(4) and (1), (2), (5), (6) with unknown $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{A}_{ij}$, $i, j \in \mathcal{N}$, $j \neq i$. The stability will be proved with the use of the generalized second Lyapunov method. In the following paragraph, some basic theorems and definitions are introduced.

3. SYSTEM STABILITY RESPECTING THE PART OF VARIABLES

The second Lyapunov method is frequently used when analysing the stability of differential equations. The authors themselves have examined the idea of analysing the stability with uncomplete state set. Further development of this idea by several mathematicians (Rumyantsev, Oziraner, Zubov, Halanay, Rouche, Peiffer, Yoshizawa ...) led to the so called generalized Lyapunov stability theory. An overview of this problem is given in [10].

In basic considerations, the system

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}, t), \quad \boldsymbol{x} \in \mathbb{R}^n \\ \dot{\boldsymbol{y}} &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, t), \quad \boldsymbol{y} \in \mathbb{R}^m \end{aligned} \tag{7}$$

is used. The $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$ functions have a unique solution for every

$$t \geqq 0, \quad \|\boldsymbol{x}\| \leqq h, \quad \|\boldsymbol{y}\| < +\infty, \quad h > 0 \tag{8}$$

whete $\|\cdot\|$ denotes the Euclidian norm.

Moreover, for all $t \geqq t_0 \geqq 0$

$$\begin{aligned} \boldsymbol{f}(\mathbf{0}, \mathbf{0}, t) &= \mathbf{0} \\ \boldsymbol{g}(\mathbf{0}, \mathbf{0}, t) &= \mathbf{0} \end{aligned} \tag{9}$$

Let $\boldsymbol{z}^{\mathrm{T}} = (\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{y}^{\mathrm{T}}) \in \mathbb{R}^{n+m}$ denote the state vector of the composite system. Then,

$$\boldsymbol{z}^{\mathrm{T}}(z_0, t_0; t) = [\boldsymbol{x}^{\mathrm{T}}(z_0, t_0; t), \quad \boldsymbol{y}^{\mathrm{T}}(z_0, t_0; t)] \tag{10}$$

represents the solution of the system (7) starting from z_0 at time t_0.

Let us now introduce some definitions and theorems which are used to analyse the stability when respecting the part of variables.

Definition 1. The equilibrium $z = \mathbf{0}$ of the system (7) is called stable with respect to the variable $\boldsymbol{x}$ when for every $\varepsilon > 0$ and $t_0 \geqq 0$, there exists $\delta(\varepsilon, t_0)$ such that the conditions $\|z_0\| < \delta$ implies $\|\boldsymbol{x}(z_0, t_0; t)\| < \varepsilon$ for every $t \geqq t_0$.

Definition 2. If the scalar function $a(r)$ is continuous and increasing for every $r \in \langle 0, h \rangle$ and $a(0) = 0$, then it belongs to the class $\mathscr{L}$ $(a(r) \in \mathscr{L})$.

Definition 3. The function $V(z, t)$ (z is given by (10)) is called positive definite with respect to the variables $\boldsymbol{x}$ if there exists a t-invariant function $w(\boldsymbol{x})$ $(w(\mathbf{0}) = 0)$ and the following relations hold on the set (8):

$$\begin{aligned} V(z, t) &\geqq w(\boldsymbol{x}) \\ w(\boldsymbol{x}) &\geqq 0 \end{aligned}$$

Theorem 1 (cf. [10]). If there exists a function $V(z, t)$ which is positive definite with respect to the variables $\boldsymbol{x}$ and its derivation $\dot{V}(z, t)$ along the solution of (7) is nonpositive ($\dot{V}(z, t) \leqq 0$), then the equilibrium $z = \mathbf{0}$ is stable with respect to the variables $\boldsymbol{x}$.

Theorem 2 (cf. [12]). If there exist functions a, $b \in \mathscr{L}$ and $V(z, t)$ satisfying

$$a(\|x\|) \leqq V(z, t) \leqq b(\|z\|) \tag{11}$$
$$\dot{V}(z, t) \leqq 0$$

for the differential equations system (7) on the set (8), then the equilibrium $z = \mathbf{0}$ is uniformly stable with respect to the variables $\boldsymbol{x}$.

