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A STOCHASTIC APPROACH TO SOME LINEAR FRACTIONAL GOAL PROGRAMMING PROBLEMS*

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This paper deals with an extension of the goal programming with linear fractional criteria and linear constraints studied by Kornbluth and Steuer [9] (see also [1]). This extension covers the case in which the objective functions or target values are random variables. Three problems are considered. Using some arguments similar to those employed by Ben-Israel, Charnes and Kirby [2] and Stancu-Minasian [15] for the linear case, it is shown that the three stochastic problems can be reduced, under certain hypotheses, to deterministic linear fractional min-max problems with linear or convex constraints. The latter problems are solved by use of the parametrical method presented in [18] and [19] or the procedures considered in [5] and [10] for the linear fractional case with linear constraints.

1. INTRODUCTION

The classical goal programming problem (Charnes and Cooper [4]) is formulated as is shown below.

Problem GP:

$$\text{Minimize } g(x, y, z)$$

subject to

$$F_i(x) - y_i + z_i = G_i \quad \text{for} \quad i \in I = \{1, 2, \ldots, r\}\,, \tag{1.1}$$

$$x \in S\,, \quad y = (y_1, y_2, \ldots, y_r) \geqq 0\,, \quad z = (z_1, z_2, \ldots, z_r) \geqq 0\,, \tag{1.2}$$

where:

(i) $F_i: S \to \mathbb{R}$, $(i \in I)$ are the objective functions;

(ii) $G_i \in \mathbb{R}$ $(i \in I)$ are the levels (goals, target values) to be reached by the objective functions;

(iii) y and z are unknown deviational vectors, whose components measure the deviation upwards and downwards, respectively, of $F_i(x)$ values from the corresponding goal values G_i $(i \in I)$;

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(iv) $S \subseteq \mathbb{R}^n$ is the set of feasible solutions to the problem;

(v) g: $\mathbb{R}_+^{n+2r} \to \mathbb{R}_+$ represents a global deviation function depending on the deviational vectors y and z.

Ordinarily, the objective function g assumes the form

$$g(x, y, z) = \sum_{i \in I} (\alpha_i y_i + \beta_i z_i) , \tag{1.3}$$

where α_i and β_i are intragoal positive weights and specify the relative penalties to be applied for over- or underachievements in G_i. However, in this paper we shall consider another form for $g(x, y, z)$ (i.e. (2.9)).

The goal programming problem has been investigated and generalized by several authors who gave many efficient solution methods. Thus, for instance, for the case when the objective functions F_i $(i \in I)$ are linear, Lee [11] suggested a modified simplex algorithm, whereas Ijiri [8] proposed a generalized inverse technique. In [1], Agrawal, Swarup and Garg considered that the objective functions F_i $(i \in I)$ are linear fractional and proposed an algorithm for solving the goal programming problem. This method is based on Swarup's procedure [17] for solving linear fractional functional programming problems. Kornbluth and Steuer [9] considered the case when F_i $(i \in I)$ are linear fractional and either (a) each linear fractional criterion is assigned to its own priority level or (b) all the criteria are at the same level. In case (a), a sequence of linear fractional programming problems has to be solved, whereas in case (b), one has to solve only one linearly constrained programming problem with a nonlinear objective function which is the sum of the linear fractional objective functions.

Stancu-Minasian [14] considers also that all the criteria are at the same priority level and a goal objective function of Chebyshev type must be minimized (see, e.g. (2.9)).

Peteanu and Ţigan [12] examined the case when all the objective functions are linear fractional and have the same priority level and, additionally, the goals G_i $(i \in I)$ are extended to the interval goals $[G_i', G_i'']$ $(i \in I)$. It is shown that the interval goal programming problem can be reduced to a min-max linear fractional problem with linear constraints. In [20], Ţigan gives an extension of the interval goal programming [12] to include objective functions with inexact data. This extension is a natural link between goal programming, multiobjective programming and inexact programming with set-inclusive constraints, as introduced by Soyster [13]. Stochastic approaches to goal programming are considered by Contini [7] and Chobot [6].

