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RATIONALITY OF THE INFORMATION EXCHANGE IN BIMATRIX GAMES

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A few brief comments on the sense of the exchange of strategic information in two-players games are proposed. It is shown that the rationality of such exchange can be investigated by means of the general coalition game model that also offers theoretical tools for the considerations on the optimal amount of the exchanged information.

0. INTRODUCTION

The exchange of at least partial information in a non-antagonistic conflict represets a primitive form of cooperation in games. It is obvious that in some games the information exchange is rational in order to prevent the players from choosing some bilaterally unadvantageous strategies. It concerns namely the non-antagonistic games and the choosing of some minimax strategies in situations where it has no advantage for any player except to guarantee certain level of profit against any theoretically possible threat.

In this paper which generalizes and completes some ideas from [5] we shall briefly mention three different ways of the manipulation with the strategic information. Namely, the search for information by one player, the offering of information also by one player, and the mutual exchange of information. We shall be interested in some conditions under which the manipulation with information is useful, in the rational price that could be paid for it, and in its rational extent. Some of the conclusions are intuitively expectable but it is useful to formulate them in an exact way. The mutual exchange of information that represents a virtual bilateral cooperation between players can be modelled by means of the general coalition games theory [3], as it is shown in this paper, too. These tools enable us to derive in a simple way some results concerning the rational forms of the information exchange. Moreover, it is possible, then, to use some general results concerning the existence of rational solutions of general coalition games [3], [4], [6] and their basic properties.

1. BIMATRIX GAME

The concept of the bimatrix game is well known from the literature (e.g. [1], [2], [7]), and we shall remember it here briefly, to introduce the notations used below.

Let us consider a two-player set $I = \{1, 2\}$. The players dispose with the sets of *pure strategies*

$$A_1 = \{a_1^{(1)}, \ldots, a_1^{(m)}\}, \quad A_2 = \{a_2^{(1)}, \ldots, a_2^{(n)}\},$$
$$m > 0, \quad n > 0,$$

and with the corresponding sets of *mixed strategies* S_i, $i = 1, 2$, where each mixed strategy $s_i \in S_i$ is a probability distribution over A_i, $i = 1, 2$. Each mixed strategy is consequently an m-dimensional, resp. n-dimensional, real valued vector. It means that any set $\bar{S}_i \subset S_i$, $i = 1, 2$ of mixed strategies is a subset of an Euclidian space, and it is possible to consider its topological properties, especially its closedness according to the usual topology.

The *pay-offs* $\boldsymbol{m}^{(i)}(a_1, a_2)$, $a_i \in A_i$, $i = 1, 2$, connected with the pure strategies form $m \times n$ matrices

$$\mathbf{M}_i = (\boldsymbol{m}_{jk}^{(i)})_{j=1,\ldots,m,k=1,\ldots,n}, \quad i = 1, 2,$$

where

$$\boldsymbol{m}_{jk}^{(i)} = \boldsymbol{m}^{(i)}(a_1^{(j)}, a_2^{(k)}), \quad j = 1, \ldots, m, \quad k = 1, \ldots, n, \quad i = 1, 2.$$

Then the pay-offs connected with mixed strategies are given by the formula

$$\boldsymbol{m}^{(i)}(s_1, s_2) = \sum_{j=1}^{m} \sum_{k=1}^{n} s_1(a_1^{(j)}) \,.\, s_2(a_2^{(k)}) \,.\, m_{jk}^{(i)}, \quad i = 1, 2, \tag{1.1}$$

where $s_1(a_1^{(j)})$ and $s_2(a_2^{(k)})$ are the probabilities of $a_1^{(j)}$ and $a_2^{(k)}$ under the probability distributions s_1 and s_2, respectively.

The quintuple $\Gamma = (I, S_1, S_2, \mathbf{M}_1, \mathbf{M}_2)$ shall be called a *bimatrix game*.

