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A NOTE ON THE DIFFERENTIABILITY IN TWO-STAGE STOCHASTIC NONLINEAR PROGRAMMING PROBLEMS

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It is well known that the optimized function in two-stage stochastic programming problems is the mathematical expectation of the optimal value of the optimized function in a deterministic optimization problem with parameter. So a close connection exists between deterministic parametric optimization problems and two-stage stochastic programming problems. In general, it is possible to say that the behaviour of a stochastic problem is determined by the properties of the corresponding deterministic parametric problem.

In this paper we deal with differentiability of the optimized function in a special case of two-stage stochastic nonlinear programming problems. More precisely, we present conditions under which the form of the classical gradient in the stochastic case follows from the form of the super-gradient in the corresponding parametric problem.

1. INTRODUCTION

The differentiability problem of two-stage stochastic programming problems has been already discussed in the literature. For example, P. Kall [1] studied this problem in the linear case. Sufficient assumptions for the existence of the gradient vector of the optimized function in nonlinear problems were presented in [2] and [4]. In this paper we are going deeper in the nonlinear case. Namely, we try to investigate the form of the gradient vector in a special (but from the practical point of view rather important) case.

Let $X \subset E_n$, $Z_1 \subset E_{s_1}$, $Z_2 \subset E_{s_2}$, $U \subset E_r$, $n, s_1, s_2, r \geqq 1$ be non-empty sets, $(\Omega, \mathscr{S}, P)$ be the probability space, $\eta = \eta(\omega)$ and $\xi = \xi(\omega)$, respectively, be s_1-dimensional and s_2-dimensional random vectors defined on $(\Omega, \mathscr{S}, P)$ such that

$$(1) \qquad P\{\omega: \eta(\omega) \in Z_1\} = P\{\omega: \xi(\omega) \in Z_2\} = 1 .$$

(E_n, $n \geqq 1$ denotes an n-dimensional Euclidean space. It is assumed in (1) that $\{\omega: \eta(\omega) \in Z_1\} \in \mathscr{S}$ and $\{\omega: \xi(\omega) \in Z_2\} \in \mathscr{S}$.)

Moreover, let $h_i(u, z_2)$, $i = 0, 1, 2, \ldots, l$ and $f_i(x, z_1)$, $i = 0, 1, 2, \ldots, l$, respectively, be real valued, continuous functions defined on $E_n \times Z_2$ and $E_n \times Z_1$. If the mapping $K(u, z_1, z_2)$ and the functions $\varphi(u, z_1, z_2)$, $\psi(u, z_1, z_2)$ are defined by

$$K(u, z_1, z_2) = \{x \in X : f_i(x, z_1) \leqq h_i(u, z_2), \quad i = 1, 2, \ldots, l\}$$

$$(2) \qquad \varphi(u, z_1, z_2) = \sup \{f_0(x, z_1) : x \in K(u, z_1, z_1)\}$$

$$\psi(u, z_1, z_2) = \varphi(u, z_1, z_2) + h_0(u, z_2)$$

for $x \in X$, $z_1 \in Z_1$, $z_2 \in Z_2$, $u \in E_r$, then we can introduce the stochastic nonlinear programming problem with a recourse as the problem to find

$$(3) \qquad \sup \{\mathsf{E}\psi(u, \eta(\omega), \xi(\omega)) : u \in U\}$$

where E denotes the operator of mathematical expectation. (In this paper we assume the validity of such conditions under which all symbols in (2) and (3) are meaningful).

The problem given by (2), (3) is a special case of the general problem of the two-stage stochastic nonlinear programming introduced in [2], [4], for example. In this paper we shall deal with this special case only.

In the sequel it will be useful to substitute

$$(4) \qquad y_i = h_i(u, z_2), \quad i = 1, 2, \ldots, l, \quad y = (y, \ldots, y_l) \in E_l$$

and to join to the problem given by (2), (3) the following parametric optimization problem:

Find

$$(5) \qquad \bar{\varphi}(z_1, y) = \sup \{f_0(x, z_1) : x \in \bar{K}(z_1, y)\}$$

where

$$\bar{K}(z_1, y) = \{x \in X : f_i(x, z_1) \leqq y_i, \ i = 1, 2, \ldots, l\},$$

$z_1 \in Z_1$, $y \in E_l$.

