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NONLINEAR PERTURBATIONS OF QUASI-LINEAR DELAY CONTROL SYSTEMS

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Sufficient conditions are established for controllability of nonlinear perturbations of quasi-linear delay systems. These conditions are obtained by solving a system of nonlinear integral equations with the help of Schauder's fixed point principle.

1. INTRODUCTION

The problem of controllability of nonlinear systems and nonlinear delay systems has been studied by several authors by means of the Schauder fixed point theorem (see the survey article by Balachandran and Dauer [3]). In [5] Dauer proved that if the linear system is completely controllable then its corresponding perturbed nonlinear system is also completely controllable provided the nonlinear function satisfies certain growth conditions. Dauer and Gahl [6] and Balachandran and Dauer [4] extended this method to nonlinear delay systems and Gahl [7] to nonlinear system of neutral type. Balachandran [1, 2] studied the problem for nonlinear perturbations of linear systems with distributed delays in control. In this paper we shall study the controllability of nonlinear perturbations of quasi-linear delay systems by suitably adopting the technique of Dauer [5].

2. PRELIMINARIES

Let us denote $C_n[-1, t_1]$, the Banach space of continuous $\mathbb{R}^n$ valued functions defined on the interval $[-1, t_1]$ with the following norm

$$\|x\| = \sup |x(t)| \quad \text{for} \quad t \in [-1, t_1] .$$

Let Q be the Banach space of all continuous functions such that

$$Q = C_n[-1, t_1] \times C_m[0, t_1]$$

with norm defined by

$$\|(x, u)\| = \|x\| + \|u\|$$

where

$$\|x\| = \sup |x(t)| \quad \text{for} \quad t \in [-1, t_1]$$
$$\|u\| = \sup |u(t)| \quad \text{for} \quad t \in [0, t_1] .$$

We consider the controllability on a bounded interval $J = [0, t_1]$ of perturbed nonlinear delay system

$$\text{(1)} \qquad \dot{x}(t) = A(t, x(t-1), x(t))\, x(t) + B(t, x(t-1), x(t))\, u(t) + \\ + f(t, x(t-1), x(t), u(t)) \\ x(t) = \phi(t) \quad \text{on} \quad [-1, 0] .$$

Here the vector function x has its values in $\mathbb{R}^n$ and the control function u has its values in $\mathbb{R}^m$. The matrix functions A and B have appropriate dimensions and are assumed to be continuous and bounded on $J \times \mathbb{R}^{2n}$.

Definition. System (1) is said to be completely controllable on the interval J if, for every continuous function ϕ defined on $[-1, 0]$ and every $x_1 \in \mathbb{R}^n$, there exists a control function u defined on J such that the solution of (1) satisfies $x(t_1) = x_1$.

For each fixed $(z, v) \in Q$, consider the following dynamical system

$$\text{(2)} \qquad \dot{x}(t) = A(t, z(t-1), z(t))\, x(t) + B(t, z(t-1), z(t))\, u(t) + \\ + f(t, z(t-1), z(t), v(t)) .$$

Let ϕ be an n-vector function which is continuous on $[-1, 0]$. Then the solution of the system (2) on J with $x(t) = \phi(t)$ for $-1 \leqq t \leqq 0$, is given by

$$\text{(3)} \qquad x(t) = X(t, 0; z(t-1), z(t))\, \phi(0) + \\ + \int_0^t X(t, s; z(s-1), z(s))\, B(s, z(s-1), z(s))\, u(s)\, \mathrm{d}s + \\ + \int_0^t X(t, s; z(s-1), z(s))\, f(s, z(s-1), z(s), v(s))\, \mathrm{d}s$$

where $X(t, s; z(t-1), z(t))$ is the transition matrix of the linear homogeneous system

$$\dot{x}(t) = A(t, z(t-1), z(t))\, x(t)$$

with $X(t, t; z(t-1), z(t)) = I$, the identity matrix and $X(t, s; z(t-1), z(t)) = 0$ for $t < s$. Define the controllability matrix W by

$$\text{(4)} \qquad W(t_1, z) = \int_0^{t_1} X(t_1, s; z(s-1), z(s))\, B(s, z(s-1), z(s))\, B^*(s, z(s-1), z(s)) \\ X^*(t_1, s; z(s-1), z(s))\, \mathrm{d}s$$

where the star denotes the matrix transpose.

3. MAIN RESULT

Now we are able to prove our main result on controllability of perturbed nonlinear delay system (1). For this we will take $p = (x, y, u) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m$ and let $|p| = |x| + |y| + |u|$.

Theorem. Let the continuous function f satisfy the condition

$$\lim_{|p|\to\infty} \frac{|f(t, p)|}{|p|} = 0$$

and suppose there exists a positive constant δ such that

$$\det W(t_1, z) \geqq \delta \quad \text{for all} \quad z \in C_n[-1, t_1]\,.$$

Then, the system (1) is completely controllable on J.

