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LARGE VARIABLE SYSTEMS OF HIGHER DEGREES IN FUZZIFICATION PROCESS

Fuzzification of Systems for Technical and Medical Practice II

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The paper deals with fuzzification of large variable systems known from technical and medical sciences. Precise definitions of these systems and description of their functioning can be formulated by means of languages having time terms and formed on the base of predicate logics of higher degrees. The fuzzification of these systems is accompanied by their models modifications enabling calculation of certainty grades associated with corresponding entities of systems.

In the article methods of calculation of certainty grades for exactly defined notions are introduced, among others those of time variable system, element of system, input and output of system. Special care is taken of their applications in medical science and practice.

This discourse follows the paper [1] and deals with precise definition and modelling of large variable systems of higher degrees.

We first define on a selected ontological sphere a "sharp system" which later is modelled by a precise language system and fuzzified. At the beginning, let us consider a case of simple systems (known from medical practice) having a class of relations given by only one relation and the universe consists of only those objects which occur in the field of this relation at a given moment. That is the case, where we classify a set of signs wanting to form a clinical consequence which is at a given moment in a given relation with this collection.

Theoretically the situation can be described as follows: there is given $(n + 1)$-member relation R with the first $(n - 1)$-tuple of elements from its field of the type $\langle a_1, \ldots, a_{n-1} \rangle$ at a moment t_i and since the relation R can be taken for mapping of the type

$$R : \langle a_1, \ldots, a_{n-1}, t_i \rangle \rightarrow \langle a_n, t_i \rangle$$

(this mapping may not be unique),
we can consider the element a_n at a moment t_i.

We may not be sure about parameters of the type $a_1, \ldots, a_{n-1}$ at the moment t_i. There are three cases possible:

1. Relation R is sharp, but relevance of given objects into n-tuples of the type $\langle a_1, \ldots, a_{n-1}, t_i \rangle$ i.e. definition domain of the mapping R, is fuzzy.
Hence

$$\mu_{R^0}(a_1, \ldots, a_{n-1}, a_n, t_i) = 1 \quad \text{for all } (n+1)\text{-tuples from the field of sharp relation } R,$$

$$\mu_{(\text{cont})^0 k, R}(a_j, t_i) < 1 \quad \text{for some } a_j \ (1 \leqq j \leqq n-1) \text{ and for some contexts of the relation } R.$$

From here fuzzification of the n-th domain of R is inducted and its value can be calculated:

$\mu_{(\text{Rg})^0 n, R}(a_m, t_i) = \max\{\min\{\mu_{(\text{cont})^0 k, R}(a_j, t_i)$ for all $a_j(1 \leqq j \leqq n-1)$, forming the k-th context $\langle a_1, \ldots, a_{n-1}, a_n, t_i \rangle \in R\} \mid$ for all contexts of the type $\langle x_1, \ldots, x_{n-1}, a_n, t_i \rangle \in R$ in which a_n occurs on the n-th place$\}$.

Example 1. Let

$$R = \{\langle a_1, a_2, a_3, t_i \rangle, \ \langle a_1, a_1, a_3, t_i \rangle, \ \langle a_2, a_3, a_3, t_i \rangle\}$$

($\langle a_1, a_2, a_3, t_i \rangle$ is the first context of the relation R,
$\langle a_1, a_1, a_3, t_i \rangle$ is the second context of the relation R,
$\langle a_2, a_3, a_3, t_i \rangle$ is the third context of the relation R).

Let

$$\mu_{(\text{cont})^0 1, R}(a_1, t_i) = 0{\cdot}7, \qquad \mu_{(\text{cont})^0 1, R}(a_2, t_i) = 0{\cdot}65$$
$$\mu_{(\text{cont})^0 2, R}(a_1, t_i) = 0{\cdot}67, \qquad \mu_{(\text{cont})^0 2, R}(a_1, t_i) = 0{\cdot}62$$
$$\mu_{(\text{cont})^0 3, R}(a_2, t_i) = 0{\cdot}8, \qquad \mu_{(\text{cont})^0 3, R}(a_3, t_i) = 0{\cdot}9$$

then

$$\mu_{(\text{Rg})^0 3, R}(a_3, t_i) = \max\{\min\{0{\cdot}7, 0{\cdot}65\}\}, \min\{0{\cdot}67, 0{\cdot}62\}, \min\{0{\cdot}8, 0{\cdot}9\}\} = 0{\cdot}8$$