It is easy to see that

$$(6) \qquad K(u, z_1, z_2) = \bar{K}(z_1, (h_1(u, z_2), \ldots, h_l(u, z_2)),$$

$$\varphi(u, z_1, z_2) = \bar{\varphi}(z_1, (h_1(u, z_2), \ldots, h_l(u, z_2)).$$

Further, we denote by $Y \subset E_l$ a set for which

$$(7) \qquad (h_1(u, z_2), \ldots, h_l(u, z_2)) \in \operatorname{int} Y \text{ for every } u \in U, \ z_2 \in Z_2.$$

2. SOME AUXILIARY ASSERTIONS

In this section we summarize some definitions and results of the deterministic parametric optimization theory and convex analysis that are necessary for our considerations.

First, we recall one well known result.

Lemma 1. Let X, Y be convex sets, $\bar{K}(z_1, y) \neq \emptyset$ for every $z_1 \in Z_1$, $y \in Y$. If for every $z_1 \in Z_1$ $f_0(x, z_1)$ and $f_i(x, z)$, $i = 1, 2, \ldots, l$, respectively are concave and convex functions on X then for every $z_1 \in Z_1$ $\bar{\varphi}(z_1, y)$ is a concave function on Y.

Remark. As it can happen that $\bar{\varphi}(\cdot, \cdot) = +\infty$ for some $z_1 \in Z_1$, $y \in Y$, we consider $\bar{\varphi}(\cdot, \cdot)$ as a generalized function mapping $Z_1 \times Y$ into $E_1 \cup \{+\infty\}$.

Proof. Since it is easy to see that

$$\lambda \bar{K}(z_1, y) + (1 - \lambda) \bar{K}(z_1, y') \subset \bar{K}(z_1, \lambda y + (1 - \lambda) y')$$

for all $z_1 \in Z_1$, $y, y' \in Y$, $\lambda \in [0, 1]$, the assertion of Lemma 1 follows immediately from the definition of supremum (the sum of sets is considered in the Minkowski sense). □

The Hausdorff distance between two subsets in E_n is defined by the following way.

Definition 1. If $X', X'' \subset E_n$, $n \geqq 1$ are two non-empty sets then the Hausdorff distance of these sets $\Delta_n(X', X'')$ is defined by

$$\Delta_n(X', X'') = \max\left[\delta_n(X', X''), \delta_n(X'', X')\right],$$
$$\delta_n(X', X'') = \sup_{x' \in X'} \inf_{x'' \in X''} \varrho_n(x', x''),$$

where ϱ_n denotes the Euclidean metric in E_n. (We usually omit the subscripts in the symbols Δ_n, ϱ_n, δ_n.)

The following assertion holds.

Lemma 2. Let X, Y be convex sets. If

(i) $y \in Y, z_1 \in Z_1$ implies $\bar{K}(z_1, y) \neq \emptyset$ and the fulfillment at least one of the following conditions
   a) $\bar{K}(z_1, y)$ is a compact set,
   b) $f_0(x, z_1)$ is a bounded function on $\bar{K}(z_1, y)$,

(ii) $f_0(x, z_1)$ is for every $z_1 \in Z_1$ a Lipschitz function on X with the Lipschitz constant C_1 not depending on $z_1 \in Z_1$,

(iii) there exists a constant C_2 such that

$$\Delta[\bar{K}(z_1, y), \bar{K}(z_1, y')] \leqq C_2 \, \varrho(y, y'),$$

for all $z_1 \in Z_1$, $y, y' \in Y$.

then for every $z_1 \in Z_1$ $\bar{\varphi}(z_1, y)$ is a Lipschitz function on Y with the Lipschitz constant $C_1[C_2 + 1]$.

Proof. The assertion of Lemma 2, for every $z_1 \in Z_1$, follows immediately from Lemma 2 of [2]. □

Lemma 3. Let the assumptions of Lemma 1 be fulfilled. Moreover, let be $-\infty < \bar{\varphi}(z_1 y) < +\infty$ for every $z_1 \in Z_1$, $y \in Y$. If int $Y \neq \emptyset$, then for every $z_1 \in Z_1$ there exists a set $R = R(z_1) \subset$ int Y of the Lebesgue measure 0 such that $\bar{\varphi}(z_1, y)$ is a continuously differentiable function on int $Y - R$.

