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RECEIVING BINARY CODED SPREAD SPECTRUM SIGNALS

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A short heuristic introduction to individual study of the literature on the reception of pseudo-noise-coded spread-spectrum signals is presented.

1. INTRODUCTION

In the past few years, the use of spread-spectrum coded signals ceased to be the job of a limited group of specialists. Substantially more workers will be interested in applications of this part of the information theory and practice in the near future [29].

There is no need to present here the motives for the practical use of spread-spectrum coded signals. They are elucidated with details e.g. in [8], [13], [24], [28].

The matter of the article is treated heuristically to give a brief survey of basic problems and solutions to readers with no prior knowledge, and a balanced sample of pertaining articles has been collected for more serious study.

2. THE ESSENCE OF USED SIGNALS

Information code $\{d_k\}$ and transmission code $\{g_l\}$ will be discerned in what follows. Their sense will be clear from an example (see (5)), and both are supposed known on the receiving side. Usually, both are formed by phase shift keying, so that the transmitted signal can be expressed as

$$s(t) = e^{j(\omega t + \varphi_k + \varphi_l)} \tag{1}$$

where ω is the carrier circular frequency (eventually, on the reception side, lowered by heterodyning). Only binary codes will be considered in what follows, so that $d_k = \exp(j\varphi_k) = +1$ or -1 over a sequence of time intervals of the same length T,

$k = 1, 2, \ldots, K$, and $\{d_k\}$ is the used 'alphabet'. Not all 2^K 'letters' need have sense and the transmitted 'information' may be ciphered. For the arbitrary origin of the transmission, one may choose the time $t = 0$. To every t unique d_k and g_l correspond with exception of isolated time instants where phase jumps occur.

The transmission code $\{g_l\}$, $l = 1, 2, \ldots, N$, with $g_l = \exp(\mathrm{j}\varphi_l) = \pm 1$ is usually a well-known pseudonoise (PN) code, repeating periodically during the transmission. The time length of every code element is τ_i and

$$T/\tau_i = N \tag{2}$$

Thus, a single d_k is superposed on a whole period $\{g_l\}$, and the spectre broadening given by (2) represents the 'processing gain' of this method of communication.

The received signal can be expressed as

$$s(t) = d_k g_l \, \mathrm{e}^{\mathrm{j}(\omega t + \varphi)} \tag{3}$$

where φ is the phase shift given by the distance between the transmitter and the receiver. In (3) and in what follows, factors representing amplifications and attenuations, and the variations of φ by the Doppler effect, are neglected.

Knowing ω, φ on the receiving side, the operation

$$s(t) \,.\, \mathrm{e}^{-\mathrm{j}(\omega t + \varphi)} = d_k g_l \tag{4}$$

could be realized.

The right side of (4) will now be expressed in more details, chosen for $\{g_l\}$ the well-known PN-7 code (in practice N is usually of the order of 10^3 to 10^4).

(5)

$\{d_k\}$	1	1	-1	...
$\{g_l\}$	1 1 1 -1 -1 1 -1	1 1 1 -1 -1 1 -1	1 1 1 -1 -1 1 -1	...
$\{d_k g_l\}$	1 1 1 -1 -1 1 -1	1 1 1 -1 -1 1 -1	-1 -1 -1 1 1 -1 1	...
t	0	T	$2T$	$3T$

Now, t in (5) is the local time of the receiver and the third row results from the received signal according to (4). Knowing τ_i and the 'phase' τ of the code $\{g_l\}$ on the receiving side (which is analogous to knowing ω, φ) makes it possible to restore the second row in (5). Multiplying now the (restored) second row and the third row in (5) is equivalent to multiplying the right side of (4) by g_l. And summing the succesive seven ± 1's gives rise to the sequence 7, 7, -7, ... thus $\{d_k\}$ with the processing gain 7.

Generating the code $\{g_l\}$ with a shift of some terms (that is knowing τ_i but not the 'phase' τ) relative to the third row in (5) and repeating the described operation, none processing gain will result, and for sufficiently great N the wanted signal will be hidden in noise.

The reception with known ω, φ, τ_i, τ (which may be called 'communication parameters') is, in communication, called 'coherent'.