A bimatrix game Γ is *antagonistic* iff for all $s_1, s_1' \in S_1$, $s_2, s_2' \in S_2$

$$\boldsymbol{m}^{(1)}(s_1, s_2) \geqq \boldsymbol{m}^{(1)}(s_1', s_2') \Leftrightarrow \boldsymbol{m}^{(2)}(s_1, s_2) \leqq \boldsymbol{m}^{(2)}(s_2', s_1').$$

In more special case, every game Γ such that $\mathbf{M}_1 = -\mathbf{M}_2$ is antagonistic.

In the following parts of this paper we shall be interested in the mixed strategies only, as each pure strategy can be considered as a special degenerated mixed strategy.

We say that the mixed strategies $s_1^* \in S_1$ and $s_2^* \in S_2$ are the *guarantee strategies* (cf. [8]) iff

$$\min_{s_2 \in S_2} \boldsymbol{m}^{(1)}(s_1^*, s_2) \geqq \min_{s_2 \in S_2} \boldsymbol{m}^{(1)}(s_1, s_2) \quad \text{for all} \quad s_1 \in S_1, \tag{1.2}$$

and

$$\min_{s_1 \in S_1} \boldsymbol{m}^{(2)}(s_1, s_2^*) \geqq \min_{s_1 \in S_1} \boldsymbol{m}^{(2)}(s_1, s_2) \quad \text{for all} \quad s_2 \in S_2. \tag{1.3}$$

The numbers

$$v_1(\Gamma) = \max_{s_1 \in S_1} \min_{s_2 \in S_2} \boldsymbol{m}^{(1)}(s_1, s_2) = \min_{s_2 \in S_2} \boldsymbol{m}^{(1)}(s_1^*, s_2) \tag{1.4}$$

and

(1.5) $$v_2(\Gamma) = \max_{s_2 \in S_2} \min_{s_1 \in S_1} \boldsymbol{m}^{(2)}(s_1, s_2) = \min_{s_1 \in S_1} \boldsymbol{m}^{(2)}(s_1, s_2^*)$$

shall be called the *values of the game Γ* for player 1 and player 2, respectively.

It is not difficult to prove that the values exist not only for the game Γ with the sets of mixed strategies S_1 and S_2 but also whenever some their non-empty and closed subsets $\bar{S}_1 \subset S_1$ and $\bar{S}_2 \subset S_2$ are considered.

The guarantee strategies play an important role. Using them the player may be sure that his pay-off cannot be worse than the corresponding value of the game, and that no other strategy can guarantee a better pay-off for him.

It was proved in the classical literature, e.g. in [2], [7] and [9], that in every antagonistic bimatrix game there exists a pair of guarantee strategies $s_1^* \in S_1$, $s_2^* \in S_2$ such that

$$\boldsymbol{v}_1(\Gamma) = \boldsymbol{m}^{(1)}(s_1^*, s_2^*), \quad \boldsymbol{v}_2(\Gamma) = \boldsymbol{m}^{(2)}(s_1^*, s_2^*)$$

and that for $s_1 \in S_1$, $s_2 \in S_2$

$$\boldsymbol{m}^{(1)}(s_1, s_2) \geqq \boldsymbol{v}_1(\Gamma) \Leftrightarrow \boldsymbol{m}^{(2)}(s_1, s_2) \leqq \boldsymbol{v}_2(\Gamma).$$

2. INFORMATION IN BIMATRIX GAME

The guarantee strategies have a good sense especially in case of antagonistic games. However, in the non-antagonistic case their advantages are not so clear. In fact, they guarantee certain pay-off against the worst possible behaviour of the anti-player but there often hardly exists a reason to expect such behaviour. If one player gets reliable information that his partner eliminated some of his strategies, he can change also his own strategy choice and to guarantee more acceptable outcome.

In the following sections of this paper we shall be interested in three types of such manipulation with the strategical information. First, we shall brieffy mention the case when one player gets an opportunity to obtain some information about the strategy of his partner. Further we shall study the possibility of offering some information about player's own strategy to the partner in order to influence his behaviour. Finally, we shall investigate the most interesting case of the mutual exchange of information between both the players.

In all cases we shall represent an information about the strategies of player $i = 1, 2$ by a non-empty and closed subset $\bar{S}_i \subset S_i$ containing the strategies that might be used by player i. Why was exactly this model chosen?