2. n-tuple of the type $\langle a_1, \ldots, a_{n-1}, t_i \rangle$ are sharp (definition domain of the mapping R is sharp), but relation R is fuzzy.
Thus

$$\mu_{(\text{cont})^0 k, R}(a_j, t_i) = 1$$

for all $a_j(1 \leqq j \leqq n-1)$ forming the k-th context $\langle a_1, \ldots, a_{n-1}, a_n, t_i \rangle \in R$ and for all contexts of the type $\langle x_1, \ldots, x_{n-1}, a_n, t_i \rangle \in R$ in which a_n is on the last place,

$$\mu_{R^0}(a_1, \ldots, a_{n-1}, a_n, t_i) < 1$$

for some $\langle a_1, a_2, \ldots, a_{n-1}, a_n, t_i \rangle$ belonging before fuzzification into R.

From here the fuzzification of the n-th domain of R is inducted and its value can be calculated:

$$\mu_{(\text{Rg})^0 n, R}(a_n, t_i) = \max\{\mu_{R^0}(x_1, \ldots, x_{n-1}, t_i) \mid \text{for all contexts of the type } \langle x_1, \ldots, x_{n-1}, a_n, t_i \rangle \in R\}.$$

Example 2. Let R be the same as in Example 1, but

$$\mu_{R^0}(a_1, a_2, a_3, t_i) = 0{\cdot}95\,, \qquad \mu_{R^0}(a_1, a_1, a_3, t_i) = 0{\cdot}82$$
$$\mu_{R^0}(a_2, a_3, a_3, t_i) = 0{\cdot}78$$

then

$$\mu_{(\mathrm{Rg})^0 3.R}(a_3, t_i) = \max\{0{\cdot}95, 0{\cdot}82, 0{\cdot}78\} = 0{\cdot}95$$

3. The relation R is fuzzy and so are its individual n-tuples of the type $\langle a_1, a_2, \ldots a_{n-1}, t_i\rangle$.
Hence

$$\mu_{R^0}(a_1, \ldots, a_{n-1}, a_n, t_i) < 1$$

for some $\langle a_1, \ldots, a_{n-1}, a_n, t_i\rangle$ belonging before fuzzification to R,

$$\mu_{(\mathrm{cont})^0 k.R}(a_j, t_i) < 1$$

for some $a_j(1 \leqq j \leqq n-1)$, for some contexts of the type $\langle x_1, \ldots, x_{n-1}, a_n, t_i\rangle \in R$.

From here the fuzzification of the n-th domain of R is inducted and its value can be calculated:

$$\mu_{(\mathrm{Rg})^0 n.R}(a_n, t_i) = \max\{\min\{\mu_{R^0}(a_1, \ldots, a_{n-1}, a_n, t_i)\,,$$
$$\mu_{(\mathrm{cont})^0 k.R}(a_j, t_i)$$

for all $a_j(1 \leqq j \leqq n-1)$, forming the k-th context $\langle a_1, \ldots, a_{n-1}, a_n, t_i\rangle \in R\}\ |$
$|$ for all contexts of the type $\langle a_1, \ldots, a_{n-1}, a_n, t_i\rangle \in R\}$.

Example 3. Let R be the same as in Examples 1 and 2, let contexts be fuzzified as in Example 1 and so relation as in Example 2.
Then

$$\mu_{(\mathrm{Rg})^0 3.R}(a_3, t_i) = \max\{\min\{0{\cdot}7, 0{\cdot}65, 0{\cdot}95\}, \min(0{\cdot}67, 0{\cdot}62, 0{\cdot}82\}\,,$$
$$\min\{0{\cdot}8, 0{\cdot}9, 0{\cdot}78\} = 0{\cdot}78$$

Let us return to the definition of term "system of the n-th degree".

D 1. An ordered pair $\langle \mathscr{U}, \mathscr{R}\rangle = \mathscr{S}$ forms a system of the n-th degree at a time interval Δt (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{Syst}^{(n)}$), iff:

$\mathscr{U}$ is a class of objects (individuals = entities of zero degree),
$\mathscr{R}$ is a collection of objects of the type ${}^{(s)}R_k^{(j)}$ where holds:
for $s = 1$: ${}^{(s)}R_k^{(j)} \subset U^{\langle j\rangle}$
for $s > 1$: there is at least one element from the collection $\{s_1, \ldots, s-1\}$ where $s-1$ exists so that ${}^{(s)}R_k^{(j)} \subset U^{\langle j_0\rangle} \times \{{}^{(s_1)}R\}^{\langle j_1\rangle} \times \ldots \times \{{}^{(s-1)}R\}^{\langle j_{s-1}\rangle}$, where $\{{}^{(s_i)}R\}^{\langle j_i\rangle}$ is the j_i-th power of the class of all relations of the s_i-th degree from the set $\mathscr{R}$, $1 \leqq i \leqq s-1$, $j = j_0 + j_1 + \ldots + j_{s-1}$.

