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ZEROTESTING BOUNDED ONE-WAY MULTICOUNTER MACHINES

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One-way multicounter machines with bounds on the number of reversals and zerotests in accepting computations are studied. The bounds are considered as functions of the length of input words. The first hieararchy results for nonconstant bounds on the number of zerotests are obtained. Further results relate the reversal complexity and zerotest complexity again in relation to nondeterminism, time, and the number of counters as additional complexity parameters.

1. INTRODUCTION AND DEFINITIONS

We shall study the hierarchy of zerotesting bounded one-way multicounter machines and their relation to reversal bounded one-way multicounter machines in this paper. We consider a multicounter machine as a multipushdown machine whose pushdown stores have a single-letter alphabet.

Informally (the formal definition can be found in [4, 5]), a *one-way multicounter machine* consists of a finite state control, a one-way reading head which reads the input from the input tape, and a finite number of counters. We can regard a counter as an arithmetic register containing an integer which can be positive or zero. In one step, a one-way multicounter machine may increase or decrease a counter by 1. The action or the choice of actions of the machine is determined by the input symbol currently scanned, the state of the machine, and the sign of each counter: positive or zero. The machine starts with all counters empty and accepts if it reaches a final state with all counters empty. The class of one-way multicounter machines working in quasirealtime (for each machine there exists such a constant d that the length of each part of any computation, in which the reading head is stationary, is bounded by d) will be denoted by *QR-COUNTER*, the deterministic version by *QR-DCOUNTER*. The class one-way multicounter machines without any restriction will be denoted by *COUNTER*, the deterministic version by *DCOUNTER*.

We shall study the reversal and zerotesting bounded versions of these machines, where the number of reversals is bounded by a function of the input word length. The constant-reversal bounded computations were studied in [5, 7], the f-reversal bounded computations, for an increasing function f, were studied in [1, 2, 8, 10, 11]. The zerotesting bounded multicounter machines have been so far considered for constant bound only [12].

For increasing functions $f(n) = o(g(n))$, where $g(n) = o(\log_2 n)$, and one-way deterministic quasirealtime multicounter machines, we prove the existence of strong hierarchy according to zerotest bounds f and g. Further, we show that there is a language L' recognized by a one-way deterministic quasirealtime multicounter machine with one zerotest which can be accepted by no one-way nondeterministic multicounter machine with $f(n) = o(\mathrm{n})$ reversal number bound. This result relates the reversal complexity and the zerotest complexity in relation to nondeterminism and time as additional complexity parameters.

Now, let us define formally the reversal, and zerotest complexity measures. Let $\mathscr{L}(M)$ be the class of languages accepted by the machines in a class M. Let A be a multicounter machine from the class M and $L(A)$ be the language accepted by A. Let f be a real function defined on natural numbers. Then $L_{Rf}(A)$ denotes the set of all words in $L(A)$ for which there is an accepting computation containing at most $f(n)$ reversals (i.e. changes from increasing to decreasing contents of a counter or vice versa), where n is the length of the input word. Let F be the class of all functions q from natural numbers to positive real numbers such that for all natural numbers $n\colon f(n) \geqq q(n)$. Then we define the classes of languages

$$\mathscr{L}_{Rf}(M) = \bigcup_{B\in M} \{L_{Rf}(B)\} \quad \text{and} \quad \mathscr{L}(M - R(f)) = \bigcup_{q\in F} \mathscr{L}_{Rq}(M)\,.$$

Now, we shall introduce the zerotesting restriction. Let f be a function from natural numbers to positive real numbers, and let A be a multicounter machine from a class M. Then $L_{Zf}(A)$ is the set of all words in $L(A)$ for which there is some accepting computation having its zerotest number at most $f(n)$ (i.e. the machine A empties the counters at most $f(n)$ times in the computation), where n is the length of the input word. Let F be the class of all functions q from natural numbers to positive real numbers such that for all natural $n\colon f(n) \geqq q(n)$. We define the following classes of languages

$$\mathscr{L}_{Zf}(M) = \bigcup_{B\in M} \{L_{Zf}(B)\} \quad \text{and} \quad \mathscr{L}(M - Z(f)) = \bigcup_{q\in F} \mathscr{L}_{Zq}(M)\,.$$

In what follows we shall often consider computations in which a multicounter machine reads a group of identical symbols whose number is greater than the number of states. Clearly, there has to be a state q which will be entered twice (or more) in different configurations in this part of computation. If these two occurrences of the state q are adjacent (no further state q and no two equal states different from q occurs inbetween) we say that this part of the computation is a *cycle* with *state characteristic* q, *reading head characteristic* – the number of symbols over which the reading head moves to the right in this cycle, and *counter characteristic*, for each counter, which is the difference between the counter contents at the beginning and at the end of the cycle. Thus, the counter characteristic can be positive, if the machine increases the contents of the counter in the cycle, it can be negative if the counter contents is decreased in this part of the computation, and obviously it can be zero.