Theorem 3 (cf. [11]). Let us suppose that $\boldsymbol{f}(z, t)$ in (7) is uniformly bounded:

$$\|\boldsymbol{f}(z, t)\| \leqq c_1 \quad (0 < c_1 = \text{const.}) \tag{12}$$

for every

$$t \geqq 0\,, \quad \|z\| < H\,, \quad (0 < H = \text{const.}) \tag{13}$$

and $a, c \in \mathscr{L}$. Then, if there exists the Lyapunov function satisfying on the set (8)

$$\begin{aligned} a(\|z\|) &\leqq V(z, t) \\ \dot{V}(z, t) &\leqq -c(\|x\|) \end{aligned} \tag{14}$$

the equilibrium $z = \mathbf{0}$ of (7) is stable and asymptotically stable with respect to the variables $\boldsymbol{x}$.

4. THE GLOBAL SYSTEM STABILITY

In this chapter, we will prove the stability of the system (1) with decentralized control (3) or (5). The dynamics of the general system (7) are the following (two considered control types are distinguished):

$$\mathscr{S}_1: \begin{aligned} \dot{\boldsymbol{x}}_i &= (\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i})\,\boldsymbol{x}_i - \alpha_i\boldsymbol{B}_i\boldsymbol{K}_{0i}\boldsymbol{x}_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \boldsymbol{A}_{ij}\boldsymbol{x}_j\,, \\ \dot{\alpha}_i &= \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{C}_i\boldsymbol{x}_i\,. \end{aligned} \quad i \in \mathcal{N} \tag{15}$$

$$\mathscr{S}_2: \begin{aligned} \dot{\boldsymbol{x}}_i &= (\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i})\,\boldsymbol{x}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i}\boldsymbol{\varkappa}_i\boldsymbol{x}_i + \sum_{\substack{j=1\\ j\neq i}}^{N} \boldsymbol{A}_{ij}\boldsymbol{x}_j\,, \\ \dot{\boldsymbol{\varkappa}}_i &\quad \text{is given by (6)}\,. \end{aligned} \quad i \in \mathcal{N} \tag{16}$$

When comparing the systems $\mathscr{S}_1$ and $\mathscr{S}_2$ with the system (7), the variables $\boldsymbol{x}$((2)) of the systems $\mathscr{S}_1$ and $\mathscr{S}_2$ can be regarded as corresponding to the $\boldsymbol{x}$ variables in (7). The $\mathscr{S}_1$ variables $\bar{\alpha}_1, \ldots, \bar{\alpha}_N$ and $\mathscr{S}_2$ variables $\bar{\boldsymbol{\varkappa}}_1, \ldots, \bar{\boldsymbol{\varkappa}}_N$ are chosen to correspond to the $\boldsymbol{y}$-variables:

$$\bar{\alpha}_i = \alpha_i - \alpha_i^*\,, \quad i \in \mathcal{N}\,.$$
$$\bar{\boldsymbol{\varkappa}}_i = \boldsymbol{\varkappa}_i - \boldsymbol{\varkappa}_i^*\,, \quad i \in \mathcal{N}\,,$$

where α_i^*, $\boldsymbol{\varkappa}_i^*$ are time-invariant finite scalars (matrices) specified later.

Theorem 4. Let the matrices $\boldsymbol{K}_{0i}$, $i \in \mathcal{N}$, in (15) be chosen such that:

a) $\quad \operatorname{Re}\{\lambda_j[\boldsymbol{A}_i - (1 + \alpha_i)\,\boldsymbol{B}_i\boldsymbol{K}_{0i}]\} < 0\,, \qquad (17)$

for every $\alpha_i \in (0, \alpha_{i\max} >, \alpha_{i\max} > 0, \quad j = 1, \ldots, n_i. \quad i \in \mathcal{N}$

b) $\quad \operatorname{Re}\{\lambda_j(\boldsymbol{B}_i\boldsymbol{K}_{0i})\} \geqq 0, \quad j = 1, \ldots, n_i, \quad i \in \mathcal{N},$

where $\lambda(\cdot)$ denotes the eigenvalue of the indicated matrix. Then the equilibrium $\boldsymbol{x}_i = 0$, $\bar{\alpha}_i = 0$, $i \in \mathcal{N}$, of the system $\mathcal{S}_1$ is stable and asymptotically stable with respect to $\boldsymbol{x}$.