Class $\mathscr{U}$ is called the system universe.

Successive fuzzification of a system can be motivated by following reasons and can be formulated by precise language model modification of formerly "sharp" system.

1. Time interval Δt can be fuzzy, because we cannot say with full reliability, if given moment t_i belongs to Δt.

A reason may be relative accuracy regarding time items about events or our own uncertainty connected with unity structure of physical time (chronometer time) and biological one (in biological systems fuzzification). In many instances we may not be sure about relevancy of time interval, in which given event occurred. We doubt in this case about relevancy of this moment to given n-tuple of objects forming an element of this fuzzy relation.

Fuzzy time interval Δt^0 is a mapping from the class of all time moments into interval $Q_{01} = \langle 0, 1 \rangle$. Membership function of the type $\mu_{\Delta t^0}(t_i)$ can have its values found at basic cases by empirical methods.

2. The system universe $\mathscr{U}$ can be fuzzy, since we are not able to decide with full certainty whether given objects from selected ontological sphere is from that. The cause of that may be limited accuracy or empirical data and variability of distinquishing characteristics classifying relevancy of the object into the defines system universe and by that coincidentally into the fields of system relations. When fuzzifying terms of systems theory, we may further meet the system universe timely relativitated (given object from the language universe belongs to the system universe at a given moment). Such a type of universe we further denote by $\mathscr{U}_T$.

Fuzzy, timely relativitated universe $\mathscr{U}_T^0$ is a mapping from a class of pairs consisting of objects from selected ontological sphere and moments from interval Δt into numerical interval Q_{01}. The membership function $\mu_{\mathscr{U}_T{}^0}(x, t_i)$ can have its values determined in empirical way.

3. Fuzzification of relations $\mathscr{R}$ collection of the system. We cannot say surely, if a given relation belongs to $\mathscr{R}$.

The first fuzzification of the class $\mathscr{R}$ is a mapping from all "sharp" relations collection over selected ontological sphere (these relations can be of distinct degrees) into the interval Q_{01}. The membership function can have its values determined empirically in basic cases.

We fuzzify further successively all objects from the field of every relation of the type ${}^{(s_j)}R_j$ and so far these objects are also relations we fuzzify even them. So we continue in further fuzzification process till the universe elements in fields of successively fuzzified relations. Successive fuzzification can be thus formed until the $2s_j$-th degree.

As a whole, the "sharp" collection $\mathscr{R} = \{{}^{(s_1)}R_1, \ldots, {}^{(s_n)}R_n\}$ can be fuzzified till the $(2s_j + 1)$-th degree, where s_j is the highest degree among degrees of relations from the collection $R : s = \max \{s_1, \ldots, s_n\}$. In such a case we shall simply write $\mathscr{R}^0 = \mathscr{R}^{0(2s_j+1)}$ and this fuzzy collection is mapping from that of mostly fuzzified relations over selected region into the interval Q_{01}.

The membership function values can be calculated if functional values are given (obtained empirically or as results of some other calculations):

$$\mu_{\mathscr{R}^{01}}({}^{(s_i)}R_i) \quad \text{for each} \quad {}^{(s_i)}R_i \in \mathscr{R} \quad (\text{in "sharp" system}),$$

further functional value $\mu_{{}^{(s_j)}R_j{}^{01}}({}^{(s_{j1})}R_{j1}, \ldots, {}^{(s_{jn})}R_{jn}, t_i)$ for each ${}^{(s_j)}R_j \in \mathscr{R}$(in "sharp" system), where $\langle {}^{(s_{j1})}R_{j_1}, \ldots, {}^{(s_{jn})}R_{j_n}, t_i \rangle \in {}^{(s_j)}R_j$. (The method of calculation has been shown in the previous paper).

4. Fuzzification of notion "system": we cannot decide with full reliability, if $\mathscr{U}_T$ is a class of selected elements from ontological region and if $\mathscr{R}$ is a collection of selected relations over that. $\mathscr{S}^0$ is thus a mapping from a class of triples consisting of a class of objects with zero type level from the selected region, a class of moments and a class of relations of various degrees defined over the region into the interval Q_{01}.