This paper extends goal programming with linear fractional criteria and linear constraints to include the case when the objective functions or target values are random variables. Three stochastic problems are considered and it is shown that these problems can be reduced, under certain hypotheses, to deterministic linear fractional min-max problems with linear or convex constraints. The latter problems can be solved by use of the parametrical methods presented in [18] and [19] or the procedures given in [5] and [10] for the linear fractional case with linear constraints.

Let us first replace Problem GP by the following equivalent Problem GP1:

$$\text{Minimize } g(x, y, z)$$

subject to

$$F_i(x) - y_i \leqq G_i\,, \quad i \in I\,, \tag{1.4}$$

$$F_i(x) + z_i \geqq G_i\,, \quad i \in I\,, \tag{1.5}$$

$$x \in S\,, \quad y_i \geqq 0\,, \quad z_i \geqq 0\,, \quad i \in I\,. \tag{1.6}$$

This formulation of the goal programming problem will be used in the next sections for the extensions obtained in case when the functions F_i or target values G_i $(i \in I)$ are assumed to be random variables. Problems amenable to this formulation are met with in many areas. For an application of a stochastic model dealing with production scheduling and investments in a firm, see [6].

It can be easily shown that problems GP and GP1 have the same optimal solution set when the function g is of the form (1.3) with positive weights. Moreover, it can be proved that GP and GP1 have the same optimal solution set when $g(x, y, z)$ is a strictly increasing function with respect to y and z.

2. RANDOM TARGET VALUES

In what follows, we shall assume that the objective functions F_i in Problem GP1 are well determined, and the target values G_i are independent random variables with known distributions $T_i(\cdot)$ $(i \in I)$.

In this situation, we consider that constraints (1.4)–(1.5) are satisfied in a proportion of cases or, in other words, with certain given probabilities, rather than always satisfied.

Like Ben-Israel, Charnes and Kirby [2] (see also Stancu-Minasian [15]), we consider the following stochastic goal.

Problem SGP:

$$\text{Minimize } g(x, y, z)$$

subject to

$$\mathsf{P}\{\omega \mid F_i(x) \geqq G_i(\omega) - z_i\} \geqq p_i\,, \quad i \in I\,, \tag{2.1}$$

$$\mathsf{P}\{\omega \mid F_i(x) < G_i(\omega) + y_i\} \geqq q_i\,, \quad i \in I\,, \tag{2.2}$$

$$x \in S\,, \quad y_i \geqq 0\,, \quad z_i \geqq 0\,, \quad i \in I\,, \tag{2.3}$$

where p_i and $q_i \in [0, 1]$ are lower bounds of these probabilities.

The other notation is as in the foregoing section.

Let us find the deterministic equivalent of Problem SGP.

Let

$$T_i^{-1}(\theta) = \inf\{\eta\colon T_i(\eta) \geqq \theta\}\,, \quad 0 \leqq \theta \leqq 1\,, \quad i \in I\,,$$

and

$$\hat{T}_i^{-1}(\theta) = \sup\{\eta\colon T_i(\eta) \leqq \theta\}\,, \quad 0 \leqq \theta \leqq 1\,, \quad i \in I\,.$$

We rewrite (2.1) as:

$$\mathsf{P}\{\omega \mid F_i(x) \geqq G_i(\omega) - z_i\} = \mathsf{P}\{\omega \mid G_i(\omega) \leqq F_i(x) + z_i\} =$$
$$= T_i(F_i(x) + z_i) \geqq p_i\,, \quad i \in I\,,$$

or, equivalently,

$$F_i(x) + z_i \geqq T_i^{-1}(p_i)\,, \quad i \in I\,. \tag{2.1'}$$

Similarly, (2.2) is re-written as

$$F_i(x) - y_i \leqq \hat{T}_i^{-1}(1 - q_i)\,, \quad i \in I\,. \tag{2.2'}$$

Thus, we have

Theorem 1. If G_i $(i \in I)$ are independent random variables with distributions $T_i(\cdot)$ $(i \in I)$, then Problem SGP is equivalent to the deterministic

Problem PGD:

$$\text{Minimize } g(x, y, z)$$

subject to

$$F_i(x) + z_i \geqq T_i^{-1}(p_i)\,, \quad i \in I\,, \tag{2.4}$$

$$F_i(x) - y_i \leqq \hat{T}_i^{-1}(1 - q_i)\,, \quad i \in I\,, \tag{2.5}$$

$$x \in S\,, \quad y_i \geqq 0\,, \quad z_i \geqq 0\,, \quad i \in I\,, \tag{2.6}$$

Further on, we need supplementary conditions on F_i and g and shall show that the stochastic goal problem SGP reduces to a linear fractional min-max problem with linear constraints.

