KYBERNETIKA — VOLUME 19 (1983), NUMBER 5

ON THE GROUP PULSE PROCESSES
I. Classification

KAREL VOKURKA

The nth order group pulse process is defined to be the group pulse process with n levels of pulse clustering. Three basic types of pulse processes, i.e., the processes with independent points, independent intervals, and constant intervals, are considered and structural formulas for these processes are introduced. Finally, mutual relations between the inner and the outer characteristics of the pulse process and among the point, pulse, and continuous processes are shortly discussed.

1. INTRODUCTION

The group pulse processes represent a generalization of the pulse processes without clustering. Some simple group pulse processes were used in the late 50's and during 60's by Bunkin [1] to model Barkhausen noise, by Furutsu and Ishida [2] to model atmospheric radio noise and by Akulichev and Olshevskii [3], [4] to approximate cavitation noise. In communication theory they were first introduced by Lee [5], Levin and Fomin [6] and Levin [7]. The latter two authors also coined the term "the group pulse process". Review of the work on the group pulse processes that was carried out in connection with pulse signals in the late 60's and early 70's may be found in the book by Konovalov and Tarasenko [8]. Recently, the group pulse processes of high complexity have been studied by Konovalov [9], [10] and Goldberg [11].

In approximating cavitation noise the author used both the processes DA^P and BA^P [12]. The derivation of the power spectra of these processes was published in [13] and [14], respectively. As for the process DA^P, the results obtained differ from those derived by Akulichev and Olshevskii [3], [4]. Recently, the process DA^PA^P has been used to model Barkhausen noise [15]. The results of the works [13]–[15] are further generalized in papers [16], [17], where the power spectra of the nth order group pulse processes A, B and D are derived.

During the work on the papers [13] – [17] the non-existence of a suitable classification, terminology and structural formulas was felt as a serious drawback. This paper is an attempt to bridge this gap.

The paper is organized as follows. The concept of the nth order group pulse process is introduced in Section 2. The basic pulse processes A, B and D are defined in Section 3, where the structural formulas of the group pulse processes are also developed. A relation between the outer and the inner statistical description of the pulse process is discussed in Section 4. At last, an attempt to clarify the position of the pulse processes among the point and continuous processes is undertaken in Section 5.

2. THE nth ORDER GROUP PULSE PROCESS

A disturbance of a limited duration will be called a *pulse*. According to the nature of the disturbance the pulses may be electrical, acoustical, optical, etc. The form of the pulses may be mathematically described by a suitable real time function $f(t)$. This function will be also called the pulse. Not every real time function $f(t)$ can be the pulse. For example, to ensure existence of first and second moments, $f(t)$ must be absolutely and square integrable.

Time sequences of pulses occurring in physical and biological systems are called either *pulse signals* or *pulse noises*. Mathematical models of these signals and noises will be referred to as the *pulse processes*.

Both the form and occurrence of pulses can be either random or deterministic. Accordingly, the pulse processes are also either random or deterministic. We shall be interested in the random pulse processes first of all because the deterministic pulse processes may be regarded as their special case.

In order to describe a pulse sequence mathematically a suitable point determining the pulse position with respect to the time axis origin must be defined. This point will be called a *pulse reference point* and an instant the pulse has a maximum may be conveniently used for this purpose.

The random *form of pulses* may be described in several ways. For example, the time function $f(t, \boldsymbol{a})$ may be given, where $\boldsymbol{a}$ is an m-dimensional random vector of m random pulse parameters such as the amplitude, the time constant, etc. Another possible way is to define a set of N functions $f_n(t)$, where $n = 1, \ldots, N$, and a probability, $P(n)$, of an event that $f_n(t)$ occurs in a realization of the pulse process.

Let us now consider pulses randomly distributed along the time axis and let us assume that the pulses occur in *groups* and these groups form group clusters and the group clusters form larger units, etc. (Fig. 1). In this work only stationary or cyclostationary [18] processes will be considered. This means that the clustering will be either "homogeneous" or "periodical" along the time axis.

It will be useful to distinguish the different *levels* on which clustering occurs.