It is the least exaggerating one concerning the form of the knowledge on the strategies. It is limited to the only information which strategies come into account and which do not. Within these limits the model suggested here offers a wide scale of possibilities, including the concentration of probabilities to some pure strategies or to only some probabilistic mixtures of strategies up to the concrete conditions of the modelled situation.

Generally, it would be possible to model the information about the chosen strategies by a probability distribution P_i over S_i, $i = 1, 2$. This attitude assumes much more knowledge on the probabilistic character about the stiategy choice, that is not always available. Moreover, the probability distribution P_i over S_i leads very easily to a probability distribution q_i over A_i, $i = 1, 2$, where q_i is a probabilistic mixture of P_i and the probabilities $s_i \in S_i$

$$(2.1) \qquad q_i(a_i^{(j)}) = \int_{S_i} s_i(a_i^{(j)}) \, \mathrm{d}P_i(s_i) \,,$$

$$j = 1, \ldots, m \quad \text{for} \quad i = 1 \,, \quad j = 1, \ldots, n \quad \text{for} \quad i = 2 \,,$$

if the well known measure theoretic assumptions are fulfilled. For us it means that q_i is a mixed strategy, $q_i \in S_i$, and that we have reduced the set S_i to one signle point. This point can represent our vague knowledge about the strategy choice but it may be essentially different from the strategy really chosen by player i. It could be a good topic for discussion which information is more preferable, the substitution of S_i by a subset $\bar{S}_i \subset S_i$ certainly containing the really chosen strategy, or the reduction of S_i to one strategy q_i that reflects our knowledge on the actually chosen strategy but may be different from it. In this paper the first one of both the possibilities is preferred.

3. INFORMATION OBTAINING

In this section we shall be interested in the simpliest one of the problems mentioned in the previous section. We shall try to define the value of the information about the strategy of one player for his partner. Let us suppose that player 1 is the player that obtains the information.

It means that there exists a set $\bar{S}$, $\emptyset \neq \bar{S}_2 \subset S_2$, containing all the mixed strategies really applicable by player 2. We suppose that $\bar{S}_2$ is closed. Then the game Γ is transformed into

$$\bar{\Gamma} = (I, S_1, \bar{S}_2, \mathbf{M}_1, \mathbf{M}_2)$$

with the values $\boldsymbol{v}_1(\bar{\Gamma})$ and $\boldsymbol{v}_2(\bar{\Gamma})$ for both the players. If player 2 keeps his guarantee attitude and chooses one of his guarantee strategies then $\boldsymbol{v}_2(\bar{\Gamma}) = \boldsymbol{v}_2(\Gamma)$. Even in this case may be $\boldsymbol{v}_1(\bar{\Gamma}) \neq \boldsymbol{v}_1(\Gamma)$.

Statement 1. For every information $\bar{S}_2$ about the strategy of players 2 the inequality $\boldsymbol{v}_1(\bar{\Gamma}) \geqq \boldsymbol{v}_1(\Gamma)$ holds.

Proof. The statement obviously follows from (1.4) applied to the games Γ and $\bar{\Gamma}$, and from the fact that $\bar{S}_2 \subset S_2$. □

The information $\bar{S}_2$ is *effective* for player 1 iff $\boldsymbol{v}_1(\bar{\Gamma}) - \boldsymbol{v}_1(\Gamma) > 0$, and it is rational to pay for it a price not greater than $\boldsymbol{v}_1(\bar{\Gamma}) - \boldsymbol{v}_1(\Gamma)$.

Statement 2. If the game Γ is antagonistic and if the player 2 plays his guarantee strategy then no information $\bar{S}_2 \subset \bar{S}_2$ can be effective for player 1.