Proof. Let $V(\boldsymbol{x}, \bar{\boldsymbol{\alpha}})$ $(\bar{\boldsymbol{\alpha}}^{\mathrm{T}} = [\bar{\alpha}_1, \ldots, \bar{\alpha}_N])$ be a Lyapunov function candidate for the system $\mathcal{S}_1$,

$$V(\boldsymbol{x}, \bar{\boldsymbol{\alpha}}) = \sum_{i=1}^{N} \{\boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{P}_i \boldsymbol{x}_i\} + \bar{\boldsymbol{\alpha}}^{\mathrm{T}} \bar{\boldsymbol{\alpha}}, \tag{18}$$

where $\boldsymbol{P}_i \in \mathcal{N}$, are symmetric positive definite matrices specified later. For the proof of stability, the third theorem will be used. We have to prove the validity of relations (12) and (14) for the function (18). The well-known properties of quadratic forms imply

$$\sum_{i=1}^{N} \lambda_{\mathrm{m}}(\boldsymbol{P}_i) \|\boldsymbol{x}_i\|^2 \leqq \sum_{i=1}^{N} \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{P}_i \boldsymbol{x}_i, \tag{19}$$

where $\lambda_{\mathrm{m}}(\cdot)$ represents the minimum eigenvalue of the indicated matrix. From (18) and (19), we obtain

$$\min_{i \in \mathcal{N}} \{\lambda_{\mathrm{m}}(\boldsymbol{P}_i), 1\} \|z\|^2 \leqq V(z) \tag{20}$$

where $z^{\mathrm{T}} = (\boldsymbol{x}^{\mathrm{T}}, \bar{\boldsymbol{\alpha}}^{\mathrm{T}})$. The time derivation of $V(\boldsymbol{x}, \bar{\boldsymbol{\alpha}})$ along the solution of $\mathcal{S}_1$ is

$$\begin{aligned} \dot{V} = \sum_{i=1}^{N} \{ & \boldsymbol{x}_i^{\mathrm{T}} [\boldsymbol{P}_i(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i} - \alpha_i \boldsymbol{B}_i \boldsymbol{K}_{0i}) + (\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i} - \alpha_i \boldsymbol{B}_i\boldsymbol{K}_{0i})^{\mathrm{T}} \boldsymbol{P}_i \\ & + 2\alpha_i \boldsymbol{C}_i] \boldsymbol{x}_i - 2\alpha_i^* \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{C}_i \boldsymbol{x}_i + \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{P}_i \sum_{\substack{j=1 \\ j \neq i}}^{N} \boldsymbol{A}_{ij} \boldsymbol{x}_j + (\boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{P}_i \sum_{\substack{j=1 \\ j \neq i}}^{N} \boldsymbol{A}_{ij} \boldsymbol{x}_j)^{\mathrm{T}} \} \end{aligned} \tag{21}$$

If the assumptions of the theorem are satisfied, then the equations

$$\begin{aligned} & \boldsymbol{P}_i(\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i} - \alpha_i \boldsymbol{B}_i\boldsymbol{K}_{0i}) + (\boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{0i} - \alpha_i \boldsymbol{B}_i\boldsymbol{K}_{0i})^{\mathrm{T}} \boldsymbol{P}_i + 2\alpha_i \boldsymbol{C}_i = 0, \\ & i \in \mathcal{N}, \end{aligned} \tag{22}$$

have a unique solution for all $\alpha_i \in (0, \alpha_{i\max})$, $\alpha_{i\max} > 0$, represented by the symmetric positive definite matrices $\boldsymbol{P}_i$. Then, (21) will be

$$\dot{V} = \sum_{i=1}^{N} \{\boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{P}_i \sum_{\substack{j=1 \\ j \neq i}}^{N} \boldsymbol{A}_{ij} \boldsymbol{x}_j + (\boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{P}_i \sum_{\substack{j=1 \\ j \neq i}}^{N} \boldsymbol{A}_{ij} \boldsymbol{x}_j)^{\mathrm{T}} - 2\alpha_i^* \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{C}_i \boldsymbol{x}_i\} \tag{23}$$

If $\bar{\boldsymbol{x}} = [\|\boldsymbol{x}_1\|, \|\boldsymbol{x}_2\|, \ldots, \|\boldsymbol{x}_N\|]^{\mathrm{T}}$, then the derivation of the Lyapunov function (18) can be bounded by

$$\dot{V}(\boldsymbol{x}, \bar{\boldsymbol{\alpha}}) \leqq -\bar{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{W} \bar{\boldsymbol{x}} \tag{24}$$

where $\boldsymbol{W} = (w_{ij})$,

$$w_{ij} = \begin{cases} 2\alpha_i^* \lambda_{\mathrm{m}}(\boldsymbol{C}_i), & i = j \\ -\|\boldsymbol{P}_i \boldsymbol{A}_{ij}\| - \|\boldsymbol{A}_{ij}^{\mathrm{T}} \boldsymbol{P}_j\|, & i \neq j \end{cases} \tag{25}$$