Practically, at the start, ω, φ, τ_i, τ are not known on the receiving side, moreover, they are variable by influence of the Doppler effect and by the varying distance between the transmitter and the receiver (e.g. for the communication with a satellite or aircraft). And, moreover, the application of coding results in carrier suppression.

By these causes, an intensive labor has been devoted to the formation of good estimates $\hat{\omega}$, $\hat{\varphi}$, $\hat{\tau}_i$, $\hat{\tau}$ on the receiving side (the parameter τ_i may in some cases be connected with ω, so that no extra estimate is needed). In what follows, some known schemas for generating these estimates will be shown.

As is clear, the code $\{d_k\}$ is not needed in this connection.

It is usefull to discern between 'acquisition' of parameters and maintaining them 'in coherence' with those of the received signal.

The schemas will be treated firstly in connection with maintaining, and only secondly (and briefly) with acquisition, because the acquisition is accomplished by modifying the function of the maintaining (locking) schemas.

3. BASIC SCHEMAS

One will begin with a general schema in Fig. 1 [24]. In it, the knowledge of φ and τ is supposed. The operations in (4) and after (5), executed on the right side of the broken line, are not interesting for us, substantial being only the function of the blocks with the name CAR and COR. These blocks are connected together with two 'opposite direction' connections (in Russian: 'perekrestnaya svyaz').

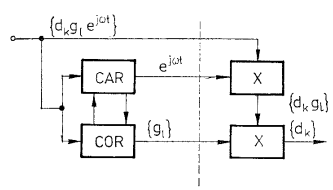

Fig. 1. CAR = carrier restoration, COR = code restoration, X = multiplier (mixer).

Thus, the problem is to find conjoitly and at the same time some signal parameters with the aid of connected blocks (in Russian the adjectives: 'sovmestnyj' [21], 'sovmechtchennyj' [20]).

The structure of both blocks CAR and COR is seen in Fig. 2 [4]. Both blocks are demarcated by broken lines. Instead of $\exp(j\omega t)$ (written with respect to (3)) one might write $\cos \omega t$, since, naturally, the signal is real. This will be arrived at in what follows. In the shift register, the well known feedback connections making of it a PN-code generator, are omitted (as is known, $N = 2^n - 1$ for PN codes). Further, in Fig. 2, the right side of both blocks is separated by horizontal dashed lines.

In the CAR block, this side represents the so called 'phase lock loop' [1], [2], [5] (the book [1], although very old, contains informations practically interesting to the present day). By the $\pi/2$ shift the function $-\sin(\hat{\omega}t + \hat{\varphi})$ is formed from

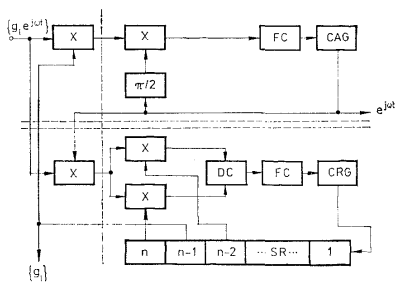

Fig. 2. FC = filter and control circuit, CAG = controlled carrier generator, $\pi/2$ = phase shift $\pi/2$, DC = difference circuit, CRG = controlled chip rate generator, SR = shift register code generator.

$\cos(\hat{\omega}t + \hat{\varphi})$, and the result of multiplication $-\cos\omega t \sin(\hat{\omega}t + \hat{\varphi})$ gives, after lowpass filtering, the discrimination characteristic (in essence $\sin\hat{\varphi}$). $\hat{\varphi}$ is the estimated phase shift between the local oscillator signal and the input signal. It serves to control the frequency of the oscilator.

The $\pi/2$ shift represents for the given trigonometric function the operation $\mathrm{d}/\mathrm{d}t$. Somewhere in the literature, this is so denoted.

In the lower block, the right hand part represents an analogy – for the code – of the phase lock loop and is named the delay lock loop. It is seen that the code is considered to be known, only τ_i can be varied (controlled) by a code 'chip rate' generator (in Russian: generator' taktovoj chastoty'). In the upper multiplier, a signal of the form $\wedge$ and width $2\tau_i$ in dependence on τ^* will be formed, τ^* being the shift violating the coherence of the input code and the locally generated code. In the lower multiplier a signal of the same form, but shifted in time by $2\tau_i$ is generated. The difference of both signals after the block DC represents the discriminator characteristic to control the chip rate generator.