Thus, for example, the clustering of pulses into groups represents one level, the clustering of pulse groups into group clusters represents another level, etc. In other words the groups on a certain level are formed of lesser units (subgroups) on a lower level and they alone may form larger units (group clusters) on a higher level.

In order to describe the position of the pulse groups and of all larger units, the

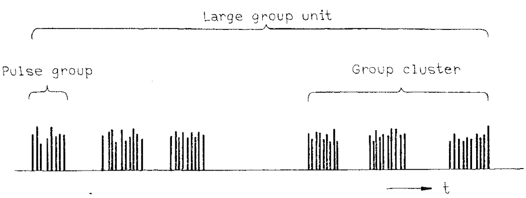

Fig. 1. Complex group pulse process.

reference points of the groups, group clusters, etc., will also be introduced. The group reference point may be conveniently identified with a reference point of some prominent subgroup in the group, e.g., with the first subgroup in the group. However, it may lay aside any subgroup reference point and may be identified, for example, with a "center of gravity" of the group.

If a certain level of the group clustering is examined in more detail, then it may be seen that the reference points of the groups in a cluster also form a group on the next higher level. This is true for any level of clustering but the highest one on which the reference points never cluster systematically. Let us therefore denote this highest level as the level zero (no clustering occurs), the first lower level as the 1st level, the next one as the 2nd level, etc., and at last the level where the pulses form the groups as the nth level (Fig. 2).

The group pulse process having n clustering levels will be referred to as the nth *order group pulse process*. Thus, for example, the second order group pulse process is formed by the clusters of the pulse groups, the first order group pulse process is formed by the pulse groups and with the pulse process of the order zero no systematic clustering occurs.

Let us now denote the pulses, the pulse groups, the group clusters, etc., by suitable subscripts. Starting with the level zero the reference points (further only points) will be denoted by the subscript i, where $i = 0, \pm 1, \pm 2, \ldots, \pm \infty$. Thus any group on the first level is also determined by the subscript i. The points inside the groups on the first level will be denoted by the subscript k_1, where $k_1 = 1, \ldots, K_i$ and K_i is a number of points in the ith group. The points inside the groups on the second level will be denoted by the subscript k_2, where $k_2 = 1, \ldots, K_{i,k_1}$ and K_{i,k_1} is a number of points in the k_1th subgroup on the second level. Similarly it will be proceeded on the next lower levels. Hence, any point on the level zero is denoted by one subscript (i), any point on the first level by two subscripts (i, k_1), on the second level by three subscripts $(i, k_1, k_2), \ldots$, and on the nth level by $n + 1$ subscripts $(i, k_1, \ldots, k_n)$ – see Fig. 3.

Let us denote the *position* of the ith point on the level zero with respect to the time axis origin by τ_i, the position of the k_1th point on the first level in the ith group with respect to the (reference) point of the ith group by φ_{i,k_1}, etc., and the position of the

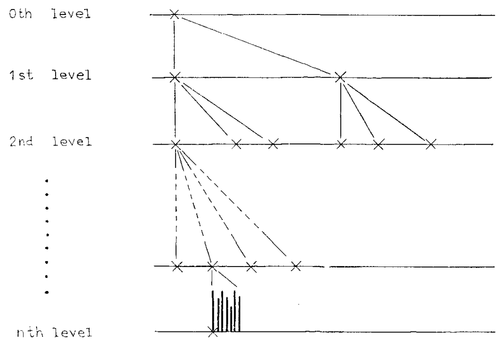

Fig.2. Levels of pulse clustering.

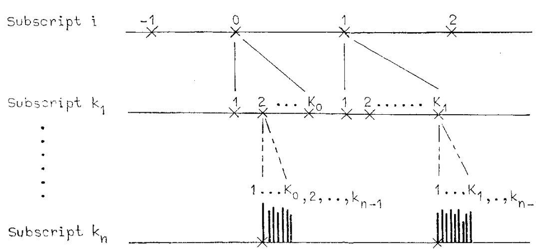

Fig. 3. Numbering of points and pulses. No assumption about the point ordering has been done so far and therefore the points can be both ordered and disordered (see Remark 1). However, for figure to be easy to survey the points are drawn in ordered sequences.

