KYBERNETIKA — VOLUME 19 (1983), NUMBER 1

SOME PROBLEMS IN MODELLING OF REAL SYSTEMS BY FORMALIZED ONES

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The paper is a continuation of the articles [1], [2], [3] and is their conclusion. It concerns some more aspects of relations "model_1", "model_2" and "model_3" among ontological and formalized systems. Particular attention is foc used to problems created by modelling of real systems by language ones.

1. MOTIVATION

In a contradiction to exact mathematical and logical approach to the theory of systems applied in papers [1], [2], [3] I would like to devote this discourse to a methodological classification and judgment of the models which kind reader met in the above mentioned papers.

An aim of a language model formation is not only precise description of a given system, but explanation and prediction of events already having taken place as well as future ones of that. Real systems, being modelled by a precise language media are dynamic ones, i.e. they have time variable character.

As a rule, such systems we know only partially, namely for two reasons:

a) We are not acquainted with all characteristics of a given system, i.e. all attributes and relations of its objects existing at a period of the system existence. Hence we often deal with only its subsystems and in many cases these ones have character input-output systems ("black box" approach).

b) Given real system is known as to its contemporary stage, but we do not know its future characteristics. Also in this case we are familiar with merely its subsystems selected with respect to "time standpoint". However, an adequate prediction requires to take into account some more subsystems for coming existentional period of the system.

In many instances we may be satisfied with input-output systems representing those desired subsystems of a system under consideration. In these cases relations "model_2" and "model_3" play more significant role than in modelling of a language system by another one. It is obvious that we can hardly talk on isomorphy between an ontological system and its – just constructed – language model.

2. CAUSES OF DYNAMIC SYSTEMS IMPERFECT DESCRIPTION

Partial character of real dynamic systems identifications and their precise language modelling has a lot of reasons.

Let us investigate some of them:

a) The first cause may be considerable complexity of a modelled system. For example, let us consider a real system which a technician-engineer operates with. The universe of such a system is a set of many objects in intervals of their relative time stability. These elements are very dishomogeneous. Besides those with physical parameters there are also ones with very qualitatively distinct parameters. We have already seen a significance of man factor in modern production with its social and psychological aspects. Variability of these elements demands very complicated partition of existence interval of those systems and in connection with that selection of subsystems existing in particular time subintervals is necessary. Large size, qualitative dishomogeneity and time variability of the system universe determine obviously extent of properties and relations set of individual elements. Attributes and relations envolved in a system do not regard only the system universe elements, but also properties of these elements properties, those of their relations etc. Properties and relations of higher orders play a remarkable role in large variable systems theory, but their exact description including that of time variability can be a source of many difficulties. It is clear that for technical reasons there is only a part of the universe elements of a system enclosed in the description of the system and merely a subset of its characteristics, i.e. properties and relations of distinct orders. This simplification can be understood as a specification of a subsystem selected on the given system.

b) The second cause is a regular distinction of our experimental equipment capacity. This limitation is often given by physical regularity of its function.

c) The third cause is time variability of described real systems characteristics. I mean by that so many times disputed continuous or discrete modifications of real objects attributes. In a process of an empirical investigation of continuous stages of an object characteristics we necessarily obtain only a discrete sequence of individual concrete knowledge. Description and modelling of such changes by a precise language systems lead to many problems which we shall meet later.

d) The fourth and principle reason is given by a utilization of a given formalized model for a prediction of modelled real system future events. In an optimized case we can form an exact prediction for contemporary events or not a far past of modelled systems, whereas we forecast their future only on the base of general laws. These laws enter language system in a form of general sentences whose complete verification (in a sense of classical requirements) is very problematic.

3. POSSIBLE COMPLICATIONS OF MODELLING

Let us consider a few complications of exact description and modelling of real systems created by previously mentioned causes.

a) Exact description of a large system by an isomorphic language system (i.e. formation of model$_1$) requires mutually unique assignment of language model simple sentences and individual facts of modelled system. If a language system with a finite alphabet should do (that is what plays an important role in the requirement of a description by algorithmic languages), the universe and number of modelled system characteristics can also have a finite extent. Let us specify just used term "fact". By a "fact" I understand an occurrence of a relation of the form

$$\langle a_1, t_i \rangle \in \mathbf{R}^{(1)} \times \Delta t \quad \text{or} \quad \langle a_1, a_2, \ldots, a_n, t_i \rangle \in \mathbf{R}^{(n)} \times \Delta t$$

Let us admit that an element a_1 changes at a moment t_i from the interval Δt its property of the type $\mathbf{R}^{(1)}$ and goes thus through the properties

$$R_1^{(1)}, R_2^{(1)}, \ldots, R_k^{(1)}$$

on the interval Δt.

