KYBERNETIKA — VOLUME 26 (1990), NUMBER 1

THE UNCERTAINTY PROBLEM IN CONTROL THEORY
Part I. Models of Theories

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The main sources of uncertainties in control problems are discussed. Practical control problems must mostly start from the measured data given with the low precision, e.g. on two decimal places. So the relevance of the present feedback control theory depends above all on the incorporation of these data in the formal apparatus. The proposed concept of the theory models, which is based on the modified two-values logic (see [4]), creates the framework for the solution of this problem.

1. INTRODUCTION

The "modern" control theory developed in the 1970's was derived on a precise mathematical basis (cf. [3], [20]). It was introduced as a universal intellectual framework which was motivated more by mathematical apparatus than by any practical problem. It was assumed that systems must have well defined state in a well defined state space and the state can be estimated from a set of measurements. Further it was assumed that control commands can be expressed in a form computable on line from the measurements. All definitions, control laws, stability conditions, etc., had to have be formulated in a fully rigorous mathematical framework. Systems with uncertain parameters or signals corrupted with different uncertainties were alien to this theory (see [20]).

In the late 1970's the experiments connected with practical tasks on small table computers have shown the importance of uncertainties. Simultaneously it became clear from these experiments that the methods of the "modern" control theory alone cannot lead to the design of controllers which are sufficiently robust in the presence of uncertainties (cf. [12], [20]). The explanation of this situation can be found in problems connected with the measurement accuracy (cf. [11]). The measurement error were imbedded in the modern control theory in a stochastic zero-mean component neglecting the fundamental fact that each real measurement

instrument generates data corrupted by an unknown bias. Let us illustrate this problem on the analysis of large scale systems. These systems have mostly hundreds of sensors with different accuracy and reliability characteristics (see [1]). The measured data are mostly given with a precision lower than two decimal places (cf. [11]). Just this limited measurement accuracy, i.e. the residual systematic errors, creates the basic part of an uncertainty band (see [11], [18]). With respect to this uncertainty band it is mostly difficult to get a mathematical description of interactions. So they must be included in the uncertainty band and decrease more and more the usable information in measured signals. Therefore all problems connected with large scale systems must be solved with regard to some uncertainty band and so they are beyond the effective reach of the modern control theory. At the present time we have no theoretical basis for the solution of these problems (see [1]). What we are lacking is a methotology or a theory that combines system dynamics and the uncertainty problem. This dilemma, i.e. the gap between the formal theories and practice, can be explained with the fact that practically all control theories were introduced as the "explanation" theories (cf. [4]).

The topic of this paper follows from the previous discussion. We focus on the relevance of the formal theories. This problem will be analyzed with respect to the uncertainty problem.

Before we pass on to the organization of the paper let us add one remark to the uncertainty in mathematics. The applicability of the formal theories resp. the applicability of pure mathematics depends on the ability to include uncertainties in its apparatus. Here the crisis of the control theory is closely connected with the crisis of mathematics which was not able to explain different anomalies in the numerical computation in the presence of a roundoff uncertainty band, e.g. the stability of numerical processes, the solution of ill-posed problems, the inversion of ill-conditioned matrices, etc. (see [9]). There were different attempts to solve this problem. Unfortunately the fundamental dilemma of all these attempts lies perhaps in the classical two-values logic (cf. [4], [7]). If we want to take into account the uncertainty band then we must accept neutral statements. But the classical logic knows only two values: it holds or it does not hold. From this viewpoint the formal theories based on the two-values logic (e.g. the methods of the modern control theory) are not compatible with an uncertainty band.

The organization of the paper is as follows. The system description is introduced in Section 2. The main sources of uncertainties in the numerical computation and in input data with respect to control problems are discussed in Section 3 resp. 4. The concept of the theory models is proposed in Section 5. The internally robust procedures are introduced and demonstrated on the DNLS estimator in Section 6 and 7.

2. SYSTEM DESCRIPTION

Consider a SISO linear continuous system by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{\beta}\,u(t), \quad y(t) = \boldsymbol{\gamma}\,\boldsymbol{x}(t), \tag{2.1}$$

where the system triple $(\boldsymbol{A}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ has the appropriate dimension. For the zero initial conditions the corresponding transfer function can be written as

$$F(s) = \boldsymbol{\gamma}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\,\boldsymbol{\beta} = \frac{M(s)}{N(s)}. \tag{2.2}$$

For the sake of simplicity we consider the class Λ of systems S with $M(s) = 1$. This is the class of the most favourable cases with respect to uncertainties. The pertinent model $\bar{S}$ ($\bar{S} \in \Lambda$) of the system S derived from measured points of the output signal $\tilde{y}(t) \in \tilde{R}$ is given by the transfer function

$$\bar{F}(s, a) = \frac{1}{\bar{N}(s)}, \tag{2.3}$$

where $\bar{N}(s) = \sum_{i=0}^{\bar{n}} a_i s^i$ is the Hurwitz polynomial.