k_nth point on the nth level in the k_{n-1}th group of the k_{n-2}th cluster, etc., with respect to the point of the k_{n-1}th group by $\varphi_{1,k_1,\ldots,k_n}$. Thus the position of the k_nth pulse in the k_{n-1}th group, ..., in the ith cluster with respect to the time axis origin is given by (Fig. 4)

(1) $$t_{i,k_1,\ldots,k_n} = \tau_i + \varphi_{i,k_1} + \ldots + \varphi_{i,k_1,\ldots,k_n}$$

and the nth order *group pulse process* $\xi(t)$ may be written in the form

$$\xi(t) = \sum_{i=-\infty}^{\infty} \sum_{k_1=1}^{K_i} \dots \sum_{k_n=1}^{K_{i,k_1,\dots,k_{n-1}}} f(t - t_{i,k_1,\dots,k_n}, \mathbf{a}_{i,k_1,\dots,k_n}) . \tag{2}$$

Here $\mathbf{a}_{i,k_1,\dots,k_n}$ is an m-dimensional random vector of m random parameters of the k_nth pulse in the k_{n-1}th group ... in the ith cluster.

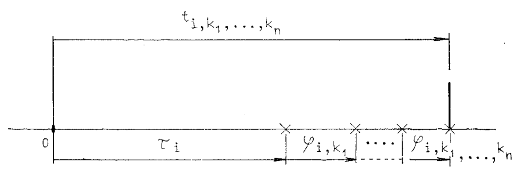

Fig. 4. Determination of pulse position with respect to the time axis origin. For figure to be easy to survey the variables φ are chosen to be positive only. However, this need not always be the case because for some reference point positions the variables φ can be both positive and negative.

3. CLASSIFICATION OF THE PULSE PROCESSES

If the process $\xi(t)$ is to be determined by (2), then it is necessary to know the mutual distribution function of all random variables occurring in this expression. Such a determination of the process $\xi(t)$ will be referred to as the determination by the inner characteristics.

In this paper only the processes for which the random variables $K_i, \dots, K_{i,k_1,\dots,k_{n-1}}$, $t_{i,k_1,\dots,k_n}$ and the random vectors $\mathbf{a}_{i,k_1,\dots,k_n}$ are mutually independent will be considered. This independence is meant to occur not only among the different variables but also among the same variables having different values of subscripts. The only exception from this assumption are the random variables $t_{i,k_1,\dots,k_n}$. They will also be assumed to be independent of the other random variables, yet between two random variables $t_{i,k_1,\dots,k_n}$ having different subscripts a certain statistical dependence, the nature of which will be explained below, may occur.

According to the expression (1) the random variables $t_{i,k_1,\dots,k_n}$ equal the sum of random variables $\tau_i, \varphi_{i,k_1,\dots}, \varphi_{i,k_1,\dots,k_n}$. In this paper only the case will be considered when the random variables τ_i, $\varphi_{i,k_1,\dots}$, $\varphi_{i,k_1,\dots,k_n}$ are mutually independent, i.e., there is no statistical dependence between the different levels. The random variables $K_{i,\dots}, K_{i,k_1,\dots,k_{n-1}}$ and the random vector $\mathbf{a}_{i,k_1,\dots,k_n}$ will also be assumed identically distributed, i.e., their distribution does not depend on the value of the subscripts.

Owing to many subscripts the *notation* used so far is rather cumbersome. Therefore, in cases where there is no danger of misunderstanding the notation will be substantially simplified. From Fig. 3 it follows that there are two main directions in which subscripts change. First, there is a vertical direction, i.e., between different levels.

Second, there is a horizontal direction, i.e., inside the groups. To denote the vertical position a subscript p $(1 \leqq p \leqq n)$ will be used. Then in the simplified notation all the subscripts but the last one, i.e., the one belonging to the level p, will be omitted. Thus, for example, we shall write K_p instead of $K_{i,k_1,\ldots,k_p}$. To denote the horizontal position (in a group) a subscript q $(1 \leqq q \leqq K_p)$ will be used. For example, we shall write t_q and φ_q instead of $t_{i,k_1,\ldots,k_p}$ and $\varphi_{i,k_1,\ldots,k_p}$, respectively. However, in this case information regarding the level that is considered must be added.