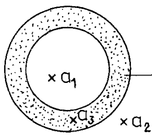
Then a formation of model$_1$ of such a system needs a formulation of complete k-tuple of simple sentences of the type:

$$\bar{R}_1^{(1)}(\bar{a}_1, \bar{t}_{i1}), \quad \bar{R}_2^{(1)}(\bar{a}_1, \bar{t}_{i2}), \ldots, \bar{R}_k^{(1)}(\bar{a}_1, \bar{t}_{ik})$$

So dynamic systems can be modelled on given intervals, when respecting the isomorphy demand, merely by large formalized systems representing subsystems of an integral real system existing at a given time period. Complexity of such a descriptive model is further increasing, if we like to form the description quantitatively. If we use for that quantitative (metric) predicates, we have to apply for their numerization numerical values from discrete and often finite sets.

b) Given simple sentences of descriptive language model$_1$ are formulated on the base of knowledge at the first approach obtained by an empirical way. Logical base of corresponding language is one from logical calculi based on extensional princip. According to that, we replace property which have given objects at a given moment by classes of ordered pairs and n-member relations in which were n-tuples of object at that moment. For a formulation of a descriptive language model$_1$ valid sentences we assume a possibility of the unique decidability, whether a given object of a related real system belongs or does not, as a member of the ordered pairs, to the given set.

If this set represents a quantitative measure of a certain property or a relation, then we often have to answer a question whether a given object at a given moment to this set really belonged or did not. Hence these elements represent for us "unidentified individuals" and those sets seem to be collection of objects with "vague borders" at a moment t_i.



Obviously: $a_1 \in \mathscr{M}$ at a moment t_i,

$a_2 \notin \mathscr{M}$ at a moment t_i,

but about membership of a_3 at a moment t_i with respect to $\mathscr{M}$ we cannot uniquely decide. Therefore we are often forced to omit from a precise description those sentences which regard facts not perfectly known for mentioned reasons.

Similar complications appear also at precise description of ontological system qualitative characteristics. By perfectioning of measurement and observation equipment the area of border vagueness of considered sets are successively narrowing, but this improvement never reaches a perfection necessary for "ideally sharp" measuring and observing. Therefore we can talk about "language descriptive model$_1$" of large real system with considerable restrain.

c) I have shown at the point a) a necessity of specification of subsystems existing within subintervals of a real system existentional period. As an example I gave "partition" of the characteristic $\mathbf{R}^{(1)}$ of an object a_1 into k its subsets $R_1^{(1)}, \ldots, R_k^{(1)}$ associated with particular development stages of the characteristic time variable within given limits. Hence sequence of descriptive language models can model time modifications of ıeal system given characteristics. This process can be compared with a trajectory of a motion picture solid moving according to individual camera views. The better a motion should be described, the more individual film shots must be made. If we assume a continuous character of a motion in accordance with a classical physical concept, then we have to admit with regret, that the best partition of a motion process within a time inteıval provides only a possibility of very poor description of that.

Similarly, the same holds true for any characterization, property or relation modifications of a real system objects changing continuously within a given finite time interval. Ideal description of those modifications would demand division into partial subsystems whose number would be infinitely increasing. Modelling language of such subsystems would have to contain the continuum of related non-logical constants. Denotations of those constants would be however undistinguishable.

This fact is a contradiction with demand of exact languages alphabet decidability. Hence also for this reason we can hardly talk about "exact language model$_1$ of dynamic real system with continuously changing characteristics".

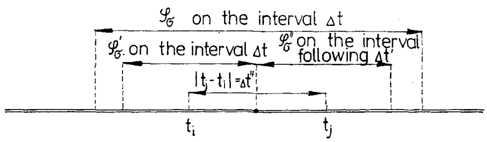
d) Simple language descriptive model$_1$ cannot fulfil a function of a language system in which we form precise prediction of real dynamic system characteristics in future developing.