The output signal $\bar{y}(t, \boldsymbol{a}) \in \bar{R}$ of the model $\bar{S}$ can be written in the form of the nonlinear regression function (cf. [14], [16])

$$\bar{y}(t, \boldsymbol{a}) = \sum_{i=0}^{\bar{n}} a_i\, v^{(i)}(t), \tag{2.4}$$

where the Laplace transforms of the sensitivity functions are

$$\mathscr{L}\{v^{(i)}(t)\} = \frac{s^i\,u(s)}{\bar{N}^2(s)}. \tag{2.5}$$

The system identification can start only from measured points of the output resp. input signals, i.e. from $\tilde{\boldsymbol{y}} = [\tilde{y}(1), \tilde{y}(2), \ldots, \tilde{y}(q)]^{\mathrm{T}}$ resp $\boldsymbol{u} = [u(1), u(2), \ldots, u(q)]^{\mathrm{T}}$ for $\tilde{y}(k) = \tilde{y}(t_k)$ resp. $u(k) = u(t_k)$. Let us emphasize that we shall consider the discrete alternative which is based on the continuous case. The reasons for such an approach are (cf. [2], [14], [16]):

(i) The real systems are continuous in nature and so we obtain rather easily physical insights in control problems.

(ii) The use of z-transformation is connected with numerical errors (cf. [2], [17]) and does not allow introducing the nonlinear regression functions in a simple way.

Thus, the discrete form of regression function (2.4) is given by

$$\bar{y}(k) = \sum_{i=0}^{\bar{n}} a_i\, v^{(i)}(k) \quad (k = 1, 2, \ldots, q). \tag{2.6}$$

3. THE UNCERTAINTY PROBLEM IN NUMERICAL PROCESSES

Uncertainty can enter at every step of the formulation and solution of problems. Let us discuss briefly the main sources of uncertainties.

The first source of uncertainty is the formulation of mathematical models. It is not possible and practical for a mathematical model to represent every aspect of a real problem. A common fallacy in problem solving is the belief that enhancing the model will automatically lead to better results (see [9], [15]).

Most of the methods and software compute things approximately. There are different methods for the same problem because their efficiency and accuracy vary considerable from example to example (see [6], [8], [9]).

The error made in truncating an infinite process is the truncation error. In many cases, the truncation error is exactly the difference between the mathematical and numerical model.

The main source of uncertainty by the computation are round-off errors. It is not easy to know in advance how much precision (i.e. number of digits in the mantissa) is needed to obtain satisfactory results from a computation (cf. [8], [9]). So the round-off error effects resp. the error propagation must be checked. It is possible to show on the case of the quadratic equation that round-off errors can ruin a very ordinary computation very quickly (see Paragraph 3.1).

A mathematical model contains more than equations and relationships; it also contains data that must come from the real world. Uncertainties in the data can have the largest effect at all. This is the case of control problems as we shall show in the next section.

3.1 The quadratic equation

The influence of a round-off uncertainty band can be demonstrated on the known example – on the quadratic equation (cf. [9])

$$\psi(x) = x^2 + 2bx + c \tag{3.1}$$

for the subset of cases $\psi(x) \in \Gamma$ with $c = 10$, and real roots $x_1 = -10^{\alpha}$ and $x_2 = -10^{(1-\alpha)}$ $(\alpha = 1, 2, \ldots)$, i.e. for $b^2 > c$. We shall analyze three alternatives:

(1) b is computed from $b = (10^{\alpha} + 10^{(1-\alpha)})/2$,

(2) b is given directly, i.e. for $\alpha = 3$ we have $b = 1000 \,.\, 01/2$,

(3) b resp. c is given with the precision two decimal digits in the floating point arithmetic, i.e. the precision of initial data is $d_a = 2$. For $\alpha = 1$ resp. $\alpha = 2$ we obtain $b = 11/2$ resp. $b = 1{\cdot}0 \,.\, 10^2/2$.

The values of the small roots $\bar{x}_{2k}$ for the considered three alternatives $(k = 1, 2, 3)$ and pertinent relative errors

$$\Theta_k = \frac{1}{|x_2|} |x_2 - \bar{x}_{2k}| \, 100\% \quad (k = 1, 2, 3) \tag{3.2}$$

were computed on the calculator TI 59 (Texas Instruments 59) which has 13 decimal digits arithmetic ($d_c = 13$) (Table 1). For alternative 1 resp. 2 it can be computed for $\alpha < 4$ accurate results, for $\alpha < 7$ the results with a small error, and for $\alpha = 7$

Table 1.