The membership function values can be calculated:

$$\mu_{\mathscr{S}^0}(\mathscr{U}_T, \mathscr{R}) = \min \{\mu_{\mathscr{U}^0{}_T}(x_i, t_i), \mu_{\mathscr{R}^0}({}^{(s_j)}R_j) \mid \text{for each } x_i \text{ and } t_i \in \Delta t$$
$$\text{for every relation of the type } {}^{(s_j)}R_j \text{ belonging to collection } \mathscr{R}\}.$$

Since existentional interval Δt of the system is fuzzy (for above mentioned reasons), fuzzification of the class $\mathscr{S}yst$ is motivated by uncertainty, if the pair $\mathscr{S} = \langle \mathscr{U}_T, \mathscr{R} \rangle$ is really a system at a time interval Δt to which belong moments appearing in relations from the class R.

The membership function values can be calculated:

$$\mu_{(\mathscr{S}yst)^0}(\mathscr{S}, \Delta t) = \min \{\mu_{\mathscr{S}^0}(\mathscr{U}_T, \mathscr{R}), \mu_{\Delta t^0}(t_i) \mid \text{for all } t_i \in \Delta t \text{ occurring}$$

on the last places in relations of the type ${}^{(s_j)}R_j \in \mathscr{R}$ and in couples of the type $\langle x_i, t_i \rangle \in$

$$\in \mathscr{U}_T \text{ for } \mathscr{S} = \langle \mathscr{U}_T, \mathscr{R} \rangle, \langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst\}.$$

E. 3.1. Example.

Let a "sharp" system be given:

$$\mathscr{S} = \langle \mathscr{U}_T, \mathscr{R} \rangle, \mathscr{U}_T = \{\langle x, t_i \rangle, \langle y, t_j \rangle\}, \quad \mathscr{R} = \{{}^{(1)}R, {}^{(2)}R, {}^{(3)}R\}$$
$$\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst, \quad t_i, t_j \in \Delta t,$$

let relations ${}^{(1)}R$, ${}^{(2)}R$, ${}^{(3)}R$ be the same as in Example E. 2.1.
Let their fuzzification be the same as in E. 2.1.
Let values of further membership functions be

$$\mu_{\Delta t^0}(t_i) = 0{\cdot}95 \quad \mu_{\Delta t^0}(t_j) = 0{\cdot}87$$

completely for t_i and t_j in Δt is then 0·87.
Let

$$\mu_{\mathscr{U}^0{}_T}(x, t_i) = 0{\cdot}75, \quad \mu_{\mathscr{U}^0{}_T}(y, t_j) = 0{\cdot}82$$

and totally for x and y in $\mathscr{U}_T$ is 0·75

$$\mu_{\mathscr{R}^{01}}({}^{(1)}R) = 0{\cdot}82 \quad \mu_{\mathscr{R}^{01}}({}^{(2)}R) = 0{\cdot}68 \quad \mu_{\mathscr{R}^{01}}({}^{(3)}R) = 0{\cdot}57.$$

Since relations from the collection $\mathscr{R}$ can be fuzzified only five times (for example relation ${}^{(3)}R$), the whole collection can be fuzzified utmost six times.

$$\mu_{\mathscr{R}01}({}^{(1)}R) = 0{\cdot}82 \quad \mu_{\mathscr{R}02}({}^{(2)}R) = 0{\cdot}68 \quad \mu_{\mathscr{R}02}({}^{(3)}R) = 0{\cdot}6$$

We can calculate for all relations:

$$\min \{\mu_{\mathscr{R}02}({}^{(1)}R),\ \mu_{\mathscr{R}02}({}^{(2)}R),\ \mu_{\mathscr{R}02}({}^{(3)}R)\} = 0{\cdot}6$$

Further calculations only in brief:

$$\mu_{\mathscr{R}02}({}^{(1)}R) = \mu_{\mathscr{R}02}({}^{(2)}R) = 0{\cdot}82 \quad \mu_{\mathscr{R}03}({}^{(2)}R) = 0{\cdot}68 \quad \mu_{\mathscr{R}03}({}^{(3)}R) = 0{\cdot}6$$

total calculation:

$$\min \{0{\cdot}82, 0{\cdot}68, 0{\cdot}6\} = 0{\cdot}6$$

further

$$\mu_{\mathscr{R}05}({}^{(1)}R) = \mu_{\mathscr{R}02}({}^{(1)}R) = 0{\cdot}82 \quad \mu_{\mathscr{R}05}({}^{(2)}R) = \mu_{\mathscr{R}04}({}^{(2)}R) = 0{\cdot}68$$