It is clear that by the multipliers the cross correlations of the received and local codes are produced.

Denoting the code in dependence of the time shift τ^* as $f(t, \tau^*)$, it is possible to conceive the discriminator characteristic as an approximation of the function $\partial f(t, \tau^*)/\partial\tau^*$. Sowewhere in the literature [16], it is so written and has in essence the given meaning. In essence, since the approximation can be realized in more ways,

e.g. in [8] $\partial/\partial\tau^*$ is formed so that instead using the outputs of the nth and $(n-2)$th stages of the shift register, the outputs of the nth and $(n-1)$th stages are taken and $\{g_l\}$ is formed from the output of the nth stage with the aid of a delay $\tau_i/2$.

4. FURTHER SCHEMAS

How is it possible to construct schemas differing from that in Fig. 2, which has been derived with accord to the known Stratonovich's theory of nonlinear filtering? Apparently so that some connections in Fig. 2 will not be used or will be replaced by other, which, eventually, may be simpler. As usually, this is coupled with a deterioration of the signal/noise ratio.

Some schemas differing by internal structure are equivalent in function, although this may not be always proven exactly. Various differences are also of patent-law significance.

In what follows some schemas will be shown, other may be found in the literature by the reader himself.

4.1. Schemas retaining both mutual connections of the blocks CAR and COR

Simplifying the schema in Fig. 1 of [6], one gets the schema in Fig. 3. The CAR block is identical with the upper block in Fig. 2. The lower block is replaced in Fig. 3 by a substantially simpler one. The delay inserted in the connection feeding $\{g_l\}$

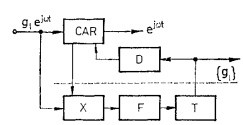

Fig. 3. D = delay line, F = filter, T = threshold ('signum' circuit).

back in the block CAR, serves to precisely synchronize this $\{g_l\}$ with the $\{g_l\}$ from the input signal. The value of the delay must be chosen experimentally, since now the block COR is without generators of the chip rate and the code, and without an internal feedback.

The schema in Fig. 3 works in principle equally well for known and unknown $\{g_l\}$. This advantage is payed for by only insubstantial deterioration of the signal/noise ratio, accordingly to [6].

Another schema quoted in many articles (and, apparently, often used) has been presented by Costas and bears his name. The schema is in Fig. 4. Its function will be described in some detail with a real signal (the function of the phase lock loop in Fig. 2 can be described similarly).

Let us imagine the feedback loop disconnected at the input of the block CAG. At the input of the schema, there is a signal $g_l \cos \omega t$, at the output of the CAG the signal $\cos [(\omega + \Omega) t + \varphi]$, where Ω is the circular frequency of detuning and φ is a constant phase angle. The description with the aid of Ω, φ seems to be heuristically more acceptable than that with $\cos \Phi(t)$, where $\Phi(t)$ is a general phase function.

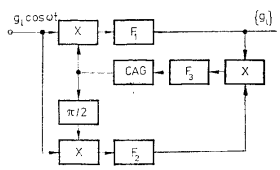

Fig. 4. F_1, F_2, F_3 = filters.

The signal from the CAG is conveyed in the multiplier at the top. The following bandpass filter F_1 with the center frequency $1/\tau_l$ serves to filter out the component ω, so that $g_l \cos (\Omega t + \varphi)$ remains.

Similarly the input from CAG to the multiplier at the bottom is $\sin [(\omega + \Omega) t + \varphi]$ and the output of F_2 is $g_l \sin (\Omega t + \varphi)$.

The output of the multiplier at the right side is $\sin 2(\Omega t + \varphi)$, since $g_l^2 = 1$. The time t is an independent variable, thus this signal is permanently 0 only in the case of $\Omega = 0$, $\varphi = 0, \pi, 2\pi, \ldots$. For low detuning frequency, the band of the lowpass filter F_3 may be small (for noise suppression). Closing the feedback loop gives the control signal for CAG, the loop stability corresponding to $\varphi = 0, 2\pi, \ldots$. For $\Omega = 0$, $\varphi = 0$, $\{g_l\}$ result on the right side at the top. The code must not be known in advance.