α	$\lvert x_2 \rvert$	$\lvert \bar{x}_{21} \rvert$	Θ_1	$\lvert \bar{x}_{22} \rvert$	Θ_2	$\lvert \bar{x}_{23} \rvert$	Θ_3
1	1	1	0	1	0	1	0
2	$1 . 10^{-1}$	$1 . 10^{-1}$	0	$1 . 10^{-1}$	0	$1{\cdot}001 . 10^{-1}$	0·1
3	$1 . 10^{-2}$	$1 . 10^{-2}$	0	$1 . 10^{-2}$	0	$1{\cdot}00001 . 10^{-2}$	0·001
5	$1 . 10^{-4}$	$1{\cdot}0001 . 10^{-4}$	0·01	$1{\cdot}0001 . 10^{-4}$	0·01	$1{\cdot}0001 . 10^{-4}$	0·01
6	$1 . 10^{-5}$	$1{\cdot}04 . 10^{-5}$	4·0	$1{\cdot}01 . 10^{-5}$	1·0	$1{\cdot}01 . 10^{-5}$	1·0
7	$1 . 10^{-6}$	$2 . 10^{-6}$	100	$2 . 10^{-6}$	100	$2 . 10^{-6}$	100
8	$1 . 10^{-7}$	$6 . 10^{-5}$	59900	0	100	0	100

the error is 100%. The difference between these two alternatives for $\alpha = 6$ resp. 8 is given by the fact that 10^{α} is computed approximately. Initial data in alternative 1 resp. 2 are given with the computation precision, i.e. $d_d = d_c$, as it is usual in numerical mathematics. Now if we start from initial data given with the precision $d_d < d_c$, e.g. $d_d = 2$ in alternative 3, then the region of the cases with errorneous results can essentially increase (cf. Table 1). This alternative is interesting for the error analysis of control problems (see later). The clue to the trouble in the considered example is that two nearly equal numbers are subtracted which allows the round-off error to become a dominant part of the result.

Let us add that a simple change in the method might eliminate adverse round-off effects. Here the condition $\bar{x}_{1k}\bar{x}_{2k} = 10$ can be used for the regularization of the considered method (cf. [9], [12]).

3.2 Well-posed problems

The uncertainty is discussed in the literature on numerical mathematics in connection with the well-posed resp. ill-posed problems. One may also speak of a computation being ill-posed, and this is the same as saying it is numerically unstable. The word stability refers here mostly (for $d_d = d_c$) to the fact that the round-off error effects are grossly magnified by the method. There were different attempts in the literature (see [8], [9]) how to define the well-posed problems. So it was asked fulfilling three requirements. The solution of the problem must exist, i.e. the sufficiently reliable result for some simple case can be computed by the pertinent algorithm (e.g. cases for $\alpha < 6$ in Table 1). The second requirement is that the computation is unique and the third is that the result of the computation should depend

Lipschitz continuously on the data with a constant not too large. Consider the point of the signal $\bar{y}(k, \boldsymbol{a})$ according to relation (2.6). Here we must ask (cf. [10])

$$|\bar{y}(k, \boldsymbol{a}) - \bar{y}(k, \boldsymbol{a} + \Delta\boldsymbol{a})| \leqq M|\Delta\boldsymbol{a}| \tag{3.3}$$

resp.

$$\left|\frac{\partial \bar{y}(k, \boldsymbol{a})}{\partial a_i}\right| \leqq M_i(\xi) \quad (i = 0, 1, \ldots \bar{n}), \tag{3.4}$$

where $\|\Delta\boldsymbol{a}\| \leqq \xi$.

"Small" changes in the coefficients a_i should result in only "small" changes in the computed point $\bar{y}(k, \boldsymbol{a})$. We see that a computing problem may be well-posed for some data, i.e. for $\|\Delta\boldsymbol{a}\| \leqq \xi$, but not for all data. Let us emphasize that it is not easy to obtain or to estimate the Lipschitz constant for more complex problems that depend on many variables, e.g. in the considered case given by relation (2.6) for $n \geqq 2$ (see [10]). So the given requirements cannot fully characterize the difficulties with the error propagation. Therefore the complementary terms reliability and robustness were introduced (cf. [9]).

The program resp. method reliability measures how well the program resp. method generates the sufficiently reliable results. The most important is the reliability of use (see [9]). A program has reliability of use p if the number of successes in its use divided by the total number of uses is p. Reliability is here defined in terms of probability. If we want to know beforehand whether the software is reliable or not then the best way is systematic testing, i.e. executing the program on a large sample of problems chosen in some way from the problem space.