Now we may return to the original problem of the pulse processes *classification*. With respect to the statistical dependence of variables t_q three basic types of processes, definitions of which follow, will be distinguished.

Definition 1. (a) Let the random variables τ_i on the level zero be mutually independent and uniformly distributed in an principal interval $(0, T)$ and let the number of points in this interval, I, be Poisson distributed. (b) Let the random variables φ_q on the pth level be mutually independent and identically distributed.

Then the statistical independence of the type (a) and (b) will be denoted by A and we shall say that on the given level the process is of the type A, i.e., with independent point occurrence times or in short with independent points.

Remark 1. With processes A it is not necessary to assume that the points are ordered, i.e., it need not hold on the pth level that

$$\varphi_q \leqq \varphi_{q+1} . \tag{3}$$

However, with the processes that will be defined below the points must always be ordered, that is, the relation (3) must hold. If the points are ordered, then the interval, ϑ_i, between two successive points on the level zero equals $\vartheta_i = \tau_{i+1} - \tau_i$ and the interval, ε_q, between two successive points in a group on the pth level equals $\varepsilon_q = \varphi_{q+1} - \varphi_q$ (Fig. 5).

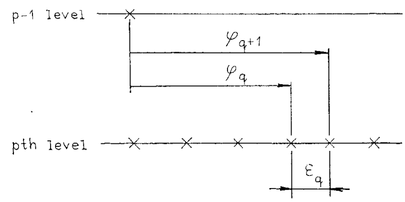

Fig. 5. Interval between two neighbouring points on the pth level.

Definition 2. (a) Let the random variables ϑ_i on the level zero be mutually independent and identically distributed. (b) Let the random variables ε_q on the pth level be mutually independent and identically distributed.

Then the statistical independence of the type (a) and (b) will be denoted by B and we shall say that on the given level the process is of the type B, i.e., with independent intervals among the points or in short with independent intervals.

Definition 3. (a) Let the random variables ϑ_i on the level zero be deterministically distributed with probability density $w_1(\vartheta) = \delta(\vartheta - \vartheta_0)$, where ϑ_0 is a given constant. (b) Let the random variables ε_q on the pth level be deterministically distributed with a probability density $w_1(\varepsilon) = \delta(\varepsilon - \varepsilon_p)$, where ε_p is a given constant.

Then such a dependence will be denoted by D and we shall say that on the given level the process is of the type D, i.e., with constant intervals among points or in short with constant intervals.

Remark 2. If it is clear from the context what is meant, the expressions "the zero order process of the type A" and "the process of the type A on the pth level" will usually be shortened to "the process A".

Though it has not been said explicitly up to now it is evident that a group pulse process can be of different types, i.e., A, B and D, and with different distributions of random variables I, K_p, τ_i, φ_p, ϑ_i and ε_p on different levels. To be able to describe the structure of a certain group pulse process in a short way the *structural formulas* will be introduced now. These may be written in a form

$$\text{LL} \dots \text{L}$$

where each L represents one level (taken from the left to the right, the first L stands for the level zero, the second one for the first level, etc., the last one representing the nth level). According to the type of the process on the given level these L can be substituted by the letters A, B and D. The use of the structural formula can be best illustrated by a simple *example*. Let us consider a second order group pulse process that is of the type B on the level zero, of the type A on the first level and of the type D on the second level. The structural formula of this process has the form

$$\text{BAD}\,.$$

It may sometimes be useful to give further data regarding the pulse process in the structural formula. This information may concern, for example, the distribution of the random variables K_p. The type of this distribution will be denoted by a superscript at the corresponding L. Here only two important distributions of the number of points in groups will be mentioned. First of all it will be the *Poisson* distribution that will be denoted by the capital P. The *deterministic* distribution represents the second important case. Now the number of points in groups is constant and equals K_p. This distribution will be denoted by the capital D. With respect to applications the case when $K_p = 1$ is quite important and occurs rather frequently. When this happens the digit 1 will be written instead of the capital D. As only single points can occur on the level zero, this digit should always be written as a superscript of the first L. However, to simplify the notation it will usually be omitted. If necessary, further distributions may be denoted by suitable capital letters.