By binary PN coding of the transmission, the carrier is suppressed, being named sometimes 'virtual carrier'. The schema in Fig. 4 is used for renewing the carrier on the receiving side also in situations, where the code modulation of the signal is removed first.

4.2. A schema retaining only the connection from the block CAR in the block COR

Such a schema is in Fig. 5 [2], [8], [17]. The code is not conveyed in the block at the top in Fig. 5, thus this block must be different from that in Fig. 2 and 3

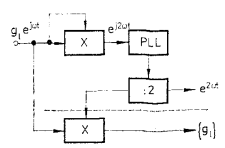

Fig. 5. PLL = phase locked loop, : 2 = frequency divider (halver).

and the whole schema is, as usual, named 'the schema with quadrator'. Now, the PLL is tuned to 2ω. Since $g_l^2 = 1$, the code modulation is removed and no connection is needed conveying $\{g_l\}$ in the block CAR. The bandpass filter around 2ω can possess small band for ω being approximately known. After the frequency halving a sign conversion can be eventually needed in communication application. In radar applications this is of no consequence. The code $\{g_l\}$ may be unknown in advance.

4.3. A schema retaining only the connection from the block COR in the block CAR

Such a schema is in Fig. 6 [8], [14]. It can be used for small signal/noise ratio, if the code is known. The schema will be compared with that in Fig. 2. No connection takes 'information' from CAR to COR, thus in Fig. 6 the first multiplier at the left

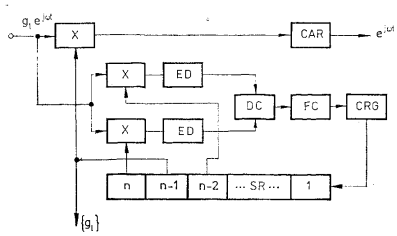

Fig. 6. ED = envelope detector.

side is omitted. The incoming signal to the code delay lock loop is $g_l \exp(j\omega t)$ and after both multipliers the signal is resp. $g_l(t)\, g_l(t + \tau_i) \exp(j\omega t)$, $g_l(t)\, g_l \,.\, (t - \tau_i) \exp(j\omega t)$, with notches. Two envelope (or quadratic) detectors inserted after both multipliers remove ω, it is thus not needed at this stage. The block CAR can be a phase lock loop similarly as in Fig. 2 or the circuit from Fig. 4 or an incoherent frequency lock loop [11], [19].

5. SIGNAL ACQUISITION

It is clear that some apriori information is needed concerning the signal to be acquired. The acquisition is accomplished by searching for the signal in some carrier frequency and code chip rate bands. The feedbacks in the phase lock loop and the code delay lock loop are opened and instead, the voltage controlled oscillators are driven by special sweeping circuits. If some signals are crossing given thresholds, coherence is indicated and switching to locking modes is executed automatically.

If the search is too slow, the frequency bands for the carrier and the code chip

rate are divided in subbands and by this multiplexing the search time can be shortened [13].

To make the search for $\hat{\tau}$ (knowing τ_i at least approximately) more rapid, a short code $\{g_i\}$ without $\{d_k\}$ modulation is used on the transmitter side. This code is named preambule. E.g. the codes known today as Barker's have been introduced by himself for this purpose. Often, not correlators, but matched filters are used to detect the preambule [8], [13].

The 'suboptimal' schemas of 4.2 and 4.3 possess an advantage of making a sequential search (firstly of ω, φ and then of τ_i, τ or in the reversed order) possible. With ω, φ already known, a rapid code search is described in [30], [31].

Now, one schema of code acquisition will be shown in Fig. 7 [8], [14], [32], corresponding to the schema of Fig. 6.

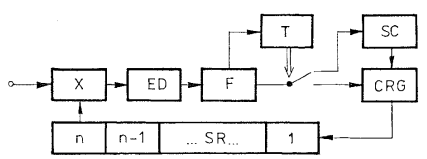

Fig. 7. SC = sweeping circuit, T = thresholds.