Software robustness is closely connected with software reliability. It measures how the performace degrades as other methods or better other subroutines are used (cf. [9]). Robustness is also a property of the program and its underlying method (cf. alt 1 and 2 for $\alpha = 8$ in Table 1). The robustness analysis is mathematically difficult; in some instances, it is impossible as it follows from the truthful remark in the literature (cf. Rice [9], pp. 443): Robustness does not have an easy mathematical definition, but it is easier to test a program for robustness.

At the present time there are not available cook-book procedures for the robustness analysis. So the robustness must be analyzed in the close connection with the error analysis as we shall show in Section 5 and 6.

4. UNCERTAINTIES IN INPUT DATA

In this section we shall analyze above all the uncertainty connected with the measurement accuracy of signals, i.e. with the special character of control experiments. Further we shall try to compare different precision levels used in the control theory and practice.

4.1 Control experiments

The control experiments differ from the experiments used by testing the numerical procedures, and from physical experiments used for corroborating physical theories.

The design of experiments in numerical mathematics is mostly based on the assumption that the precision of input data d_d is near to the computation precision d_c, i.e. $d_d = d_c$, as it was shown by the analysis of the quadratic equation in Paragraph 3.1. On the other side the design of control experiments must always start from two different precision levels as we shall show in Paragraph 4.3.

Now let us discuss the difference between physical and control experiments. Here the situation by the feedback control will be assumed. The main properties of physical experiments with respect to possible uncertainties are:

(1a) The most accurate measurement instruments are used.

(2a) The duration of an experiment is not limited. Experiments can be repeated to obtain better results, e.g. in sense of mean values.

(3a) The physical experiment can be prepared a long time, e.g. weeks, to get the best starting conditions.

(4a) An experiment resp. experiments serve for corroborating the hypothetical-deductive theories. The result should be the theory holds or does not hold (cf. [4], [17]).

The control experiments are physical experiments with these special properties:

(1b) The operational measurement instruments with necessary low precision are used as a rule. Now if we control on the required value of the controlled variable then for small deviations we must start from practically zero precision.

(2a) The duration of control experiments is limited. Experiments cannot be repeated (see point (4b)).

(3b) Control experiments are determined with operational conditions.

(4b) An optimal control if possible must be derived from these experiments, i.e. from the evaluation of a small number of measured points, in the time stress (see point (2b)).

It follows from this comparison that it is above all the uncertainty band which designs the character of control experiments and in this way the applicability of the control theory. Till now the formal control theories were derived as the pure physical theories, i.e. the hypothetical-deductive theories (cf. [4], [7]). The fact, that all applications of the feedback control theory must start from control experiments, was ignored (cf. Introduction). If the control theory is to be applicable then it must respect the character of control experiments and so it must be at least partially an inductive theory. It means that the validity of formal theories in the presence of uncertainties is not given beforehand and so it must be always tested with respect to the input data precision. This problem we shall discuss in detail in the next section.

4.2 Modelling the uncertainty band

First let us introduce the measured output signal

$$\tilde{y}(k) = y(k) + \sum_{l=1}^{g} n_l(k) \quad (k = 1, 2, \ldots, q), \tag{4.1}$$

where $n_l(k)$ $(l = 1, 2, \ldots, g)$ are the components of an additive noise. This total noise describes not only the ideal noise, as it is considered in formal theories, but plenty of other errors, e.g. systematic errors of measurement instruments, interactions, approximation error, sampling error, etc. The further limitations must be respected by input signals. Only simple well measurable input signals can be used for the system identification (see Paragraph 6.2 and Section 7). The total noise in relation (4.1) has two parts: the reducible and irreducible errors. The uncertainty band is given above all by these irreducible errors.

Different models of the total noise in relation (4.1) can be found in the literature. In this paper we shall use modelling the total noise with the round-off errors. This approach has some advantages:

(i) It can be simply realized on digital computers. It is closely connected with the measuring scales and so we obtain the uniform scale for different precision levels.

(ii) The results of experiments are unambiguous. Experiments can be simply repeated. That is the main reason why the classical statistics is not suited.

(iii) This model work better in the region of data given with a small precision. The maximal error is here bounded.

Let us discuss the relation of this approach to the precision measure ω of the measuring scale. So we obtain for $d_d = 2$ the precision measure $\omega = 10^{-d} = 10^{-2}$. The dimensionless scale of the unit signal channel is then given by the following levels

$$0{\cdot}00\ 0{\cdot}01\ 0{\cdot}02 \ldots 0{\cdot}99\ 1{\cdot}00. \tag{4.2}$$

Mathematically we have an equivalence over the set Ω of real numbers on the unit interval and sequence (4.2) is the system of the residual class representatives. Therefore the measured points of signals $\tilde{y}(k)$, i.e. $0 \leqq \tilde{y}(k) \leqq 1$ for $k = 1, 2, \ldots, q$, are given with these representatives, i.e. $\tilde{y}(d_d) \in \tilde{R}(\tilde{y}, d_d)$ (see relation (5.1)). We shall assume that the sampling error is always smaller then the measurement error.