From (24), (25), it follows that there exist such finite $\alpha_i^* > 0$, $i \in \mathcal{N}$, that the symmetric matrix W will be positive definite, and hence

$$\dot{V}(x, \bar{\alpha}) \leqq -\lambda_m(W) \|x\|^2 \tag{26}$$

Therefore, the third condition of Theorem 3 is satisfied. Condition (12) is satisfied for $t \geqq 0$, $\|z\| < H$. This proves that the differential equations system (15) is asymptotically stable with respect to the variable x. □

Theorem 5. Let the matrices K_{0i}, $i \in \mathcal{N}$, in (16) be chosen such that the conditions

$$\operatorname{Re}\{\lambda_j[A_i - B_i K_{0i}(E_i + \varkappa_i)]\} < 0\,,$$

for every $\varkappa_i \in (0, \varkappa_{i\max}\rangle$, $\varkappa_{i\max} > 0$, $j = 1, \ldots, n_i$, $i \in \mathcal{N}$ are fulfilled. Then the equilibrium $x_i = 0$, $\bar{\varkappa}_i = 0$, $i \in \mathcal{N}$, of the system $\mathcal{S}_2$ is stable and asymptotically stable with respect to x.

Proof. We will consider the following Lyapunov function candidate for $\mathcal{S}_2$:

$$V(x, \bar{\varkappa}_1, \ldots, \bar{\varkappa}_N) = \sum_{i=1}^{N} \{x_i^T P_i x_i + \operatorname{tr}(\bar{\varkappa}_i)^2\} \tag{27}$$

In the condition (20), z is

$$z^T = (x^T, \bar{\varkappa}^T)$$

where $\bar{\varkappa}$ is the vector collecting the diagonals of $\bar{\varkappa}_i$, $i \in \mathcal{N}$. The time derivation of (27) along the trajectory of $\mathcal{S}_2$ is

$$\dot{V} = \sum_{i=1}^{N} \{x_i^T[P_i(A_i - B_i K_{0i} - B_i K_{0i}\varkappa_i) + (A_i - B_i K_{0i} - B_i K_{0i}\varkappa_i)^T P_i + \\ + 2\varkappa_i D_i]\, x_i - 2x_i^T \varkappa_i^* D_i x_i + x_i^T P_i \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j + (x_i^T P_i \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j)^T\} \tag{28}$$

The Lyapunov function derivation (28) can be bounded similarly as in previous case:

$$\dot{V}(x, \bar{\varkappa}) \leqq -\bar{x}^T W \bar{x} \tag{29}$$

$$w_{ij} = \begin{cases} 2\lambda_m(\varkappa_i^* D_i) & i = j \\ -\|P_i A_{ij}\| - \|A_{ji}^T P_j\| & i \neq j \end{cases} \tag{30}$$

where P_i, $i \in \mathcal{N}$, are solutions of the Lyapunov equations

$$P_i[A_i - B_i K_{0i}(I + \varkappa_i)] + [A_i - B_i K_{0i}(I + \varkappa_i)]^T P_i + 2\varkappa_i D_i = 0\,, \\ i \in \mathcal{N} \tag{31}$$

From (29), (30), we get that there exist finite $\varkappa_i^* > 0$, $i \in \mathcal{N}$, such that the matrix W is positive definite. This completes our proof. □

Remark. For the constructions of the mentioned adaptive laws, the K_{0i} matrices are needed stabilizing the isolated subsystems. Moreover, other conditions defined in Theorems 4 and 5 have to be fulfilled. Hence, the exact knowledge of subsystems dynamics may help to determine appropriate K_{0i}, $i \in \mathcal{N}$. We note that often such K_{0i}, $i \in \mathcal{N}$, can be found when the amount of prior information is less significant.

5. EXAMPLE

We consider the system consisting of two subsystems

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ -1 & -0{\cdot}2 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_2 \\ \dot{x}_2 &= -0{\cdot}1 x_2 + u_2 + [0, -1]\, x_1 \end{aligned} \tag{32}$$

Local state feedbacks are chosen as

$$K_{01} = [2, 3]\,, \quad K_{02} = 3 \tag{33}$$

The following control is considered:

$$u_1 = -(1 + \alpha_1)\, K_{01} x_1\,, \quad u_2 = -(1 + \alpha_2)\, K_{02} x_2 \tag{34.a}$$

$$u_1 = -K_{01} x_1 - K_{01} \varkappa_1 x_1\,, \quad u_2 = -K_{02} x_2 - K_{02} \varkappa_2 x_2 \tag{34.b}$$