Thus, we assume that:

(H1) The objective functions are linear fractional:

$$F_i(x) = \frac{c_i x + c_{i0}}{d_i x + d_{i0}}\,, \quad \text{for all} \quad x \in S\,, \tag{2.7}$$

where $c_i \in \mathbb{R}^n$, $d_i \in \mathbb{R}^n$, c_{i0}, $d_{i0} \in \mathbb{R}$.

(H2) For each $i \in I$:

$$d_i x + d_{i0} > 0\,, \quad \text{for all} \quad x \in S\,. \tag{2.8}$$

(H3) The global deviation function g is of Chebyshev type, i.e.

$$g(x, y, z) = \max_{i \in I} (\alpha_i y_i + \beta_i z_i)\,, \quad (\alpha_i, \beta_i > 0\,, \quad i \in I)\,. \tag{2.9}$$

Such functions were considered, for example, in [12], [14] and [20].

(H4) The set S is polyhedral nonempty and bounded subset of $\mathbb{R}^n$, defined by

$$S = \{x \in \mathbb{R}^n \mid Ax = b,\, x \geqq 0\}\,, \tag{2.10}$$

where A is a given $m \times n$ real matrix and $b \in \mathbb{R}^m$.

Using these statements, problem PGD becomes

Problem PGD1: Find

$$\min \max_{i \in I} (\alpha_i y_i + \beta_i z_i)$$

subject to

$$\frac{c_i x + c_{i0}}{d_i x + d_{i0}} + z_i \geqq T_i^{-1}(p_i), \quad i \in I, \tag{2.11}$$

$$\frac{c_i x + c_{i0}}{d_i x + d_{i0}} - y_i \leqq \hat{T}_i^{-1}(1 - q_i), \quad i \in I, \tag{2.12}$$

$$x \in S, \quad y_i \geqq 0, \quad z_i \geqq 0, \quad i \in I. \tag{2.13}$$

Using assumption (H2), we transform constraints (2.11) and (2.12) into the following constraints:

$$(c_i - T_i^{-1}(p_i)\, d_i)\, x + z_i(d_i x + d_{i0}) \geqq -c_{i0} + d_{i0} T_i^{-1}(p_i), \quad i \in I, \tag{2.11'}$$

$$(c_i - \hat{T}_i^{-1}(1 - q_i)\, d_i)\, x - y_i(d_i x + d_{i0}) \leqq -c_{i0} + d_{i0} \hat{T}_i^{-1}(1 - q_i), \quad i \in I. \tag{2.12'}$$

Therefore, problem PGD1 becomes

Problem PGD2: Find

$$\min \max_{i \in I} (\alpha_i y_i + \beta_i z_i)$$

subject to constraints (2.11′), (2.12′) and (2.13).

The constraints of Problem PGD2 are nonlinear, but we can linearize them by the Kornbluth-Steuer transformation [10]:

$$u_i = z_i(d_i x + d_{i0}); \quad v_i = y_i(d_i x + d_{i0}), \quad i \in I. \tag{2.14}$$

Employing this change of variable, problem PGD2 becomes a deterministic linear fractional min-max problem with linear constraints:

Problem PM: Find

$$\min \max_{i \in I} \frac{\alpha_i v_i + \beta_i u_i}{d_i x + d_{i0}}$$

subject to

$$(c_i - T_i^{-1}(p_i)\, d_i)\, x + u_i \geqq -c_{i0} + d_{i0} T_i^{-1}(p_i), \quad i \in I,$$

$$(c_i - \hat{T}_i^{-1}(1 - q_i)\, d_i)\, x - v_i \leqq -c_{i0} + d_{i0} \hat{T}_i^{-1}(1 - q_i), \quad i \in I,$$

$$x \in S, \quad u = (u_1, u_2, \ldots, u_r) \geqq 0, \quad v = (v_1, v_2, \ldots, v_r) \geqq 0.$$