Another important information which may be written in the structural formula concerns the distribution of the random variables φ_p (in the case of the process A) or ε_p and ϑ_i (in the case of the process B and D). This distribution will be denoted by a subscript at the corresponding L. The *normal* distribution will be denoted by the capital G, the *exponential* distribution by the capital E, the *uniform* distribution by the capital T, the *deterministic* distribution by the capital D and at last the *gamma* distribution by the capital Γ. According to the distribution of the random variables τ_i or ϑ_i on the level zero four basic types of the pulse processes were defined in [14]. These are the *periodic*, the *quasiperiodic*, the *aperiodic* and the *homogeneous* processes. The aperiodic processes may further be subdivided into the processes with an *attractive* and *repulsive* correlation between points. The quasiperiodic process will be denoted by the capital Q, the aperiodic process by the capital A. In the case of the attractive correlation this capital A will be suplemented by the symbol + while the symbol − is reserved for the repulsive correlation. Hence the structural formula of the quasiperiodic process begins with B_Q and that of the periodic process with D or B_D. In the case of the aperiodic process with the attractive or repulsive correlation the first L is replaced by B_{A+} or B_{A-}, respectively, and finally, in the case of the Poisson homogeneous process by A or B_E. Similar notation and terminology can certainly be applied to the higher levels, too.

In the end of this section examples of the structural formulas of three special processes will be given. The structural formula of the *pure periodical process* has the form

$$DD^D \ldots D^D \quad \text{or} \quad B_D B_D^D \ldots B_D^D .$$

It is the periodicity of points that is considered here. However, if all components of the random vector $\boldsymbol{a}$ have a deterministic distribution (the pulses with non-random form), the pure periodical process will pass into the deterministic process which is the periodic function in the usual mathematical sense.

The structural formula of the *pure homogeneous process* has the form

$$AA^P \ldots A^P .$$

The pure homogeneous process of the order zero is the well known homogeneous Poisson process [13].

The structural formula of the *Poisson periodic process* has the form ($n \geqq 1$)

$$DD^D \ldots D^D A^P \quad \text{or} \quad B_D B_D^D \ldots B_D^D A^P ,$$

where the process A^P must always be present on the nth level.

4. THE INNER AND THE OUTER CHARACTERISTICS OF THE PULSE PROCESS

It was mentioned in the preceding sections that if the function $f(t, \boldsymbol{a})$, the probability densities $w_m(\boldsymbol{a})$, $w_1(K_1), \ldots, w_1(K_n)$ and the probability densities $w_1(\tau)$, $w_1(\varphi_1), \ldots, w_1(\varphi_n)$ (in the case of the process A) or the probability densities $w_1(\vartheta)$, $w_1(\varepsilon_1), \ldots, w_1(\varepsilon_n)$ (in the case of the process B and D) are known, then from the statistical point of view the process $\xi(t)$ is in certain sense sufficiently described*. Since this description characterizes the inner structure of the pulse process, i.e., the statistical properties of individual pulses and their clusters, it will be referred to as the *inner description* and the corresponding distributions as the *inner characteristics*.

However, the random pulse processes may also be characterized by the sequence of the probability densities $w(x_1; t_1)$, $w(x_1, x_2; t_1, t_2), \ldots, w(x_1, \ldots, x_n; t_1, \ldots, t_n)$. Here, for example, the first probability density $w(x_1; t_1)$ gives a probability of an event that at time t_1 the process $\xi(t)$ is found in an interval $(x_1, x_1 + dx_1)$. Similarly, the second probability density $w(x_1, x_2; t_1, t_2)$ gives a probability of an event that at time t_1 the process $\xi(t)$ is found in an interval $(x_1, x_1 + dx_1)$ and at time t_2 it is found in an interval $(x_2, x_2 + dx_2)$. The higher probability density is known, the more one knows about the process $\xi(t)$. In fact, often only basic moments, such as the mean, the variance or the autocorrelation function are known. All these characteristics describe the pulse process $\xi(t)$ as viewed from outside and say very little about the individual pulses and their clusters directly. Therefore they will be referred to as the *outer description* and the corresponding probability densities or their moments as the *outer characteristics*.