Let us assume existence of such a system $\mathcal{S}_\sigma = \langle U, \mathcal{R} \rangle$ at a time period Δt. Let us further suppose that in a subinterval $\Delta t' \subset \Delta t (\Delta t, \Delta t'$ have common origin$)$ there is a subsystem $\mathcal{S}'_\sigma = \langle U', \mathcal{R}' \rangle$ so that a characteristic $R_k'^{(j)} \in \mathcal{R}'$ of objects $a_1, a_2, \ldots, a_j$ at a moment t_i $(\langle a_1, a_2, \ldots, a_j, t_i \rangle \in R_k'^{(j)} \times \Delta t', t_i \in \Delta t')$ t_i is time variable and changes respectively so that at a following moment t_j $(t_i < t_j, (t_j - t_i) = |\Delta t''|)$ the objects $a_1, \ldots, a_j$ will be in a relation $R_k''^{(j)}$, but no more in the relation $R_k^{(j)}$:

$$\langle a_1, \ldots, a_j, t_j \rangle \in R_k''^{(j)} \times \Delta t\,, \quad R_k''^{(j)} \notin \mathcal{R}'$$

$$\langle a_1, \ldots, a_j, t_j \rangle \notin R_k'^{(j)} \times \Delta t\,, \quad R_k'^{(j)} \in \mathcal{R}'$$

Let us demonstrate the system relations as follows:



$a_1, \ldots, a_j$ are at a moment t_j in relation $R_k''^{(j)}$, but no more in the relation $R_k'^{(j)}$;
$a_1, \ldots, a_j$ are at a moment t_i in relation $R_k'^{(j)}$, but not yet in relation $R_k''^{(j)}$.

In the illustration there is a subsystem $\mathcal{S}''_\sigma$ selected from $\mathcal{S}_\sigma$ and distinct from $\mathcal{S}'_\sigma$ just by replacement $R_k''^{(j)}$ instead of $R_k'^{(j)}$, $\langle a_1, \ldots, a_j, t_j \rangle \in R_k''^{(j)} \times |\Delta t - \Delta t'|$ where $|\Delta t - \Delta t'|$ is set difference of both intervals.

Let the subsystem $\mathcal{S}'_\sigma$ be modelled by a descriptive language system $\mathcal{S}'_L$ envolving besides others, also a sentence:

$$\bar{R}_k'^{(j)}(\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_j, \bar{t}_i)$$

Let us extend $\mathcal{S}'_L$ so that its universe U'_L gets one more $(j + 1)$-th predicate constant $\bar{R}_k''^{(j)}$ and the set of all its valid sentences obtains following general sentence:

$$\text{(1)} \qquad \forall \bar{x}_1, \ldots, \forall \bar{x}_j \, \forall \bar{t}_i [\bar{R}'^{(j)}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_j, \bar{t}_i)] \rightarrow \bar{R}_k''^{(j)}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_j, \overline{t_i + |\Delta t''|})$$

Under given circumstances it is possible to deduce by substitution $\bar{x}_1/\bar{a}_1, \ldots, \bar{x}_j/\bar{a}_j$ and detachment a sentence

$$\bar{R}_k''^{(j)}(\bar{a}_1, \ldots, \bar{a}_j, \bar{t}_j)$$

belonging to the set of all valid sentences of the system $\mathcal{S}_L$. The language system $\mathcal{S}_L$ is obviously no more a descriptive language model$_1$ of subsystem $\mathcal{S}'_\sigma$.
The sentence (1) surely does not have in $\mathcal{S}'_\sigma$ a "correlated isomorphic partner". The system $\mathcal{S}_L$ is also no more – for the same reason – a descriptive language model$_1$ of the subsystem $\mathcal{S}''_\sigma$. What is now relation between $\mathcal{S}_L$ and $\mathcal{S}_\sigma$?
The general sentence (1) does not have obviously an isomorphic partner even in the integral system $\mathcal{S}_\sigma$, because asserts validity between relations $R_k'^{(j)}$ and $R_k''^{(j)}$ for every moment t_i of time structure $\tau(\Delta t \subset \tau)$, while modelled system $\mathcal{S}_\sigma$ exists merely on the interval Δt.

The system $\mathcal{S}'_L$ is obviously a subsystem of $\mathcal{S}_L$ (defining mapping assigns $\{\bar{R}''^{(j)}_{L,k,1}\}$, $\bar{R}''^{(j)}_{Lk,1} = R_k''^{(j)}(a_1, \ldots, a_j, t_j)$ empty set and a set, which can be uniquely characterized in $\mathcal{S}_L$ by general sentence (1) also empty set).
When applying mentioned specifications, we can characterize relations between considered systems as follows:

$$\langle \mathcal{S}_L, \mathcal{S}'_\sigma, \Delta t' \rangle \in \mathcal{M}od_2$$

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$$\langle \mathcal{S}_L, \mathcal{S}_\sigma, \Delta t \rangle \in \mathcal{M}od_3$$

The system $\mathcal{S}_L$ has been formed for the characteristics prediction of objects $a_1, \ldots, a_j$ created in the interval $\Delta t'$ and concerning future of the system $\mathcal{S}_\sigma$ at a time t_j. So it is clear how important role play language models$_2$ and models$_3$ for realizations of systems prediction.