Under assumptions (H1)–(H4), the following theorem is immediate:

Theorem 2. If G_i are independent random variables and if (x^*, u^*, v^*) is an optimal solution of Problem PM, then (x^*, y^*, z^*) with

$$y_i^* = \frac{v_i^*}{d_i x^* + d_{i0}}, \quad z_i^* = \frac{u_i^*}{d_i x^* + d_{i0}}, \quad i \in I,$$

is an optimal solution to Problem SGP and conversely.

Proof. The proof follows from the fact that Problem SGP is equivalent to Problem PGD (Theorem 1), Problem PGD is equivalent to Problem PGD1 or Problem PGD2 by relations (2.7)–(2.9) and Problem PGD2 reduces to Problem PM by the variable transformation (2.14). □

3. RANDOM OBJECTIVE FUNCTIONS

In this section, we assume that the target values G_i $(i \in I)$ in Problem GP1 are well determined, but the objective functions F_i are random.

We adopt the following assumption:

(H5) The objective functions F_i $(i \in I)$ are of the form:

$$F_i(x, \omega) = \frac{c_i(\omega)\, x + c_{i0}(\omega)}{d_i x + d_{i0}}, \tag{3.1}$$

where the components of the vectors $(c_i, c_{i0}) \in \mathbb{R}^{n+1}$ $(i \in I)$ are normal independent random variables with expectations $\bar{c}_{ij}$ and variances σ_{ij}^2 $(i \in I, j = 0, 1, \ldots \ldots, n)$.

We also suppose that assumptions (H2)–(H4) in Section 2 are true.

With the notations adopted in Section 2, the stochastic goal programming problem is

Problem GPF: Find

$$\min \max_{i \in I} (\alpha_i y_i + \beta_i z_i)$$

subject to

$$\mathsf{P}\{\omega \mid F_i(x, \omega) \geqq G_i - z_i\} \geqq p_i, \quad i \in I, \tag{3.2}$$

$$\mathsf{P}\{\omega \mid F_i(x, \omega) < G_i + y_i\} \geqq q_i, \quad i \in I, \tag{3.3}$$

$$x \in S, \quad y_i \geqq 0, \quad z_i \geqq 0, \quad i \in I. \tag{3.4}$$

Having in view (2.8) and (3.1), constraints (3.2) and (3.3) can be written as

$$\mathsf{P}\{\omega \mid c_i(\omega)\, x + c_{i0}(\omega) \geqq G_i(d_i x + d_{i0}) - z_i(d_i x + d_{i0})\} \geqq p_i, \quad i \in I, \tag{3.5}$$

$$\mathsf{P}\{\omega \mid c_i(\omega)\, x + c_{i0}(\omega) < G_i(d_i x + d_{i0}) + y_i(d_i x + d_{i0})\} \geqq q_i, \quad i \in I, \tag{3.6}$$

or, equivalently, as

$$\frac{G_i(d_i x + d_{i0}) - z_i(d_i x + d_{i0}) - \bar{c}_i x - \bar{c}_{i0}}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 x_j^2 + \sigma_{i0}^2\right)^{1/2}} \leqq \phi^{-1}(1 - p_i), \quad i \in I, \tag{3.7}$$

$$\frac{G_i(d_i x + d_{i0}) + y_i(d_i x + d_{i0}) - \bar{c}_i x - \bar{c}_{i0}}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 x_j^2 + \sigma_{i0}^2\right)^{1/2}} \geqq \phi^{-1}(q_i), \quad i \in I, \tag{3.8}$$

where $\phi(\cdot)$ is the probability distribution function of the standard normal variable $N(0, 1)$.

Thus, problem GPF is

Problem GPF1: Find

$$\min \max_{i \in I} (\alpha_i y_i + \beta_i z_i)$$

subject to constraints (3.7), (3.8) and (3.4).

Employing the change of variable (2.14), problem GPF1 becomes a deterministic linear fractional min-max problem with nonlinear constraints.