Proof. If $v_1(\bar{\Gamma}) > v_1(\Gamma)$ then there exists $\bar{s}_1 \in S_1$ such that

$$\min_{s_2 \in \bar{S}_2} m^{(1)}(\bar{s}_1, s_2) > v_1(\Gamma) = \max_{s_1 \in S_1} \min_{s_2 \in \bar{S}_2} m^{(1)}(s_1, s_2) .$$

But it means that

$$\max_{s_2 \in \bar{S}_2} \min_{s_1 \in S_1} m^{(2)}(s_1, s_2) < v_2(\bar{\Gamma}) , \tag{3.1}$$

according to the definition of the antagonistic game. It is a contradiction to the assumption that $\bar{S}_2$ contains at least one guarantee strategy, and that there should be equality in (3.1). □

4. INFORMATION OFFERING

The second subject connected with the utilization of the information in bimatrix games is rather more complicated. One of the both players, we suppose that it is player 1, may offer an information about his strategy choice to his partner. In the following explanation we suppose that the information can be reliably checked and that it is always true, consequently.

Player 1 offers the information when he supposes that his partner uses a guarantee strategy and it is useful to turn this guarantee attitude towards another, for player 1 more acceptable, strategy.

Let us denote by $\bar{S}_1$ the information about the strategy choice of player 1, $\emptyset \neq \bar{S}_1 \subset S_1$, $\bar{S}$ is closed. Let us denote by $\bar{S}_2 \subset S_2$ the set of all guarantee strategies of player 2 in the game $\bar{\Gamma} = (I, \bar{S}_1, S_2, \mathbf{M}_1, \mathbf{M}_2)$. Then $\bar{S}_2$ is obviously also closed and we can consider another game

$$\Gamma^* = (I, \bar{S}_1, \bar{S}_2, \mathbf{M}_1, \mathbf{M}_2)$$

with the values $v_1(\Gamma^*)$ and $v_2(\Gamma^*)$ for both players. Let us remember that $v_1(\Gamma)$ and $v_2(\Gamma)$ are the values of the original game Γ. Then we can formulate the following statement.

Statement 3. If the information offering procedure is realized in the way described above then $v_2(\Gamma^*) \geqq v_2(\Gamma)$.

Proof. It follows from (1.5) and from the fact that $\bar{S}_2$ contains exactly all gurantee strategies of player 2 that $v_2(\Gamma^*) = v_2(\bar{\Gamma})$ is the value of both games, $\bar{\Gamma}$ and Γ^*, for player 2. If $\bar{S}_1 \subset S_1$ then (1.5) implies that the value $v_2(\bar{\Gamma})$ of $\bar{\Gamma}$ for player 2 cannot be less than the value $v_2(\Gamma)$ of Γ. □

Statement 4. There exists at least one information offering $\bar{S}_1 \subset S_1$ such that $v_1(\Gamma^*) \geqq v_1(\Gamma)$.

Proof. It is sufficient to choose $\bar{S}_1 = S_1$ which is a trivial case, or $\bar{S}_1 = \{s_1 \in S_1 : s_1$ *is a guarantee strategy*$\}$. □

The information offering $\bar{S}_1$ is *effective* for player 1 iff $v_1(\Gamma^*) > v_1(\Gamma)$ and the difference $v_1(\Gamma^*) - v_1(\Gamma)$ limits the reasonable price that player 1 can pay for the possibility to offer the information $\bar{S}_1$.

Statement 5. If the game Γ is antagonistic then there is no information offering that would be effective.

Proof. It follows from Statement 3 that there exists a strategy $\bar{s}_2 \in S_2$ such that $m^{(2)}(s_1, \bar{s}_2) \geqq v_2(\Gamma)$ for all $s_1 \in S_1$. It means, according to the definition of the antagonistic property, that

$$m^{(1)}(s_1, \bar{s}_2) \leqq v_1(\Gamma) \quad \text{for all} \quad s_1 \in \bar{S}_1$$

and then also $v_1(\Gamma^*) \leqq v_1(\Gamma)$. □

5. MUTUAL INFORMATION EXCHANGE

In the remaining sections we shall deal with the case of the mutual exchange of information between both players. Such information exchange, in fact a bilateral information offering, should satisfy both players. It means that it should be connected with an increase of pay-offs of both of them and respect their preferences. In this sense it represents a very elementary form of cooperation.