Now how this uncertainty – this measurement error – can be incorporated in the model of the total noise. Let us explain this problem on an example. Let l be some length and l_k $(k = 1, 2, \ldots, q)$ measured values in millimeters. Then we obtain the best possible estimate $l = (\sum_{k=1}^{q} l_k)/q$ only with an accuracy of millimeters even if the experiment will be infinitely times repeated (cf. [10], [11], [12]). So it must exist some limit number of experiments q_L by which the irreducible errors are respected. Let us revert to control problems and let us show how the introduced precision measure ω is connected with the accuracy of measurement instruments. The instru-

ment of the accuracy class 1% indicates the measured values $\tilde{y}(k)$ $(k = 1, 2, \ldots, q)$ with the reliability $\pm 1\%$, i.e. the ideal values $y(k)$ $(k = 1, 2, \ldots, q)$ lie in the band $\pm 1\%$ around the indicated values practically safely. So only 50 levels in the unit channel, i.e. 50 representatives in sense of sequence (4.2), can be distinguished and the limit precision measure is given by $\omega_L = 1/50 = 0{\cdot}02$.

We have obtained the uniform error distribution in both cases. To be able to assess the reliability of the estimates a_i $(i = 0, 1, \ldots, \bar{n})$ the connection of these two models to the case with the normal distribution must be at least approximately established. We set out from the demand of the same information contents of the rectangular and Gaussian error distribution with a standard deviation σ resp σ_L (cf. [10], [11]) and we obtain for the total noise

$$\sigma = \omega/\sqrt{(2\pi e)} = 10^{-d}/\sqrt{(2\pi e)} \tag{4.3}$$

resp. for the uncertainty band

$$\sigma_L = \omega_L/\sqrt{(2\pi e)}\,. \tag{4.4}$$

Thus we can calculate for the measurement instrument of the accuracy class 1%, i.e. for $\omega_L = 0{\cdot}02$, the standard deviation $\sigma_L = 4{\cdot}84 \,.\, 10^{-3}$. Then the ideal value of the measured parameter will lie in the uncertainty band $\pm 2{\cdot}066\sigma_L = \pm 0{\cdot}01$ around the measured value practically safely. The estimates in the uncertainty band are all equivalent with respect to the available information. So the other confidence intervals, e.g. $\pm\sigma_L$, cannot be used. This is in accordance with the practical control tasks. We are here interested only in the conclusion: the derived estimates a_i $(i = 0, 1, \ldots, \bar{n})$ can or cannot be used for the solution of the goal problem in sense of the modified two values logic as we shall show later.

Let us add that the model of the uncertainty band pertinent to the measurement instrument of the accuracy class 1%, i.e. for $\omega_L = 0{\cdot}02$, can be derived from the signal $\tilde{y}(d_d = 1)$, i.e. $\tilde{y}(d_d = 1) = 10^{-1}/\sqrt{(2\pi e)}$, for the limit number of measured points

$$q_L = \sigma^2(d_d = 1)/\sigma_L^2(\omega_L = 0{\cdot}02) = 25\,. \tag{4.5}$$

4.3 The precision levels

Now we can analyze different precision levels used by the solution of practical problems. The most important cases are given in Table 2. Here the number of the scale levels D is given by $D = 1/\omega$. The values σ_L make sense only in the last three rows (cf. relation (4.5)). The formal theories are based on the Cantor set theory, i.e. on the absolute measurement precision given by the uncountable infinity $\aleph = 2^{\aleph_0}$ where $\aleph_0$ is the countable infinity. Here we can say the absolute measurement accuracy. Roughly speaking the uncertainty band in Table 2 is the difference between the accuracy and precision.

The double precision of digital computers is appropriate to the range of possible

Table 2.

		D	d	σ_L
Precision of formal theories (CST)		∞	∞	0
Precision of digital computers	double precision	10^{16}	16	$(2{\cdot}4 \,.\, 10^{-17})$
	simple precision	10^{7}	7	$(2{\cdot}4 \,.\, 10^{-8})$
Precision of analog computers		10^{3}	3	$(2{\cdot}4 \,.\, 10^{-4})$
Precision of measurement instruments	0·5%	10^{2}	2	$2{\cdot}4 \,.\, 10^{-3}$
	1%	50	(1·7)	$4{\cdot}8 \,.\, 10^{-3}$
	5%	10	1	$2{\cdot}4 \,.\, 10^{-2}$

physical problems as the experience has had shown. This precision can be simply increased. This is the difference to the precision of the control measurement instruments. It shows the values pertinent to the analog computers which establish the upper boundary by the signal measurement. So increasing the precision of the measurement instruments is hardly solvable problem. Let us emphasize in this connection that if we control some parameter on the required value then we must start for small deviations from practically zero precision.