Problem GPF2: Find

$$\min \max_{i \in I} \frac{\alpha_i v_i + \beta_i u_i}{d_i x + d_{i0}}$$

subject to

$$\frac{G_i(d_i x + d_{i0}) - u_i - \bar{c}_i x - \bar{c}_{i0}}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 x_j^2 + \sigma_{i0}^2\right)^{1/2}} \leqq \phi^{-1}(1 - p_i), \quad i \in I, \tag{3.9}$$

$$\frac{G_i(d_i x + d_{i0}) + v_i - \bar{c}_i x - \bar{c}_{i0}}{\left(\sum_{j=1}^{n} \sigma_{ij}^2 x_j^2 + \sigma_{i0}^2\right)^{1/2}} \geqq \phi^{-1}(q_i), \quad i \in I \tag{3.10}$$

$$x \in S, \quad u_i \geqq 0, \quad v_i \geqq 0, \quad i \in I. \tag{3.11}$$

It is reasonable to choose $p_i > 0{\cdot}5$ and $q_i > 0{\cdot}5$ such that the constraints of Problem GPF2 define a convex set.

Hence, we have

Theorem 3. If the objective functions F_i $(i \in I)$ are random of the form (3.1) and assumptions (H2)–(H4) are true and if (x^*, u^*, v^*) is an optimal solution of Problem GPF2, then (x^*, y^*, z^*), with

$$y_i^* = \frac{v_i^*}{d_i x^* + d_{i0}}, \quad z_i^* = \frac{u_i^*}{d_i x^* + d_{i0}}, \quad i \in I,$$

is an optimal solution to Problem GPF and conversely.

The proof is similar to that of Theorem 2 and so, it will be omitted.

4. SIMPLE RANDOMIZATION OF OBJECTIVE FUNCTIONS

Throughout this section, we shall adopt the following assumptions for Problem GPF:

(H6) The objective functions F_i $(i \in I)$, given by (3.1), are simply randomized, i.e.,

$$c_i(\omega) = c_i' + t_i(\omega)\, c_i'' \,, \quad c_{i0}(\omega) = c_{i0}' + t_i(\omega)\, c_{i0}'' \,, \quad i \in I \,, \tag{4.1}$$

where $c_i', c_i'' \in \mathbb{R}^n$ $(i \in I)$ are constant vectors, $c_{i0}', c_{i0}'' \in \mathbb{R}$ $(i \in I)$ are scalar constants and $t_i(\cdot)$ $(i \in I)$ are random variables on a probability space (Ω, K, P) with continuous and strictly increasing distribution functions $T_i(\cdot)$ $(i \in I)$.

We note that the stochastic programming problems with simple randomization were examined by Bereanu [3] in the linear case and Stancu-Minasian [15], Stancu-Minasian and Ţigan [16], [21] for some special classes of nonlinear minimum-risk problems.

(H7) The global function g assumes the form

$$g(x, y, z) = \max_{i \in I} (\alpha_i y_i + \beta_i z_i) \tag{4.2}$$

(H8) $d_i x + d_{i0} > 0$, for all $i \in I$ and for all $x \in S$.

(H9) $c_i'' x + c_{i0}'' > 0$, for all $i \in I$ and for all $x \in S$.

Hence, problem GPF becomes

Problem GPSR: Find

$$\min \max_{i \in I} (\alpha_i y_i + \beta_i z_i)$$

subject to

$$\mathsf{P}\left\{\omega \,\middle|\, \frac{(c_i' + t_i(\omega)\, c_i'')\, x + (c_{i0}' + t_i(\omega)\, c_{i0}'')}{d_i x + d_{i0}} \geqq G_i - z_i\right\} \geqq p_i \,, \quad i \in I \,, \tag{4.3}$$

$$\mathsf{P}\left\{\omega \,\middle|\, \frac{(c_i' + t_i(\omega)\, c_i'')\, x + (c_{i0}' + t_i(\omega)\, c_{i0}'')}{d_i x + d_{i0}} < G_i + y_i\right\} \geqq q_i \,, \quad i \in I \,, \tag{4.4}$$

$$x \in S \,, \quad y_i \geqq 0 \,, \quad z_i \geqq 0 \,, \quad i \in I \,. \tag{4.5}$$