One of the main tasks in the theory of the pulse processes is to find a mutual relation between the inner and the outer characteristics. It is the purpose of this section to discuss this relation in more detail.

Quite generally, the inner characteristics contain more information about the pulse process than the outer ones. It is possible, for example, to determine the outer characteristics of the pulse process $\xi(t)$ from the inner ones. However, during this determination the statistical averaging accompanied by the irreversible lost of information is used. Hence, the determination of the inner characteristics from the outer ones may be done only in some simple cases and it is usually necessary to obtain some further information about the pulse process. An example of this problem may be found in Bell's paper [19].

*) In some applications it may also be useful to know some other statistics. These may be, for example, the probability distribution (or its moments) of the number, N, of pulses occurring in an interval $(0, T)$ (counting statistics), the probability density function for the forward recurrence time (time from an arbitrary moment to the first pulse) and the multifold statistics, including the joint probability distribution of the numbers of pulses $N_1, N_2, \ldots$ in multiple adjacent time intervals, and their moments.

The inner characteristics have direct relation to the underlying physical mechanism that generates the pulse noise. This is the reason why in the noise research efforts are made to determine them. If the pulses do not overlap, it is possible to measure the inner characteristics directly. However, this is usually not the case because the noises with non-overlapping pulses are very rare. One exception is Barkhausen noise at extremely slow magnetization [15]. Impulsive noise ("statics", "interferences", etc.) in radiocommunications can serve as another example of the noise with relatively low density of pulses [2]. Nevertheless, the most pulse noises are formed by pulses with very high density of overlapping (e.g., shot noise and cavitation noise) so that direct determination (measurement) of the inner characteristics is impossible. The outer characteristics are the only ones that can be measured directly and, when supplemented with further information, they serve as the starting point in search for the inner characteristics.

There is also another limitation. Both in experiments and in theoretical reasonings it is usually possible to work only with simpler outer characteristics, such as the autocorrelation function, the power spectrum or the first probability density. This is due to the tremendous mathematical and experimental difficulties that accompany the determination of the more complex characteristics. It was shown in numerous papers that the derivation of the power spectrum from the inner characteristics does not represent any serious problem for many pulse processes. In [16] and [17] we want to show this also for the nth order group pulse processes A, B and D. The autocorrelation function can be either computed directly from the inner characteristics or can be obtained from the power spectrum using the Fourier transform. However, already the derivation of the first probability density function can represent a difficult task and in literature it was found only for some simple pulse processes such as the homogeneous Poisson process and only under certain limitations [20] – [22]. Similar picture may be seen in laboratories where only spectrum analyzers, correlators, and amplitude probability analyzers (the first probability density) are common. In addition, during a measurement of the statistical characteristics (e.g., of spectrograms) only more or less exact estimates can be obtained [23].

The last limitation is due to the fact that the theory of the pulse processes is still in its beginning and up till now only a few simple pulse processes with relatively few correlations among different pulse parameters and pulses have been analysed.

As a consequence of the mentioned difficulties, the knowledge of the inner characteristics of the most noises is still rather poor at present time. Shot noise (more exactly: the shot noise model) is believed to be understood rather well [24]. There is also some knowledge of the inner characteristics of Barkhausen noise [15]. It is much worse in the case of cavitation noise [12], [25]. At last, $1/f$ noise represents an example, where, in spite of an enormous experimental and theoretical effort almost nothing is known about the inner characteristics at present [26] – [28].

5. POINT PROCESSES, PULSE PROCESSES, AND CONTINUOUS PROCESSES

In this section the mutual relation among the point, pulse, and continuous processes will be discussed briefly, so that the position of the pulse processes in the frame of all random processes could be more exactly specified.

Let us consider a realization of a *pulse process*. If the pulse reference points are taken into account only, then these points represent a realization of a *point process*. On the other hand if the envelope of the superposed pulses is taken into account only, then this envelope represents a realization of a *continuous process**. It follows that the pulse processes are closely related both with the point and continuous processes. However, it should be stressed here that they represent an independent class of random processes with its own tasks, methods, terminology and area of applications. It also follows that the outer description defined in the preceding section coincides with the statistical description of the continuous processes. On the other hand, the statistical description of the point processes forms only a part of the inner characteristics.