It is obvious that the *information exchange* can be described by a pair of closed sets $(\bar{S}_1, \bar{S}_2)$, where $\emptyset \neq \bar{S}_1 \subset S_1$, $\emptyset \neq \bar{S}_2 \subset S_2$, containing the strategies of both players that come into account to be applied in the game. The original game Γ with values $v_1(\Gamma)$, $v_2(\Gamma)$ is reduced to another game

$$\bar{\Gamma} = (I, \bar{S}_1, \bar{S}_2, \mathbf{M}_1, \mathbf{M}_2)$$

with the values $v_1(\bar{\Gamma})$ and $v_2(\bar{\Gamma})$ for player 1 and 2, respectively.

We say that the information exchange $(\bar{S}_1, \bar{S}_2)$ is *effective* iff there is no other information exchange (S_1^*, S_2^*) such that for the values $v_1(\Gamma^*)$, $v_2(\Gamma^*)$ of the game $\Gamma^* = (I, S_1^*, S_2^*, \mathbf{M}_1, \mathbf{M}_2)$ the following relations hold

$$v_i(\Gamma^*) \geqq v_i(\bar{\Gamma}) \quad \text{for both} \quad i = 1, 2,$$

$$v_i(\Gamma^*) > v_i(\bar{\Gamma}) \quad \text{for at least one} \quad i \in \{1, 2\}.$$

The effectivity concept, namely the differences $v_i(\bar{\Gamma}) - v_i(\Gamma)$, $i = 1, 2$, indicate the rational price that could be paid by players for the possibility to exchange the information.

The simple form of the cooperation hidden in the information exchange leads to the idea to apply some methods of the coalition games theory. We shall do so and use the general coalition games concept to investigate the situation described above.

6. GENERAL COALITION GAME

The notion of the general coalition game was suggested in [3] in order to find a common model for a wide scale of different more special cooperative situations.

A *general coalition game* is a pair (I, V) where I is a finite and non-empty set of *players* and V a mapping prescribing to every set of players $K \subset I$ a subset $V(K)$ of $\mathbb{R}^I$ (where $\mathbb{R}$ is the set of all real numbers) such that

$$(6.1) \qquad V(K) \quad \text{is closed}\,,$$

$$(6.2) \qquad \text{if} \quad x \in V(K)\,, \quad y \in \mathbb{R}^I\,, \quad x_i \geqq y_i \quad \text{for all} \quad i \in K\,, \quad \text{then} \quad y \in V(K)\,,$$

$$(6.3) \qquad V(K) \neq \emptyset\,,$$

$$(6.4) \qquad V(K) = \mathbb{R}^I \Leftrightarrow K - \emptyset\,.$$

The sets $K \subset I$ are called *coalitions*, any partition $\mathscr{K}$ of I into non-empty coalitions is a *coalition structure*, and every vector $x \in \mathbb{R}^I$ is called an *imputation*.

If $x, y \in \mathbb{R}^I$ then we say that x *dominates* y *via* K and write $x \operatorname{dom}_K y$ iff $x_i \geqq y_i$ for all $i \in K$ and $x_i > y_i$ for some $i \in K$. For every $K \subset I$ we define

$$(6.5) \qquad V^*(K) = \{y \in \mathbb{R}^I\colon \textit{there is no } x \in V(K) \textit{ such that } x \operatorname{dom}_K y\}\,.$$

An imputation $x \in \mathbb{R}^I$ is said to be ***strongly stable*** iff there exists a coalition structure $\mathscr{K}$ such that $x \in V(K)$ for all $K \in \mathscr{K}$, and, moreover, $x \in V^*(L)$ for all $L \subset I$.

The general coalition game (I, V) is *superadditive* iff for any pair of disjoint coalitions $K, L \subset I$, $K \cap L = \emptyset$, the inclusion

$$(6.6) \qquad V(K) \cap V(L) \subset V(K \cup L)$$

holds, and it is called *additive* iff for any pair of disjoint coalitions $K, L \subset I, K \cap L = \emptyset$, inclusions (6.6) and

$$(6.7) \qquad V^*(K) \cap V^*(L) \subset V^*(K \cup L)$$

hold.