Let us find the deterministic equivalent of Problem GPSR. Under assumptions (H6), (H7), (H8) and (H9), we transform constraints (4.3) and (4.4) into the following constraints:

$$\mathsf{P}\left\{\omega \mid t_i(\omega) \geqq \frac{G_i(d_i x + d_{i0}) - z_i(d_i x + d_{i0}) - c_i' x - c_{i0}'}{c_i'' x + c_{i0}''}\right\} \geqq p_i \,, \quad i \in I \,, \tag{4.6}$$

$$\mathsf{P}\left\{\omega \mid t_i(\omega) < \frac{G_i(d_i x + d_{i0}) + y_i(d_i x + d_{i0}) - c_i' x - c_{i0}'}{c_i'' x + c_{i0}''}\right\} \geqq q_i \,, \quad i \in I \,, \tag{4.7}$$

or

$$(4.8) \qquad \frac{G_i(d_i x + d_{i0}) - z_i(d_i x + d_{i0}) - c_i' x - c_{i0}'}{c_i'' x + c_{i0}''} \leqq T_i^{-1}(1 - p_i), \quad i \in I,$$

$$(4.9) \qquad \frac{G_i(d_i x + d_{i0}) + y_i(d_i x + d_{i0}) - c_i' x - c_{i0}'}{c_i'' x + c_{i0}''} \geqq T_i^{-1}(q_i), \quad i \in I.$$

According to assumption (H9), constraints (4.8) and (4.9) become

$$(4.10) \qquad (G_i d_i - c_i' - T_i^{-1}(1 - p_i)\, c_i'')\, x - z_i(d_i x + d_{i0}) \leqq$$
$$\leqq c_{i0}' + c_{i0}'' T_i^{-1}(1 - p_i) - d_{i0} G_i, \quad i \in I,$$

$$(4.11) \qquad (G_i d_i - c_i' - T_i^{-1}(q_i)\, c_i'')\, x + y_i(d_i x + d_{i0}) \geqq$$
$$\geqq c_{i0}' + c_{i0}'' T_i^{-1}(q_i) - d_{i0} G_i, \quad i \in I.$$

Therefore, problem GPSR can be formulated as follows:

Problem GPSR1: Find

$$\min \max_{i \in I} (\alpha_i y_i + \beta_i z_i)$$

subject to constraints (4.5), (4.10) and (4.11).

By the change of variable (2.14), Problem GPSR1 becomes:

Problem GPSR2: Find

$$\min \max_{i \in I} \frac{\alpha_i v_i + \beta_i u_i}{d_i x + d_{i0}}$$

subject to

$$(4.12) \qquad (G_i d_i - c_i' - T_i^{-1}(1 - p_i)\, c_i'')\, x - u_i \leqq$$
$$\leqq c_{i0}' + c_{i0}'' T_i^{-1}(1 - p_i) - d_{i0} G_i, \quad i \in I,$$

$$(4.13) \quad (G_i d_i - c_i' - T_i^{-1}(q_i)\, c_i'')\, x + v_i \geqq c_{i0}' + c_{i0}'' T_i^{-1}(q_i) - d_{i0} G_i, \quad i \in I,$$

$$(4.14) \qquad x \in S, \quad u_i \geqq 0, \quad v_i \geqq 0, \quad i \in I.$$

We have

Theorem 4. If assumptions (H4)–(H9) are true, and if (x^*, u^*, v^*) is an optimal solution of the min-max problem GPSR2, then (x^*, y^*, z^*), where

$$y_i^* = \frac{v_i^*}{d_i x^* + d_{i0}}, \quad z_i^* = \frac{u_i^*}{d_i x^* + d_{i0}}, \quad i \in I,$$

is an optimal solution of the stochastic goal programming problem GPSR, and conversely.

5. CONCLUSIONS

Three classes of stochastic goal programming problems were considered. Deterministic linear fractional min-max problems with linear or convex constraints (as in the case of Problem GPF2) were obtained for each of these problems.

These deterministic problems can be solved by use of the parametrical method presented in [18], [19] or the procedures given in [5], [10] for the linear fractional case with linear constraints.

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