Proof. The assertion of Lemma 3 is a consequence of Theorem 25.5 of [7] and Lemma 1 of this paper. □

The problem given by (5) is a parametric optimization problem with the parameters $z_1 \in Z_1$, $y \in Y$. However, we can also consider this problem separately for every $z_1 \in Z_1$, $y \in Y$ and define the Lagrangian function $L(x, v \mid z_1, y)$ and the Kuhn-Tucker vector $v(z_1, y)$. So let $z_1 \in Z_1$, $y \in Y$ be arbitrary given points. Then

(8) $$L(x, v \mid z_1, y) = f_0(x, z_1) + \sum_{i=1}^{l} v_i[y_i - f_i(x, z_1)]\,,$$
$$x \in E_n\,, \quad v = (v_1, \ldots, v_l) \in E_l\,.$$

Definition 2. A vector $v = v(z_1, y)$, $v \geqq 0$, $v \in E_l$ is the Kuhn-Tucker vector of the problem (5) if
$$\bar{\varphi}(z_1, y) = \sup \{L(x, v \mid z_1, y): x \in X\}$$
($v = (v_1, \ldots, v_l) \geqq 0$ denotes $v_i \geqq 0$ for all $i = 1, 2, \ldots, l$.)

Moreover, if for every $z_1 \in Z_1$ $\bar{\varphi}(z_1, y)$ is a concave function on Y then the supergradient $\partial_y \bar{\varphi}(z_1, y)$ of the function $\bar{\varphi}(z_1, y)$ with respect to y can be defined by the following way:

(9) $$\partial_y \bar{\varphi}(z_1, y) = \{v \in E_l: \bar{\varphi}(z_1, y + y') - \bar{\varphi}(z_1, y) \leqq \langle v, y' \rangle \text{ for every } y' \in E_l\}\,.$$

($\langle \cdot, \cdot \rangle$ denotes the usual scalar product in E_l, $\langle v, y \rangle = \sum_{i=1}^{l} v_i v_i$, $v = (v_1, \ldots, v_l)$, $y = (y_1, \ldots, y_l)$).

Lemma 4. Let X, Y be convex sets. Furthermore, let for every $z_1 \in Z_1$ $f_0(x, z_1)$ and $f_i(x, z_1)$, $i = 1, 2, \ldots, l$, respectively, be concave and convex functions on E_n. If $\bar{\varphi}(z_1, y) \in (-\infty, +\infty)$ then the vector $v = v(z_1, y) \in E_l$ is the Kuhn-Tucker vector of the problem (5) (at the parameter points z_1, y) if and only if $v \in \partial_y \bar{\varphi}(z_1, y)$.

Proof. First, it follows from Lemma 1 that $\bar{\varphi}(z_1, y)$ is a concave function on Y for every $z_1 \in Z_1$.

Further if $z_1 \in Z_1$, $y \in Y$ are arbitrary points we get successively
$$\{v \in E_l: v \in \partial_y \bar{\varphi}(z_1, y), \bar{\varphi}(z_1, y) \in (-\infty, +\infty)\} \Leftrightarrow$$
$$\Leftrightarrow \{v \in E_l: \bar{\varphi}(z_1, y + y') - \bar{\varphi}(z_1, y) \leqq$$
$$\leqq \langle v, y' \rangle \text{ for every } y' \in E_l, \bar{\varphi}(z_1, y) \in (-\infty, +\infty)\} \Leftrightarrow$$
$$\Leftrightarrow \{v \in E_l: \sup_{y'} \sup_{x} [f_0(x, z_1) - \langle v, y' \rangle: x \in X, y' \in E_l, f_i(x, z_1) - y_i \leqq y'_i, i = 1, \ldots, l] =$$
$$= \bar{\varphi}(z_1, y), \bar{\varphi}(z_1, y) \in (-\infty, +\infty)\} \Leftrightarrow$$
$$\Leftrightarrow \{v \in E_l, v \geqq 0, \sup [f_0(x, z_1) + \sum_{i=1}^{l} v_i[y_i - f_i(x, z_1)]: x \in X] =$$
$$= \bar{\varphi}(z_1, y), \bar{\varphi}(z, y) \in (-\infty, +\infty)\}\,.$$

This completes the proof of Lemma 4. □

Remark. The corresponding assertion for convex functions and subgradients is proved in [6].

Lemma 5. If the vectors $v^* \geqq 0$, $v^* \in E_l$, $x^* \in X_n$ fulfil the inequalities

$$L(x, v^* \mid z_1, y) \leqq L(x^*, v^* \mid z_1, y) \leqq L(x^*, v \mid z_1, y) \tag{10}$$

for some $z_1 \in Z_1$, $y \in Y$, $v \geqq 0$, $v \in E_l$, then v^* is the Kuhn-Tucker vector of the problem given by (5) (at the parameter point z_1, y).