7. MODEL OF THE INFORMATION EXCHANGE

In this section we shall use the apparatus of the general coalition games model to derive some properties of the rational information exchange in bimatrix games. Let us consider a bimatrix game

$$\Gamma = (I, S_1, S_2, \mathbf{M}_1, \mathbf{M}_2)\,, \quad I = \{1, 2\}\,,$$

and let us denote

1) $V(\{i\}) = \{x \in \mathbb{R}^2\colon x_i \leqq v_i(\Gamma)\}$, $i = 1, 2$,

(7.2) $\quad V(I) = \{x \in \mathbb{R}^2$: *there exist closed sets* $\bar{S}_1, \bar{S}_2$ *such that* $\emptyset \neq \bar{S}_1 \subset S_1$, $\emptyset \neq \bar{S}_2 \subset S_2$ *and bimatrix game* $\bar{\Gamma} = (I, \bar{S}_1, \bar{S}_2, \mathbf{M}_1, \mathbf{M}_2)$ *such that* $x_1 \leqq v_1(\bar{\Gamma})$ *and* $x_2 \leqq v_2(\bar{\Gamma})\}$,

(7.3) $\quad V(\emptyset) = \mathbb{R}^2$.

Statement 5. The pair (I, V) where $I = \{1, 2\}$ and the sets $V(K)$ for $K \subset I$ are defined by (7.1), (7.2) and (7.3) forms a general coalition game.

Proof. It can be easily verified that the sets $V(K)$ are closed and fulfil condition (6.2). The existence of the values $v_i(\bar{\Gamma})$ and $v_i(\Gamma)$, $i = 1, 2$, implies the validity of (6.3) and the finiteness of the values $v_i(\Gamma)$ together with (7.3) implies the validity of (6.4). □

Statement 6. The general coalition game (I, V) is superadditive.

Proof. It is possible to put also $\bar{S}_1 = S_1$ and $\bar{S}_2 = S_2$ in (7.2). Consequently, for all $y \in \mathbb{R}^2$ such that $y_1 \leqq v_1(\Gamma)$, $y_2 \leqq v_2(\Gamma)$, also $y \in V(I)$. □

Statement 7. If the original bimatrix game $\Gamma = (I, S_1, S_2, \mathbf{M}_1, \mathbf{M}_2)$ is antagonistic then the general coalition game (I, V) is additive.

Proof. As inclusion (6.6) follows from Statement 6, it is sufficient to prove the validity of (6.7). In case of 2-person game it means to prove the inclusion

$$V^*(\{1\}) \cap V^*(\{2\}) \subset V^*(I) .$$

Let us consider $x = (x_1, x_2) \in V^*(\{1\}) \cap V^*(\{2\})$. It means that $x_1 \geqq v_1(\Gamma)$, $x_2 \geqq v_2(\Gamma)$. If $x \notin V^*(I)$ then there exists $y \in V(I)$ such that $y \operatorname{dom}_I x$, i.e. there exist $\bar{S}_1 \subset \bar{S}_1$, $\bar{S}_2 \subset S_2$, closed, non-empty, and $\bar{\Gamma} = (I, \bar{S}_1, \bar{S}_2, \mathbf{M}_1, \mathbf{M}_2)$ such that

$$(v_1(\bar{\Gamma}), v_2(\bar{\Gamma})) \operatorname{dom}_I(v_1(\Gamma), v(\Gamma)) .$$

Then, according to the theorems on antagonistic bimatrix games known from the literature (cf. [2], [7], [8]), there exist mixed strategies $s_1', s_1 \in \bar{S}_1 \subset S_1$ and $s_2', s_2 \in \bar{S}_2 \subset S_2$ such that

$$v_1(\bar{\Gamma}) = m^{(1)}(s_1, s_2) \geqq v_1(\Gamma) = m^{(1)}(s_1', s_2') ,$$

$$v_2(\bar{\Gamma}) = m^{(2)}(s_1, s_2) \geqq v_2(\Gamma) = m^{(2)}(s_1', s_2') ,$$

and the inequality is strict in at least one of both cases. It contradicts to the assumption of the antagonicity of the game Γ. □

Statement 8. There always exist strongly stable imputations in the general coalition game (I, V).