We call attention to the fact that if a cycle occurs in a computation of a one-way deterministic multicounter machine reading a group of identical symbols it follows that this cycle will be executed repeatedly until the reading head reaches some different symbols (i.e. until the reading head reads through the whole group of identical symbols) or a counter reaches zero.

Let s be the number of states of a multicounter machine A and k be the number of A counters. Then we can bound the number of all cycles with different characteristics by

$$s(s+1)(2s+1)^k .$$

Now, we introduce the following notation. Let d be a real number. Then $\{d\}$ is the smallest natural number k such that $d \leqq k$, and $[d]$ is the largest natural number m such that $d \geqq m$. Let f, and g be functions defined on natural numbers. The fact that $\lim_{n\to\infty} f(n)/g(n) = 0$ will be denoted by $f(n) = o(g(n))$.

2. RESULTS

Using modifications of the proof technique developed in [3, 7, 10] a hieararchy of one-way quasirealtime deterministic multicounter machines according to zerotest bound is esatblished and a relation between reversal and zerotest complexity measures is shown.

Theorem 1. Let f and g be increasing functions from naturals to positive reals such that $g(n) \geqq f(n)$ for all natural n, $f(n) = o(g(n))$ and $g(n) = o(\log_2 n)$. Then $\mathscr{L}(QR\text{-}DCOUNTER\text{-}Z(f)) \subsetneq \mathscr{L}(QR\text{-}DCOUNTER\text{-}Z(g))$.

Proof. Let us consider the language $L_g = \{u_1 u_2 \dots u_p \mid p \leqq g(|u_1 u_2 \dots u_p|)$ and $u_i = a^{n_i} b^{n_i}$ for all $i = 1, \dots, p$, where $n_i > 0\}$. Clearly, this language can be accepted by a $QR\text{-}COUNTER$ machine B with one counter such that $L_{Zg}(B) = L_g$.

Now, we shall show by contradiction that the language L_g does not belong to $\mathscr{L}(QR\text{-}DCOUNTER\text{-}Z(f))$. Let h be a function from natural numbers to positive real numbers such that $h(n) \leqq f(n)$ and let there exist a $QR\text{-}COUNTER$ machine A such that $L_{Zh}(A) = L_g$. Let A have s states and let the length of each part of any computation, in which the reading head is stationary, be bounded by a constant c. We shall consider the word

$$x = a^{n_1} b^{n_1} a^{n_2} b^{n_2} \dots a^{n_{g(n)}} b^{n_{g(n)}}$$

where $n_1 \geqq s + 1$, $n_i \geqq 2sc \sum_{j=1}^{i-1} n_j$ for $i = 1, \ldots, g(n)$. Since $g(n) = o(\log_2 n)$ we can assume that such a word exists and belongs to L_g.

Since $f(n) = o(g(n))$ there exists such subword

$$y = a^{n_j} b^{n_j} \ldots a^{n_k} b^{n_k}$$

of the word x that $k - j$ is greater than the number of all cycles of A with different characteristics, and the machine A empties no counter during the computation on y. It is easy to see that, for $i = j, j + 1, \ldots, k$, the counter characteristics of all cycles used in the computation on a^{n_i} must be equal or greater than 0. It follows from the fact that if one of the counter characteristics of a cycle is negative in the computation on a^{n_r}, for some $j \leqq r \leqq k$, then the corresponding counter must be emptied in the computation on a^{n_r}, what is the consequence of the assumption

$$n_r \geqq 2sc \sum_{j=1}^{r-1} n_j .$$

Since $k - j$ is greater than the number of all cycles of A with different characteristics there exist such two numbers d, z: $j \leqq d < z \leqq k$ that the machine A works on a^{n_d} in a cycle s_1 and on a^{n_z} in a cycle s_2, where s_1 and s_2 have the same characteristics. Let m be the reading head characteristic of these cycles. Clearly, we can assume $m > 0$ because no quasirealtime computation of a deterministic multicounter machine can involve a cycle with the reading head characteristic 0. It can be simply seen that if A accepts the word $x = v_1 y v_2$ then A must accept the word

$$x' = v_1 a^{n_j} b^{n_j} \ldots a^{n_d + m} b^{n_d} \ldots a^{n_z - m} b^{n_z} \ldots a^{n_k} b^{n_k} v_2$$

what proves our assertion because x' does not belong to L_g, and if $w = w_1 s_1 w_2 s_2 w_3$ is the accepting computation on x then $w' = w_1 s_1 s_1 w_2 w_3$ is the accepting computation of A on x'. □

Now, we show that quasirealtime, determinism, and one zerotest can be sufficient for the recognition of a language, for which nondeterminism, unbounded time, and $f(n) = o(n)$ reversals do not suffice.