Now the terms as the uncertainty band resp. robustness, which are used for the characterization of the present crisis of the control theory, have clear meaning. The key problem of the present control theory is how to apply the formal theories starting from the Cantor set theory based on the continuum of real numbers, i.e. from infinite precision $d_d = d_c = \aleph$, on input data with the precision $d_d(q_L) = d_{dL} \doteq 2$ (cf. relation (4.5)). The present control theory is not able to respect the different precision of measurement instruments, i.e. the different situations by the solution of practical tasks, as they are given in the last three rows of Table 2.

5. MODELS OF THE THEORIES

Let us summarize the results of the last two sections:

(i) The formal theories are based on the Cantor set theory, i.e. on the absolute precision $d_d = d_c = \aleph$.

(ii) We can assume that the double precision, i.e. $d_c = 16$, is mostly an acceptable precision level for the solution of practical tasks (in physical sense).

(iii) The main problem of the present control theory is how to include the input data, i.e. data with precision $d_d \doteq 2$, in formal theories.

The white noise, as it is used in formal control theories, is a mathematical fiction (cf. [1]). The engineer usually knows much more about the uncertainty and so he

has to make a judgement on how to model uncertainties. There are not available cook-book procedures for doing this as it follows from Sections 3 and 4. The success of the design depends on the ability of the engineer to know the physics of his problem and to subjectively translate this into mathematical models. But this approach is not simple on the formal level as it follows from the literature ([4], pp. 63): It is consequently impossible, by means of classical logic, to deduce any empirical from any theoretical proposition. Briefly, empirical and theoretical discourse are logically disconnected.

Now the measured data in practical tasks are empirical and so we must consider empirical inexact classes with neutral candidates for the class-membership. This situation is quite foreign to classical two-values logic. So here different ways were proposed and used, e.g. fuzzy sets, bounded but unknown uncertainty, etc.. All these approaches cannot explain the substance of difficulties with the uncertainty, i.e. the difference between the empirical and mathematical propositions. The much more promising way in this direction is the use of the modified two-values (MTV) logic which is based on the following idea ([4], pp. 35): Classical logic does not admit neutral candidates for membership. This does not mean that inexact classes are therefore incapable of precise logical treatment. An imprecise logic would be a contradiction in terms. A precise logic of inexact classes is not.

5.1 The MTV logic

The MTV logic sets out from the following main rules (see [4]):

(i) It is not important the origin of inexactness and neutrality.

(ii) If the propositions are not neutral and classes are exact then they reduce to the rules of classical logic.

(iii) Neutrality is not an independent third truth-value. It may be turned into either of the other two, i.e. truth or falsehood. This is the difference to the 3-valued logic.

(iv) The MTV logic cannot in general serve as an instrument for deducing from neutral statements other such statements in conformity with the usual rules of inference – especially by means of modus ponens (cf. [4]).

Now if $\tilde{R}$ is an inexact class and $\tilde{y}_0$ is the measured signal one of three cases may hold: (i) $(\tilde{y}_0 \in \tilde{R})$, i.e. $\tilde{y}_0$ is a member of $\tilde{R}$; (ii) $(\tilde{y}_0 \notin \tilde{R})$, i.e. $\tilde{y}_0$ is a non-member of $\tilde{R}$; (iii) $(\tilde{y}_0 \in * \tilde{R})$, i.e. $\tilde{y}_0$ is a neutral candidate for membership of $\tilde{R}$.

The solution of control problems is closely connected with inclusions. Then the provisional inclusion $[\tilde{R} \subset \tilde{P}]$, which ignores neutral candidates, is defined as follows: $\tilde{R}$ and $\tilde{P}$ have at least one common member, and no member of $\tilde{R}$ is a non-member of $\tilde{P}$. The possible final inclusion $\{\tilde{R} \subset \tilde{P}\}$ is defined in the same way as the provisional ones, except that now the terms "member" and "non-member" include neutral candidates which have be turned into member or non-member respectively. If the provisional and final inclusions simultaneously hold then it can be introduced the

inclusion $(\tilde{R} \subset \tilde{P})$ or simply $\tilde{R} \subset \tilde{P}$ which can be identified with the subspace of mathematical functions

$$\tilde{y} \in \tilde{R} \cong y(C, \sigma_L) \in R(y, C, \sigma_L), \tag{5.1}$$

where σ_L characterizes the uncertainty band (see Paragraph 4.3), the sign $\cong$ denotes the equality in sense of the MTV logic and C are the conditions (e.g. the structure of the system pertinent to the class of output signals y, the form of an input signal, parameters of a controller, etc.).