4. CONCLUSIONS

On the base of well-known – perhaps already classical – reasonings regarding verification problems of general scientific sentences enclosing theoretical predicates, a further generalization can be made. In accordance with that, it is not possible on the base of any empirically adopted experience to declare an axiomatic language system in which its theorems are deducted from general postulates having scientific laws character, as a language model$_1$ of a real system existing at a given finite time interval ($|\Delta t|$ is a finite number). Such language systems can be classified utmost as models$_2$, however almost as models$_3$ of those real systems.

Significance of this conclusion resolves from the following reasoning. We often use for a realization of large real systems exact prediction automata. In engineering practice these are almost computers operating as deterministic automata. These automata represent a certain type of input-output systems. Determination of an adequate language model$_3$ for a given real system satisfies in a given case only the first part of the task. Requested "simplification" of a real system represents often a selection of its subsystem, which is a real input-ouput system. From the determined language $model_3$ we further specify a convenient input-output language subsystem and that one is latei modelled by a ieal input-output system which is a computer. This system-computer is — in an optimized case — a $model_1$ of the selected input-output language subsystem.

More precisely it can be expressed as follows: let a prediction concern a real system $\mathcal{S}_\sigma$ existing at a period Δt. We model this system by a system $\mathcal{S}_L$ enclosing also law-like propositions.

Hence let $\langle \mathcal{S}_L, \mathcal{S}_\sigma, \Delta t \rangle \in \mathcal{M}od_3$.

From the system $\mathcal{S}_L$ let us select its subsystem $\mathcal{S}'_L$ which is isomorphic with — for us interesting — subsystem $\mathcal{S}'_\sigma$ of the system $\mathcal{S}_\sigma$. The subsystem $\mathcal{S}'_L$ does not contain general propositions necessary for the realization of prediction deduction. Its extension by these propositions is a subsystem $\mathcal{S}''_L$.

Hence $\langle \mathcal{S}''_L, \mathcal{S}_L, t_i \rangle \in subsyst$ at every moment t_i of the interval Δt. Corresponding characteristics of $\mathcal{S}''_L$ are modelled by means of automaton (input-output system) $\mathcal{S}_A$. Let it hold in an optimized case: $\langle \mathcal{S}_A, \mathcal{S}''_L, \Delta t' \rangle \in \mathcal{M}od_1$ on an interval $\Delta t'$ is a realization of a solution. This interval, for practical reasons, is a subinterval of Δt. The system $\mathcal{S}_A$ is no more a model of $\mathcal{S}_L$, but it is the $model_2$ of the system $\mathcal{S}'_\sigma$.

The system $\mathcal{S}_A$ is coincidentally the $model_3$ of the system $\mathcal{S}_\sigma$ at a related time interval.

All these conclusions depend obviously on a successful formation of a system $\mathcal{S}_A$ as the $model_1$ of a language system $\mathcal{S}''_L$ enclosing also general sentences. I suppose it is possible only on the base of the following assumptions: the universe U''_L of the system $\mathcal{S}''_L$ must be finite and we have to suppose a finite and discrete time structure for the formulation of $\mathcal{S}''_L$ sentences. In this time structure all events are taking place which are described by means of the language system $\mathcal{S}''_L$.

It must be so, because automata operate at a discrete time, their inputs and outputs can take on merely a finite number of input and output values respectively. The same holds for automata stages. Characterization of the relation „$model_3$" between the system $\mathcal{S}_A$ and original real system $\mathcal{S}_\sigma$ does not exclude a possibility that some characteristics of the system $\mathcal{S}_A$ $model_3$ will not have in the system $\mathcal{S}_\sigma$ a correlating partner. Therefore we have to accept predictions regarding facts about the system and realized by the automaton $\mathcal{S}_A$ with a certain restrain.

This conclusion is interesting. Results obtained on a model of the mentioned kind can, but do not have to be, true. For this reason it is usually necessary to realize a sequence of experiments with computer when solving problems of the given type and after "multi-repeated success" we may use this procedure for solving a concrete task of this type.

(Received February 19, 1982.)

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