Theorem 2. Let $f(n) = o(n)$ be a function from naturals to positive reals. Then $\mathscr{L}(QR\text{-}DCOUNTER\text{-}Z(1)) - \mathscr{L}(COUNTER\text{-}R(f)) \neq \emptyset$.

Proof. Let us consider the language $L' = \{w \mid w \text{ in } \{a, b\}^* \text{ such that } \#a(w) = \#b(w) \text{ and for all } x, y \text{ in } \{a, b\}^+, w = xy \text{ implies } \#a(x) > \#b(x)\}$. This lan uage can be accepted by a *QR-DCOUNTER* machine B' with one counter. The machine B' increases (decreases) the contents of its counter for all $a(b)$ on the input tape. If B' computing on an input word will empty its counter, and is not scanning the last symbol of the input word then B' does not accept. If B' will empty its counter and is scanning the last symbol on the input word B' accepts.

Now, we shall prove by contradiction that L' does not belong to $\mathscr{L}(COUNTER$-

-$R(f)$). Let h be a function from natural numbers to positive real numbers such that $h(n) \leqq f(n)$. Let us assume that there exists a *COUNTER* machine B such that $L_{Rh}(B) = L'$. Let B have s states, k counters, and the number of all cycles with different characteristics be bounded by a constant d. We shall consider the input word

$$x = a(a^{s+1}b^{s+1})^{c(s+1)(k+1)\{f(n)\}}\, a^m b^{m+1}$$

in L', where $c > d$. Since $f(n) = o(n)$, for a sufficiently large n, there is a nonnegative integer m such that the length of x is $n = 2 + 2(s+1)\,c(k+1)\,\{f(n)\} + 2m$. So, there exists a subword

$$x_1 = (a^{s+1}b^{s+1})^{c(s+1)(k+1)}$$

of x such that B computing on x_1 reversals no counter. It is easy to see that in the same way as in the proof of Theorem 1 of [10] we can construct a word x' which does not belong to L' but which is accepted by B. Obviously x' will be constructed from x in such way that we first take some amount of a's of an a^{s+1} of x, and then pump a group of a's of x, what breaks the prefix property of x.

Corollary 1. Let $f(n) = o(n)$ be a function defined on natural numbers. Then
$\mathscr{L}$(*QR-COUNTER-R*(f)-*Z*(1)) $\subsetneq$ $\mathscr{L}$(*QR-COUNTER-Z*(1))
$\mathscr{L}$(*COUNTER-R*(f)-*Z*(1)) $\subsetneq$ $\mathscr{L}$(*COUNTER-Z*(1)).

We call attention to the fact that the hierarchy results formulated in Corrolary 1 can be formulated for deterministic machines, and machines with different time restrictions too.

Concluding this paper we give some open problems concerning zerotesting multi-counter machines.

Open Problems. Let $f(n) \leqq g(n)$ be functions such that $f(n) = o(g(n))$. What is the relation between:

1. $\mathscr{L}$(*DCOUNTER-Z*(g)) and $\mathscr{L}$(*DCOUNTER-Z*(f))
2. $\mathscr{L}$(*QR-DCOUNTER-Z*(g)) and $\mathscr{L}$(*QR-DCOUNTER-Z*(f)) for $g(n) \geqq \log_2 n$
3. $\mathscr{L}$(*QR-DCOUNTER-Z*(f)) and $\mathscr{L}$(*DCOUNTER-Z*(f))
4. $\mathscr{L}$(*DCOUNTER-Z*(f)) and $\mathscr{L}$(*COUNTER-Z*(f))
5. $\mathscr{L}$(*QR-DCOUNTER-Z*(f)) and $\mathscr{L}$(*QR-COUNTER-Z*(f))?

We note that Problems 1, 2, 3 can be formulated for nondeterministic machines too. We conjecture that we shall be able to solve Problems 3, 5, and Problem 2 for $g(n) \leqq n$. On the other hand we have no idea which can help to solve the additional problems.

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