Development of both the point processes theory and the pulse processes theory was in many respects independent. As a result, there is a partially different terminology. For example, the pulse processes with independent intervals have their counterpart in the renewal point processes [29] and the group pulse processes in the point clustering processes [30], [31] or branching (cascade) point processes [32]. The processes AA and AB_E are known in the point processes theory as the Neyman-Scott and Bartlett-Lewis models [33], respectively. Some authors even consider the pulse processes as a special class of the point processes and call them filtered point processes [34], [35], marked and filtered point processes [36] and so on. Unfortunately, even in the frame of the pulse processes theory the terminology is not standardized yet and several concurrent notations and terms are in use.

6. SUMMARY

In the paper the concept of the nth order group pulse process was introduced and three simple pulse processes, i.e., the processes A, B and D were defined and classified. There is a number of other pulse processes described in the literature that were not considered here. These are mostly the pulse processes with some kind of correlation among pulses as, for example, the processes without overlapping. It is believed that these processes may be included in the classification later when a better insight into their structure is gained.

(Received April 8, 1982.)

* From the mathematical point of view this envelope need not always be continuous. This is the case, for example, when the pulses have a form of rectangular or delta functions. However, from the physical point of view the envelope will always be continuous. It is this view that gave the name to this important class of random processes suitable to model all real signals and noises.

REFERENCES

[1] F. V. Bunkin: Shumy ciklicheskogo peremagnichivaniya ferromagnetikov. Zh. Tekh. Fiz. *26* (1956), 8, 1790—1798.

[2] K. Furutsu and T. Ishida: On the theory of amplitude distribution of impulsive random noise. J. Appl. Phys. *32* (1961), 7, 1206—1221.

[3] V. A. Akulichev and V. V. Olshevskii: Svyaz statisticheskich charakteristik akusticheskoi kavitacii i kavitacionnogo shuma. Akust. Zh. *14* (1968), 1, 30—36. English translation: Relationship between statistical characteristics of acoustic and cavitation noise. Sov. Phys.-Acoust. *14* (1968), 1, 22.

[4] V. A. Akulichev and V. V. Olshevskii: Nekotorye statisticheskie charakteristiki kavitacionnych yavlenii. Akust. Zh. *14* (1968), 2, 163—169. English translation: Statistical characteristics of cavitation phenomena. Sov. Phys. - Acoust. *14* (1968), 2, 135.

[5] Y. W. Lee: Statistical Theory of Communication. Wiley, New York 1960, Chapter 8, Problems 7, 11 and 12.

[6] B. R. Levin and Ya. A. Fomin: Energeticheskie spektry gruppovych signalov v mnogokanalnych impulsnych sistemach. Elektrosvyaz (1963), 2, 3—10.

[7] B. R. Levin: Nekotorye obobshcheniya v teorii impulsnych sluchainych processov. Elektrosvyaz (1963), 2, 21—32.

[8] G. V. Konovalov and E. M. Tarasenko: Impulsnye sluchainye processy v elektrosvyazi. Svyaz, Moskva 1973.

[9] G. V. Konovalov: Slozhnye impulsnye sluchainye processy. Radiotekh. & Elektron. *21* (1976), 7, 1403—1407.

[10] G. V. Konovalov: Spektralno-korrelyacionnyi analiz slozhnych impulsnych sluchainych processov. Radiotekh. & Elektron. *25* (1980), 12, 2566—2574.

[11] B. S. Goldberg: Rekurentnye sootnosheniya v spektralnoi teorii slozhnych impulsnych sluchainych processov. Radiotekh. & Elektron. *24* (1979), 5, 1073—1076.

[12] K. Vokurka: A Contribution to Analysis of Some Physical Processes in Liquids by the Study of Emitted Ultrasonic Waves. Ph. D. Dissertation, ČVUT, FEL, Praha 1979. In Czech.

[13] K. Vokurka: Power spectrum of the periodic group pulse process. Kybernetika *16* (1980), 5, 462—471.