We see that the way from measured data to mathematical formulation is relatively complicated. Let us summarize this procedure with respect to the physics. The idealization of inexact empirical into exact non-empirical predicates is the price we pay for simplifying our inferences (cf. [4]). This conclusion is valid in physics, i.e. for physical experiments (cf. Paragraph 4.1), where the sufficiently precise measurement instruments can be used to be able to get near the sample-inclusion $((\tilde{R} \subset \tilde{P}))$ which is completely testable inclusion for the considered physical problem. This situation cannot be assured in control problems as we have shown in Paragraph 4.1 (see the conditions for control experiments) and Paragraph 4.3 (see Table 2). Here the gap, i.e. the Humean gap (see [4]), between the completely testable inclusion $((\tilde{R} \subset \tilde{P}))$ and the incompletely testable idealization $\tilde{R} \subset \tilde{P}$ is so broad that the new approach must be sought to bridge over this gap.

This discussion shows that the gap between the theory and practice is broader as it is stated in the literature. So the very simple problems are open, e.g. it was not explained till now why the classical controllers, i.e. so called Proportional (P) resp. Integral (I) resp. Derivative (D) control, are effective without the identification of the systems and without the optimization of the control processes. The previous discussion together with these remarks explain why the innovation of the present control theory is so difficult or better why it is so difficult to speculate on some directions in which the field may move on.

5.2 The uncertainty problem

The uncertainty band of measured data is directly used above all in identification algorithms. The system analysis is the basis for all other control problems and so we shall focus on this problem in the rest of this paper.

Let us demonstrate the uncertainty problem on the use of some estimator. We can start from the following relation

$$y(C, \sigma_L) \in R(y, C, \sigma_L) \,\&\, T((\cdot), C, \sigma_L, d_c) \Rightarrow \bar{y}(a, \sigma_L, d_c) \in \bar{R}(\bar{y}, C, \sigma_L, d_c). \tag{5.2}$$

This relation describes in the framework of the MTV logic the identification of the system with transfer function (2.3) by means of some estimator given by the formal theory $T_f = T((\cdot), C, \sigma_L = 0, d_c = \aleph)$ from the measured signal $y(C, \sigma_L)$. We assume that the conditions C of the identification experiment are known.

We obtain for $\sigma_L = 0$ and $d_c = \aleph$ mapping $R(y, C, \sigma_L = 0) \Rightarrow \bar{R}(\bar{y}, C, \sigma_L = 0,$

$d_c = \aleph$), i.e. the formal theory – the ideal estimator, which is based on the two-values logic in sense of the literature ([5], [19]). Let us emphasize that we cannot derive some theory by means of the MTV logic as it follows from the discussion in Paragraph 5.1. So if we solve some problem by means of the MTV logic then we must always set out from the pertinent formal theory.

The difference between the solution in the literature ([5], [19]) and by means of relation (5.2) is given by uncertainties. The estimators in ([5], [19]) are considered as the asymptotic theories in sense of the Kolmogorov concept of probability, i.e. all errors are imbedded in a stochastic zero-mean component. Therefore the analysis by means of relation (5.2) is more realistic with respect to practical tasks. So in this paper two new problems with respect to methods resp. theories in the literature ([5], [19]) will be analyzed.

The first problem, i.e. the uncertainty of measured data given above all by systematic errors (cf. Paragraph 4.2 and 4.3), can be analyzed by means of the MTV logic (cf. discussion in Paragraph 5.1).

The second problem is the stability of numerical processes in sense of the discussion in Section 3. Here we shall speak about the model $T((\cdot), C, \sigma_L, d_c)$ of the formal theory T_f in a strict sense. The notation the model of the theory in broader sense can be used for the whole problem, i.e. for all problems connected with relation (5.2). As we have shown in Section 3 the stability of numerical processes is an open problem and so we shall be interested in the next sections above all in the subspace of problems $R(y, C, \sigma_L > 0)$ resp. in the subspace of results $\bar{R}(\bar{y}, C, \sigma_L > 0, d_c < \aleph)$ which can be deliminated from the spaces $R(y, C, \sigma_L = 0)$ resp. $\bar{R}(\bar{y}, C, \sigma_L = 0, d_c = \aleph)$ with respect to the given uncertainty band and for which mapping $R \Rightarrow \bar{R}$ approximately holds (see later).