Proof. Let ϕ be continuous on $[-1, 0]$, and let $x_1 \in \mathbb{R}^n$. Define

$$T: Q \to Q \quad \text{by} \quad T(z, v) = (x, u)$$

where

(5) $$u(t) = B^*(t, z(t-1), z(t))\, X^*(t_1, t; z(t-1), z(t))\, W^{-1}(t_1, z)\, [x_1 - X(t_1, 0; z(t-1), z(t))\, \phi(0) - \int_0^{t_1} X(t_1, s; z(s-1), z(s))\,.\, f(s, z(s-1), z(s), v(s))\, ds]$$

and

(6) $$x(t) = X(t, 0; z(t-1), z(t))\, \phi(0) + \int_0^t X(t, s; z(s-1), z(s))\,.\, B(s, z(s-1), z(s))\, u(s)\, ds + \int_0^t X(t, s; z(s-1), z(s))\, f(s, z(s-1), z(s), v(s))\, ds \quad \text{for} \quad t \in J$$

with $$x(t) = \phi(t) \quad \text{for} \quad t \in [-1, 0]\,.$$

Let
$$\begin{aligned}
a_1 &= \sup |X(t, s; z(s-1), z(s))\, B(s, z(s-1), z(s))| \quad \text{for} \quad 0 \leqq s \leqq t \leqq t_1\\
a_2 &= \sup |W^{-1}(t_1; z)|\\
a_3 &= \sup |X(t, 0; z(t-1), z(t))\, \phi(0)| + |x_1| \quad \text{for} \quad 0 \leqq t \leqq t_1\\
a_4 &= \sup |X(t, s; z(s-1), z(s))| \quad \text{for} \quad (t, s) \in J \times J\\
b &= \max \{t_1 a_1, 1\}\\
c_1 &= 6 b a_1 a_2 a_4 t_1\,, \quad c_2 = 6 a_4 t_1\\
d_1 &= 6 a_1 a_2 a_3 b\,, \quad d_2 = 6 a_3\\
c &= \max \{c_1, c_2\}\,, \quad d = \max \{d_1, d_2\}
\end{aligned}$$

$$\sup |f| = \sup [|f(s, z(s), z(s-1), v(s))| : s \in J]\,.$$

Then

$$|u(t)| \leqq a_1 a_2 [a_3 + a_4 t_1 \sup |f|] = d_1/6b + (c_1/6b) \sup |f| \leqq 1/6b\, [d + c \sup |f|]$$

and $$|x(t)| \leqq a_3 + t_1 a_1 \|u\| + a_4 t_1 \sup |f| \leqq b\|u\| + \tfrac{1}{6} d + \tfrac{1}{6} c \sup |f|\,.$$

By Proposition 1 in [5], f satisfies the following condition: for each pair of positive constants c and d there exists a positive constant r such that, if $|p| \leqq r$, then

(7) $$c|f(t, p)| + d \leqq r \quad \text{for all} \quad t \in J\,.$$

Also for given c and d, if r is a constant such that the implication in condition (7) is satisfied, then any r_1 such that $r < r_1$ will also satisfy the implication in (7). Now, take c and d as given above, and let r be chosen so that the implication in (7) is satisfied and $\sup_{-1 \leqq t \leqq 0} |\phi(t)| \leqq \frac{1}{3}r$. Therefore if, $\|z\| \leqq \frac{1}{3}r$ and $\|v\| \leqq \frac{1}{3}r$ then $|z(s)| + |z(s-1)| + |v(s)| \leqq r$ for all $s \in J$. It follows that

$$d + c \sup |f| \leqq r .$$

Therefore,

$$|u(t)| \leqq r/6b \quad \text{for all} \quad t \in J \quad \text{and hence} \quad \|u\| \leqq r/6b .$$

It follows that $|x(t)| \leqq \frac{1}{6}r + \frac{1}{6}r$ for all $t \in J$, and hence that $\|x\| \leqq \frac{1}{3}r$. Thus we have shown that, if

$$H = [(z, v) \in Q: \|z\| \leqq \tfrac{1}{3}r \quad \text{and} \quad \|v\| \leqq \tfrac{1}{3}r]$$

then T maps H into itself. Since all the functions are continuous, it follows that T is continuous. Using the Arzela-Ascoli theorem it is easy to prove that T is completely continuous. Since H is closed, bounded and convex, the Schauder fixed point theorem guarantees that T has a fixed point $(z, v) \in H$ such that

$$(x, u) = T(z, v) = (z, v) . \tag{8}$$

It follows that

$$\begin{aligned} x(t) = {} & X(t, 0; x(t-1), x(t))\, \phi(0) + \\ & + \textstyle\int_0^t X(t, s; x(s-1), x(s))\, B(s, x(s-1), x(s))\, u(s)\, \mathrm{d}s + \\ & + \textstyle\int_0^t X(t, s; x(s-1), x(s))\, f(s, x(s-1), x(s), u(s))\, \mathrm{d}s \quad \text{for } t \in J \end{aligned} \tag{9}$$

and $x(t) = \phi(t)$ on $[-1, 0]$. Hence $x(t)$ is a solution of the system (1) and substituting (5) into (9), we get $x(t_1) = x_1$. Hence, (1) is completely controllable.

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