Proof. Since $y_i - f_i(x^*, z_1) \geqq 0$, $i = 1, 2, \ldots, l$, follows from the right-hand side of inequalities (10) and simultaneously $\sum_{i=1}^{l} v_i^*[y_i - f_i(x^*, z_1)] = 0$, we get according to the left-hand side of (10)

$$f_0(x, z_1) + \sum_{i=1}^{l} v_i^*[y_i - f_i(x, z_1)] \leqq f_0(x^*, z_1) \text{ for every } x \in X .$$

If we set x^* instead of x into the last relation we get immediately the assertion of Lemma 5. □

3. MAIN RESULTS

The aim of this paper is to present the form of the gradient of the optimized function in the stochastic problem given by (2), (3). However, before doing this we denote by $P_{.|.}$ and $\mathsf{E}_{.|.}$, respectively, the conditional probability measure and conditional mathematical expectation (we tacitly take $P_{\eta|\xi}$ independent of the parameter u). Further, we define the vector function $h(u, z)$ by the relation

$$h(u, z_2) = (h_1(u, z_2), \ldots, h_l(u, z_2)) \quad \text{for} \quad u \in U , \quad z_2 \in Z_2$$

Let $X(\varepsilon)$ for $\varepsilon > 0$ be defined by $X(\varepsilon) = X + B(\varepsilon) = \{x = x_1 + x_2 : x_1 \in X, x_2 \in B(\varepsilon)\}$, where $B(\varepsilon)$ denotes ε-surroundings of $0 \in E_n$. Now we can present the theorem.

Theorem. Let X, U, Y be convex sets, and $h_i(u, z_2)$, $i = 1, 2, \ldots, l$ be differentiable functions on U for every $z_2 \in Z_2$. If relations (1) are fulfilled and if

1) $h(u, \xi(\omega))$ is for every $u \in U$ a random vector such that the conditional probability measure $P_{h|\eta}$ is absolutely continuous with respect to the Lebesgue measure in E_l,
2) there exists a constant $g \in E_1$ such that the condition
$$\varrho[h(u, z_2), h(u', z_2)] \leqq g\, \varrho(u, u')$$
is fulfilled for every $u', u \in U$, $z_2 \in Z_2$,
3) $y \in Y$, $z_1 \in Z_1$ implies $\bar{K}(z_1, y) \neq \emptyset$ and the fulfillment of at least one of the two conditions
   a) $\bar{K}(z_1, y)$ is a compact set,
   b) $f_0(x, z_1)$ is a bounded function on $\bar{K}(z_1, y)$,
4) $f_0(x, z_1)$ is for every $z_1 \in Z_1$ a Lipschitz function on E_n with the Lipschitz constant c_1 not depending on $z_1 \in Z_1$,

5) there exists a constant c_2 such that

$$\Delta[\bar{K}(z_1, y), \bar{K}(z_1, y')] \leqq c_2\, \varrho(y, y')$$

for every $z_1 \in Z_1$, $y, y' \in Y$,

6) $f_0(x, z_1)$ and $f_i(x, z_1)$, $i = 1, 2, \ldots, l$, respectively, are, concave and convex functions on E_n for every $z_1 \in Z_1$,

7) there exists finite $\mathsf{E}\, \bar{\varphi}(\eta(\omega), h(u, \xi(\omega))$ for every $u \in \operatorname{int} U$,

8) there exists finite, differentiable $\mathsf{E} h_0(u, \eta(\omega))$ for every $u \in \operatorname{int} U$,

then there exists the vector of the partial derivatives

$$(11) \qquad \frac{\partial \mathsf{E} \psi(u, \eta(\omega), \xi(\omega))}{\partial u_j} = \frac{\partial \mathsf{E} h_0(u, \eta(\omega))}{\partial u_j} +$$

$$+ \mathsf{E}\left[\sum_{i=1}^{l} v_i(\eta(\omega), h(u, \xi(\omega)) \frac{\partial h_i(u, \xi(\omega))}{\partial u_j}\right], \quad j = 1, 2, \ldots, l\,,$$

where $v = v(z_1, y) = (v_1(z_1, y), \ldots, v_l(z_1, y))$ is a Kuhn-Tucker vector of the parametric optimization problem to find

$$(12) \qquad \sup\{f_0(x, z_1): x \in X, f_i(x, z_1) \leqq y_i,\ i = 1, 2, \ldots, l\}\,.$$