[14] K. Vokurka: Power spectrum of the quasiperiodic and the aperiodic group pulse process. Kybernetika *17* (1981), 4, 298—309.

[15] K. Vokurka: Application of the group pulse processes in the theory of Barkhausen noise. Czech. J. Phys. *B 32* (1982), 12, 1384—1398.

[16] K. Vokurka: On the group pulse processes. II. Power spectrum of the processes with independent points. Kybernetika *19* (1983), 6 (to appear).

[17] K. Vokurka: On the group pulse processes. III. Power spectrum of the processes with independent intervals. Kybernetika *20* (1984), 1 (to appear).

[18] W. A. Gardner and L. E. Franks: Characterization of cyclostationary random signal processes. IEEE Trans. Inform. Theory *IT-21* (1975), 1, 4—14.

[19] T. H. Bell, Jr.: Representation of random noise by random pulses. J. Appl. Phys. *45* (1974), 4, 1902—1904.

[20] S. O. Rice: Mathematical analysis of random noise. Bell Syst. Tech. J. *23* (1944), 3, 282—332.

[21] E. N. Gilbert and H. O. Pollak: Amplitude distribution of shot noise. Bell Syst. Tech. J. *39* (1960), 2, 333—350.

[22] W. J. Richter, Jr. and T. J. Smits: Numerical evaluation of Rice's integral representation

of the probability density function for Poisson impulsive noise. J. Acoust. Soc. Am. *56* (1974), 2, 481—496.

[23] D. G. Childers, J. I. Aunon and C. D. McGillem: Spectral analysis: Prediction and extrapolation. CRC Crit. Rev. Bioeng. *6* (1981), 2, 133—175.

[24] W. B. Davenport, Jr. and W. L. Root: Introduction to Random Signals and Noise. McGraw-Hill, New York 1958.

[25] K. Vokurka: Power spectrum of the cavitation noise at ultrasonic cavitation. XXth Int. Conf. on Acoustics: Ultrasound, ČSVTS, Praha 1981, 53—55.

[26] D. Wolf: $1/f$— noise. In: Noise in Physical Systems. Proc. of the Vth Int. Conf. on Noise, Bad Nauheim, 1978. (D. Wolf ed.) Springer-Verlag, Berlin 1978.

[27] A. van der Ziel: Flicker noise in electronic devices. Adv. Electronics & Electron Phys. *49* (1979), 225—297.

[28] R. F. Voss: $1/f$ (Flicker) Noise: A Brief Review. IBM Research Report, RC 7732, 1979.

[29] F. J. Beutler and O. A. Z. Leneman: The theory of stationary point processes. Acta Mathematica *116* (1966), 159—197.

[30] J. Neyman and E. L. Scott: Processes of clustering and applications. In: Stochastic Point Processes: Statistical Analysis, Theory, and Applications. (P. A. W. Lewis, ed.), Wiley-Interscience, New York 1972, 646—681.

[31] K. Fleischmann: Continuity properties of clustered stochastic point processes and applications to spatially homogeneous branching processes. Colloquia Mathematica Societatis János Bolyai: 24. Point Processes and Queuing Problems. Keszthey 1978, 49—67.

[32] P. A. W. Lewis: A branching Poisson process model for the analysis of computer failure patterns. J. Roy. Statist. Soc. Ser. B *26* (1964), 398—456.

[33] B. A. E. Saleh and M. C. Teich: Multiplied-Poisson noise in pulse, particle, and photon detection. Proc. IEEE *70* (1982), 3, 229—245.

[34] E. Parzen: Stochastic Processes. Holden-Day, San Francisco 1964.

[35] D. L. Snyder: Random Point Processes. Wiley, New York 1975.

[36] W. A. Gardner: An equivalent linear model for marked and filtered doubly stochastic Poisson processes with application to MMSE linear estimation for synchronous m-ary optical data signals. IEEE Trans. Commun. *COM-24* (1976), 8, 917—921.

Ing. Karel Vokurka, CSc., oddělení výzkumu a vývoje LIAZ, n. p. (Department of Research and Development — LIAZ works), tř. V. Kopeckého 400, 466 05 Jablonec n. N. Czechoslovakia.