Let us discuss the use of relation (5.2) in detail. The first point with respect to the delimination of the subspace of results $\bar{R}$ must start from the decision that the solution of the considered problem, i.e. the estimate $\bar{y}(a, \sigma_L, d_c)$, exists for the considered uncertainty band given by the limit standard deviation σ_L. The demarcation of solvable cases in dependence on the precision of measured data can be determined according to the following inclusion for the sufficiently high computation precision d_c (see later) and for the data precision $\sigma_{L2} > \sigma_{L1}$

$$\bar{R}(\bar{y}(a), C, \sigma_{L2}, d_c) \subseteq \bar{R}(\bar{y}(a), C, \sigma_{L1}, d_c) . \tag{5.3}$$

Therefore the subspace of results $\bar{R}(\bar{y}(a), C, \sigma_L, d_c)$ degenerates with decreasing the data precision. We can simply find so large limit deviation $\sigma_L = \sigma_{Lm}$ that the subspace $\bar{R}(\bar{y}(a), C, \sigma_{Lm}, d_c)$ is empty, i.e. the solution cannot exist on this information level. Here we see the fundamental difference between the formal theories and they models. Therefore this conclusion, i.e. $\bar{R}(\bar{y}(a), C, \sigma_{Lm}, d_c) = \emptyset$, is alien to the present formal control theory where problems are solved with respect to the subspace $\bar{R}(\bar{y}(a), d_d = d_c = \aleph)$.

The second point is connected with the delimination of the subspace $\bar{R}$ with the

sufficiently reliable results. Two ways can be used. We can start from the empirical propositions of the type $((\tilde{R} \subset \tilde{P}))$ resp. $\{\tilde{R} \subset \tilde{P}\}$ which are completely or incompletely testable by observation (expert systems, trial and error methods). The second method consists in ideal propositions, grounded in and tested by ideal mathematical constructions (cf. [4]). This way should be used preferably by the theory models. The concept of the limit identifiability of control systems (see [10], [11], [12]) will be useful for the solution of this problem. It is the backwards analysis starting from the given limit noise deviation σ_L in sense of Paragraph 4.2 and the functional characteristics of the considered systems.

The system given by transfer function (2.3) is identifiable if all coefficients a_i $(i = 0, 1, \ldots, \bar{n})$ are identifiable. The coefficient a_i is identifiable if it holds for the pertinent relative standard deviations

$$s_i(C, \sigma_L) = \sqrt{(D(a_i, C, \sigma_L))}/a_i \leqq \varepsilon_M \,, \tag{5.4}$$

where $D(a_i, C, \sigma_L)$ is the variance of the coefficient a_i and ε_M must be chosen appropriate for the solution of the subsequent synthesis problem. If we start from the Neyman-Pearson concept of statistics then we test for $\varepsilon_M = 1$ by means of the null-hypothesis, i.e. for $s_i \geqq 1$ the coefficient $a_i = 0$ or the measurement of the pertinent signal was not sufficiently accurate to establish its value. In control problems we ask $a_i > 0$ and so we usually start from the condition $\varepsilon_M \leqq 0{\cdot}5$ (see [12], [13]).

The solution of the third point, i.e. the estimation of the limit standard deviation σ_L of the noise, is the most difficult as it follows from the discussion of relation (4.1). So this problem must be solved in practical tasks case by case (see Section 7). The success depends on the ability of the engineer to analyze the physical background of the considered problem with respect to uncertainties.

Let us start the discussion of the second part of the uncertainty problem, i.e. the stability of numerical processes, with the following definition:

Definition 5.1. The model $T((\cdot), C, d_d, d_c)$ of the formal theory $T_f = T((\cdot), C, d_d = d_c = \aleph)$ in a strict sense is given by the precision of input data $d_d < \aleph$ and by the computation precision $d_c < \aleph$. Here $(\cdot)$ in the model $T((\cdot), C, d_d, d_c)$ denotes the numerical algorithm pertinent to the transformation $(\cdot)$ in the formal theory T_f and C are the conditions of the pertinent experiments (cf. relation (5.1)).

So we need some concept for the applicability appreciation of the methods resp. theories which are formulated in sense of the robustness as it is discussed in the literature (see [9], [17]). The following attempt is based on the Conditionally Robust (CR) methods resp. theories.

Definition 5.2. The theory resp. the pertinent method is conditionally robust with respect to the uncertainty band of measured data if the pertinent theory model $T((\cdot), C, \sigma_L, d_c)$ fulfils for problems from the subspace $R((\cdot), C, \sigma_L)$ the following conditions:

(i) the solution of each problem from $R((\cdot), C, \sigma_L)$ exists for the double precision, i.e. for $d_d = d_c = d_{cg} = 16$,

(ii) this solution is unique,

(iii) the procedure for testing the stability of numerical processes for $d_d < d_{cg}$ and $d_c \leqq d_{cg}$ is known.

According to this definition the considered theory is the CR theory if the pertinent algorithm was successful tested for the condition of Hypothesis (i) and the numerical process is convergent. Nearly all present formal theories fulfil these requirements for the existing physical problems. So the third point is crucial for the CR theories. But here we must state according to the discussion in Section 3 that the error resp. the robustness analysis is an open problem. In the next section we shall try to show that this problem can be solved under certain assumptions.

(Received May 4, 1989.)

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