Proof. Let $u_0 \in \operatorname{int} U$, $z_1 \in Z_1$ be arbitrary points. It follows from Lemma 1 and Lemma 3 that there exists a set $R(z_1) \subset \operatorname{int} Y$ of the Lebesgue measure 0 such that $\bar{\varphi}(z_1, y)$ is a continuously differentiable function on $\operatorname{int} Y - R(z_1)$. It means, according to the theory of supergradients, that there exists only one element of the set $\partial_y \bar{\varphi}(z_1, y)$ for every $y \in \operatorname{int} Y - R(z_1)$. This element is equal to the gradient vector of the function $\bar{\varphi}(z_1, y)$ with respect to the components of the vector y.

Further, since we get from the assumptions 3 that $\bar{\varphi}(z_1, y) \in (-\infty, +\infty)$ for all $y \in Y$, it follows from Lemma 4 that the gradient vector of the function $\bar{\varphi}(z_1, y)$ for every $y \in \operatorname{int} Y - R(z_1)$ is determined by the equations

$$(13) \qquad \frac{\partial \bar{\varphi}(z_1, y)}{\partial y_i} = v_i(z_1, y)\,, \quad i = 1, 2, \ldots, l$$

where $v = (v_1, \ldots, v_l)$, $v_i = v_i(z_1, y)$, $i = 1, 2, \ldots, l$ is the Kuhn-Tucker vector of the problem (12).

If we denote $Z_2(u_0) = \{z_2 \in Z_2\colon h(u_0, z_2) \in R(z_1)\}$, it is easy to see, according to relations (6), that $\varphi(u_0, z_1, z_2)$ is a continuously differentiable function at the point u_0 for every $z_2 \in Z_2(u_0)$. Moreover, (13) implies

$$\frac{\partial \varphi(u_0, z_1, z_2)}{\partial u_j} = \sum_{i=1}^{l} v_i(z_1, h(u_0, z_2)) \frac{\partial h_i(u_0, z_2)}{\partial u_j}\,, \quad j = 1, 2, \ldots, l\,.$$

Since we get from assumption 1 that $P_{\xi|\eta}\{\omega\colon \xi(\omega) \in Z_2(u_0)\} = 0$, and since by Lemma 2 and assumption 2 $\varphi(u, z_1, z_2)$ is a Lipschitz function on U with the Lipschitz constant not depending on $z_2 \in Z_2$, we can employ the Lebesgue limit theorem.

Using this theorem we get

$$\frac{\partial \mathsf{E}_{\xi|\eta}\varphi(u_0, \eta(\omega), \xi(\omega))}{\partial u_j} = \mathsf{E}_{\xi|\eta} \sum_{i=1}^{l} v_i(\eta(\omega), h(u, \xi(\omega)) \frac{\partial h_i(u_0, \xi(\omega))}{\partial u_j}, \quad j = 1, 2, \ldots, l.$$

Further, it is easy to see that we can use the Lebesgue limit theorem once more to get

$$(14) \quad \frac{\partial \mathsf{E}\varphi(u, \eta(\omega), \xi(\omega))}{\partial u_j} = \mathsf{E} \sum_{i=1}^{l} v_i(\eta(\omega), h(u_0, \xi(\omega)) \frac{\partial h_i(u_0, \xi(\omega))}{\partial u_j}, \quad j = 1, 2, \ldots, l.$$

As $\boldsymbol{u}_0 \in \operatorname{int} \boldsymbol{U}$ is an arbitrary point we can obtain immediately the assertion of Theorem from the last relation and from assumption 8). □

In Theorem the form of the gradient vector of the optimized function in the special case of two-stage stochastic nonlinear programming problems is presented. However some of its assumptions are rather complicated. We try to find conditions under which the hard verifiable assumptions are fulfilled.

First, we present two groups of assumptions under which there exists the finite $\mathsf{E}\varphi(u, \eta(\omega), \xi(\omega))$. It can be seen that these assumptions are satified in many practical problems.

Lemma 6. Let X be a convex, compact set and let U, Y, Z_1, Z_2 be compact sets. If assumption 6 of Theorem is fulfilled and if

a) $\{x \in X : f_i(x, z_1) < y_i,\ i = 1, 2, \ldots, l\} \neq \emptyset$
for every $z_1 \in Z_1$, $y = (y_1, \ldots, y_l) \in E_l$,

b) $X \supset \{x \in E_n : f_i(x, z_i) \leqq y_i,\ i = 1, 2, \ldots, l$ for an $y \in Y,\ z_1 \in Z_1\}$,

then there exists the finite $\mathsf{E}\varphi(u, \eta(\omega), \xi(\omega))$ for every $u \in U$.