More than one threshold and a more complicated verification (of synchronization) logic can be used [13]. The acquisition mode (Fig. 7) is switched to the lock mode (Fig. 6). It is seen that many blocks are common in both operational modes.

The acquisition procedure is based on crosscorrelating the input signal with the locally generated one (replica). Since τ_i of the local code is varied, the name of 'sliding correlator' is used. Sometimes, τ_i (known) is not varied, and instead the code generated is cyclically shifted in steps, the result being analogous as before.

More detailed treating of acquisition operation may be found in [2], [8], [9], [10], [13], [14], [25].

6. SOME RECENT TRENDS

In communication, a recent trend is the incoherent reception using a frequency lock loop instead of the phase lock loop [1], [11], [19], [21], [25]. Pertaining schemas are, in principle, not differing from that of Fig. 6 (see e.g. Fig. 1 of [19]). The incoherent reception is useful for communication with aircrafts, where the influence of the Doppler shift is considerable.

Another recent trend in communication is the use of an autocorrelation receiver, which has been applied as yet (as it seems) more frequently to frequency and code analysis of radar impulses [15], [22], [20].

Sometimes, the acquisition of the transmission code 'chip rate' is useful [17]. Now, this acquisition with the aid of the autocorrelation receiver will be described heuristically.

Suppose that the transmission code is a 'white' sequence $\{\xi_n\}$ of independent random variables, each $\xi_i = \pm 1$ with probabilities $\frac{1}{2}, \frac{1}{2}$. Let τ_i be given. A random staircase function $x(t)$ will be formed from $\{\xi_n\}$ with the aid of the rule

$$x(t) = \xi_n \quad \text{for} \quad n\tau_i < t \leqq (n+1)\,\tau_i\,. \tag{6}$$

Another random function $X(t)$ will be formed from $x(t)$ according to

$$X(t) = x\left(t - \frac{\tau_i}{2}\right) x(t)\,. \tag{7}$$

Let us write the expected value

$$\mathsf{E}[X(t)] = \sum \mathsf{E}\left[x\left(t - \frac{\tau_i}{2}\right) x(t) \mid x\left(t - \frac{\tau_i}{2}\right)\right] \mathsf{P}\left(x\left(t - \frac{\tau_i}{2}\right)\right). \tag{8}$$

The sum on the right side possesses only two terms, namely

a) For t fulfilling

$$n\tau_i < t \leqq (n + \tfrac{1}{2})\,\tau_i \tag{9}$$

$x(t - \tau_i/2)$ and $x(t)$ are independent with expected values 0, thus $\mathsf{E}[X(t)] = 0$.

b) For t fulfilling

$$(n + \tfrac{1}{2})\,\tau_i < t \leqq (n+1)\,\tau_i \tag{10}$$

$x(t - \tau_i/2) = x(t)$, thus both first factors in the sum on the right side of (8) are 1 and thus $\mathsf{E}[X(t)] = 1$.

Thus the value of the autocorrelation function is periodic, having for $\tau_i/2$ the form of a meandre with known Fourier expansion

$$\mathsf{E}[X(t)] = R\left(t, \frac{\tau_i}{2}\right) = \frac{1}{2} + \frac{2}{\pi}\left(\sin \Omega_i t + \tfrac{1}{3} \sin 3\Omega_i t + \ldots\right) \tag{11}$$

where $\Omega_i = 2\pi/\tau_i$. The powers of harmonic components are

(12)

harmonic component	power
0	$1/4$
1	$4/\pi^2$
2	0
3	$4/9\pi^2$
⋮	⋮

The sum of powers is $\frac{3}{4}$. The strongest component is the first harmonic, frequency $1/\tau_i$, and it can be revealed by filtering the function in (7).

For a delay in (7) not too different from $\tau_i/2$, the results are similar, as the reader can convince himself. For a pseudonoise code, the situation remain in principle the same, although the precise formulae are substantially more complicated [7]. Thus, the 'chip rate' $1/\tau_i$ can be detected.

6. CONCLUDING REMARKS

Readers interested in problems of detection of signals with incomplete apriori information about its parameters, may find some aspect in [7], [15], [17], [18], [22].

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