Proof. As the elements of the matrices $\mathbf{M}_1$, $\mathbf{M}_2$ are limited, the set

$$W(I) = V(I) \cap \{x \in \mathbb{R}^2 : x_1 \geqq v_1(\Gamma),\ x_2 \geqq v_2(\Gamma)\}$$

is bounded and closed. It means that for every $x \in W(I) - V^*(I)$ there exists $y \in W(I) \cap V^*(I)$ such that $y \operatorname{dom}_I x$. Such y belongs to the sets $V(I) \cap V^*(I)$ and $V^*(\{1\}) \cap V^*(\{2\})$ and, consequently, it is strongly stable. □

Statement 9. An information exchange (S_1, S_2), where S_1, S_2 are closed sets, $\emptyset \neq \bar{S}_1 \subset S_1$, $\emptyset \neq \bar{S}_2 \subset S_2$, is effective if and only if the values $v_1(\bar{\Gamma})$, $v_2(\bar{\Gamma})$ of the bimatrix game $\bar{\Gamma} = (I, \bar{S}_1, \bar{S}_2, \mathbf{M}_1, \mathbf{M}_2)$ form a strongly stable imputation in the general coalition game $(I, \boldsymbol{V})$.

Proof. If the information exchange $(\bar{S}_1, \bar{S}_2)$ is effective then the pair $(v_1(\bar{\Gamma}), v_2(\bar{\Gamma}))$ is strongly stable in $(I, \boldsymbol{V})$ as follows from (7.2) and from the definitions of the strong stability and of the effectivity. On the other hand, according to (7.2) and to the definition of the strong stability, to every strongly stable imputation $x = (x_1, x_2)$ in $(I, \boldsymbol{V})$ there necessarily exists an information exchange (S_1^*, S_2^*) and a bimatrix game

$$\Gamma^* = (I, S_1^*, S_2^*, \mathbf{M}_1, \mathbf{M}_2)$$

such that $x_i = v_i(\Gamma^*)$, $i = 1, 2$. Moreover, the strong stability and (7.2) imply that there is no other information exchange $S_1^\dagger \subset S_1$, $S_2^\dagger \subset S_2$ and no corresponding bimatrix game $\Gamma^\dagger$ such that

$$(v_1(\Gamma^\dagger), v_2(\Gamma^\dagger)) \operatorname{dom}_I (v_1(\Gamma^*), v_2(\Gamma^*)),$$

and, consequently, (S_1^*, S_2^*) is effective. □

Statement 10. There always exists at least one effective information exchange in any bimatrix game.

Proof. The statement follows from Statement 8 and Statement 9 immediately. □

Statement 11. If the considered bimatrix game is antagonistic then the trivial information exchange (S_1, S_2) is effective.

Proof. The statement follows from Statement 10 and Statement 7. It is connected with the properties of additive games presented in [4]. □

8. CONCLUSIVE REMARKS

Three forms of the manipulation with the strategic information in bimatrix games were presented above. The basic idea about the character of the information as a subset of the set of mixed strategies is sufficiently free to include more of special cases. For example, the theory may be applied also to the information concerning the pure strategy only.

If the theory is applied to the mixed strategies then it is useful to check in any specific case the conditions under which the information on mixed strategies, i.e. on probability distributions, has its practical sense. It means under which conditions it is reasonable to consider the probabilities and to use probabilistic methods.

In some situations the concept of the effectivity might seem to be rather weak. Then it is possible to substitute it by its stronger modification, where an information exchange $(\bar{S}_1, \bar{S}_2)$ is *strongly effective* iff

$$v_1(\bar{\Gamma}) > v_1(\Gamma) \quad \text{and} \quad v_2(\bar{\Gamma}) > v_2(\Gamma)$$

(preserving the notations used in Sections 5 and 7). Such strong effectivity concept is equivalent to so called strong domination between imputation in the general coalition game (I, V). It is possible to use analogous methods of investigation, and also the results would be ıather analogous. The essential difference consists in the fact that the existence of the strongly effective information exchange is not generally guaranteed, and e.g. in antagonistic bimatrix games it is excluded.

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