Proof. Continuity of the function $\bar{\varphi}(z_1, y)$ on $Z_1 \times Y$ follows immediately from [4] (Theorem 2). However as Z_1, Y are compact sets, the function $\bar{\varphi}(z, y)$ is bounded on $Z_1 \times Y$. Further, it is easy to see according to relation (6) that $\varphi(\boldsymbol{u}, z_1, z_2)$ is a continuous bounded function on the compact set $U \times Z_1 \times Z_2$. This completes practically the proof. □

Lemma 7. Let X, Y be convex sets. If assumptions 3), 5) of Theorem are satisfied and if

1') there exists function $\bar{f}_i(x)$, $i = 1, 2, \ldots, l$ defined on E_n such that $f_i(x, z_1) = f_i(x)$ for every $x \in X$, $z_1 \in Z_1$,

2') $f_0(x, z_1)$ is a Lipschitz function on $X \times Z_1$,

3') $h_i(u, \xi(\omega))$, $i = 1, 2, \ldots, l$ are square integrable random variables for every $u \in U$,

4') the components of the random vector $\eta(\omega)$ are square integrable.

then the finite $\mathsf{E}\varphi(u, \eta(\omega), \xi(\omega))$ exists for every $u \in U$.

Proof. We get according to [2], Lemma 2, that

$$|\bar{\varphi}(z_1', y') - \varphi(z_1, y)| \leqq L\varrho[(z_1', y'), (z_1, y)]$$

for every $z_1, z'_1 \in Z_1$, $y, y' \in Y$ and a constant $L \in E_1$. Using the triangular inequality and relation (6) we obtain

$$(15) \qquad |\varphi(u, z_1, z_2) - \varphi(u, z'_1, z'_2)| \leqq L[\varrho(z_1, z'_1) + \varrho(h(u, z_2), h(u, z'_2))]$$

for all $u \in U$, $z_1, z'_1 \in Z_1$, $z_2, z'_2 \in Z_2$ and a constant L. The measurability of the function $\varphi(u, \eta(\omega), \xi(\omega))$ for every $u \in U$ follows immediately from the above facts. Now, it is already easy to see that using the Schwartz inequality we get the assertion of Lemma 7. □

Lemma 6 and Lemma 7 deal with the existence of the finite $\mathsf{E}\varphi(u, \eta(\omega), \xi(\omega))$ for $u \in U$. However, assumption (5) of Theorem is hardly verifiable too. At the end of the paper we shall present some conditions under which this assumption is fulfilled.

Lemma 8. Let X be a compact, convex set, Y be a convex set. If $f_i(x, z_1)$, $i = 1, 2, \ldots$ $\ldots, l$ are convex function on X for every $z_1 \in Z_1$ and $\varepsilon_0 > 0$, and if $\bar{K}(z_1, y) \neq \emptyset$ for every $y \in Y(\varepsilon_0)$ $z_1 \in Z_1$, then there exists a constant $C \in E_1$ such that

$$\Delta[\bar{K}(z_1, y), \bar{K}(z_1, y')] \leqq C\varrho(y, y') \quad \text{for every} \quad y, y' \in Y, \quad z_1 \in Z_1 .$$

Proof. Let $z_1 \in Z_1$ be arbitrary. It was proved in [3] (Lemma 1) that there exists a constant $C(z_1) \in E_1$ such that

$$(16) \qquad \Delta[\bar{K}(z_1, y), \bar{K}(z_1, y')] \leqq C(z_1)\, \varrho(y, y') \quad \text{for all} \quad y, y' \in Y .$$

However, as the constant $C(z_1)$ does not depend on $z_1 \in Z_1$ by [3] (cf. proof of Lemma 1), relation (16) proves Lemma 7 too. □

4. CONCLUSION

In this paper we have dealt with the differentiability of the optimized function in a special case of two-stage stochastic nonlinear programming problems. More precisely, we have obtained the form of the gradient vector in this special case. By this we have generalized the corresponding results obtained in [1] for the linear case and the existence theorem presented for a more general case in [2], and [4]. The introduced results can be employed for some approximative methods of solution or for finding the necessary and sufficient optimality conditions. However these possibilities are not discussed in the present paper.

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