In the adaptation laws (4), (6), we choose

$$C_1 = D_1 = \operatorname{diag}(5)\,, \quad C_2 = D_2 = 5$$

The simulations results are shown in Fig. 1 – Fig. 3. The disturbance was simulated

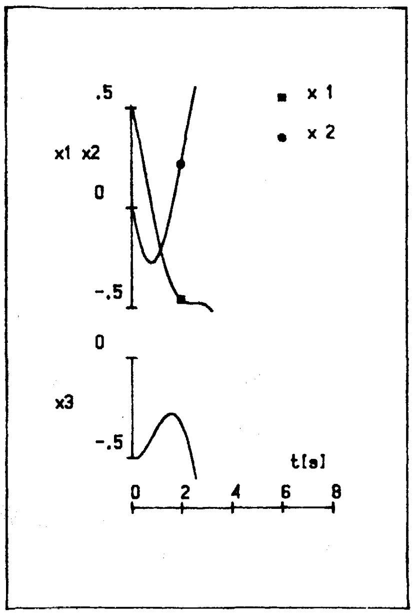

Fig. 1. System responses, nonadaptive state feedback.

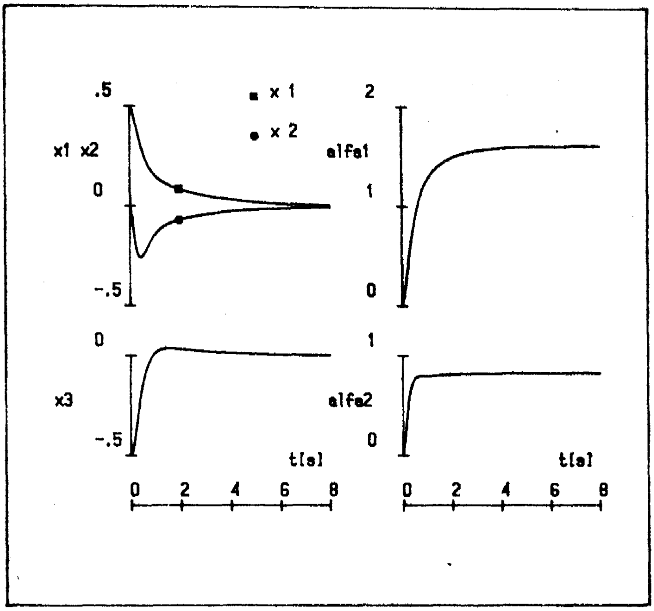

Fig. 2. Systems responses, adaptive control law (34.a).

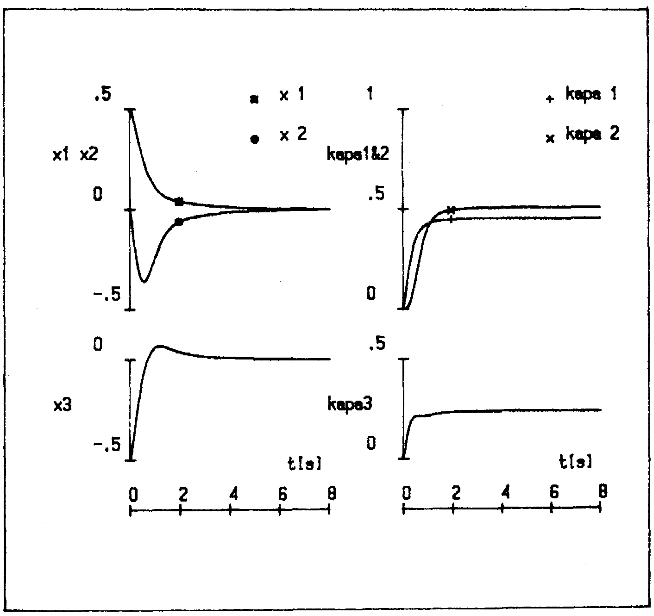

Fig. 3. Systems responses, adaptive control law (34.b).

as the changement of the second diagonal element of A_1 from the value $-0{\cdot}2$ to the value 1, and the A_2 element from $-0{\cdot}1$ to 1 at time $t = 0$. This important disturbance results in unstable behaviour when nonadaptive decentralized control is applied ((34.a); $\alpha_1, \alpha_2 = 0$) as it is shown in Fig. 1. Fig. 2 and Fig. 3. show the results obtained for the adaptive laws (34.a) and (34.b), respectively.

6. CONCLUSION

In this contribution, very simple adaptive decentralized control method for linear dynamic systems consisting of interconnected subsystems is given. The adaptive controlled system convergence is proved by the generalized Lyapunov theory of stability assuming that certain conditions can be fulfilled.

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