$$\mu_{\mathscr{R}05}({}^{(3)}R) = 0{\cdot}6$$

then

$$\min \{0{\cdot}82, 0{\cdot}68, 0{\cdot}6\} = 0{\cdot}6$$

and

$$\mu_{\mathscr{R}06}({}^{(1)}R) = \mu_{\mathscr{R}02}({}^{(1)}R) = 0{\cdot}82 = \mu_{\mathscr{R}0}({}^{(1)}R)$$

$$\mu_{\mathscr{R}06}({}^{(2)}R) = \mu_{\mathscr{R}04}({}^{(2)}R) = 0{\cdot}68 = \mu_{\mathscr{R}0}({}^{(2)}R)$$

$$\mu_{\mathscr{R}06}({}^{(3)}R) = 0{\cdot}6 = \mu_{\mathscr{R}0}({}^{(3)}R)$$

totally

$$\min \{0{\cdot}6, 0{\cdot}68, 0{\cdot}6\} = 0{\cdot}6$$

and

$$\mu_{\mathscr{S}0}(\mathscr{U}_T, \mathscr{R}) = 0{\cdot}6 \quad \mu_{(\mathscr{S}yst)0}(\mathscr{S}, \Delta t) = 0{\cdot}6$$

Remark. We have a right to ask a question, if a couple of the type $\langle \mathscr{U}_T, \mathscr{R} \rangle$ characterized by its membership function value relatively very small (for example in interval $(0, 0{\cdot}5)$) with respect to relevancy to class $\mathscr{S}yst^{(n)}$, is to be still considered as a system and further operated with as that. However we cannot lift up our attention in clinical practice from those couples which are doubtful as to the system formation. Majority of diseases begins namely with very indefinite predroms. To recognize a kind of illness beyond these symptoms is very important. Relation of these signs often resolves in a system with a low grade of membership. With increasing time formerly unidentified symptoms are developing to characteristic ones whose physiological significance is well known. (Symptoms of a disease can be understood as input values of system defined on a related ontological sphere). Definition of system with vague characteristic terms can be therefore very important for prevention and prophylacs of developing sickness.

D 2. An object x is said to be element of system $\mathscr{S}$ at a moment $t_i \in \Delta t$ (abbreviated:

$\langle x, \mathcal{S}, t_i\rangle \in \mathcal{E}\ell)$ iff:

$$\langle \mathcal{S}, \Delta t\} \in \mathcal{S}\mathit{yst}$$

$$\mathcal{S} = \langle \mathcal{U}_T, \mathcal{R}\rangle\,, \quad \langle x, t_i\rangle \in \mathcal{U}_T\,, \quad t_i \in \Delta t\,.$$

Fuzzification of class of system elements can be motivated by several grounds. First we do not have certainty regarding relevancy of object x to the system universe $\mathcal{U}_T$ and moreover we may not be quite sure about membership of formerly stated class $\mathcal{U}_T$ as the system $\mathcal{S}$ universe at all moments of interval Δt.

Further we may not insist in membership of object x to a field of some relation from collection $\mathcal{R}$ at a moment t_i and then the collection $\mathcal{R}$ is really a collection of system $\mathcal{S}$ relations. For already known reason we neednot be sure whether moment t_i belongs to the interval Δt.

Fuzzy set $(\mathcal{E}\ell)^0$ is a mapping of the class consisting of triples of the type $\langle x, \mathcal{S}, t_i\rangle$ into the interval Q_{01}. Calculation of membership function values for the triple $\langle x, \mathcal{S}, t_i\rangle$ can be done by the following algorithm:

Algorithm.

Step 1: Determine max value from membership functions values for all contexts of the type $\text{cont}_{k,R}$ in which element x occurs at a moment $t_i \in \Delta t$ for all relations of the type $R \in \mathcal{R}$.

Step 2: Determine min from the step 1 and from membership function value $\mu_{\mathcal{R}^0}(R)$.

Step 3: Determine min from the step 2 and from the values $\mu_{\mathcal{R}^0}(R)$ for all relations of the type R in which x occurs at t_i.

Step 4: Determine max of the result from the step 3 for all relations of the type $R \in \mathcal{R}$ in which x appears at $t_i \in \Delta t$, $\langle \mathcal{S}, \Delta t\rangle \in \mathcal{S}\mathit{yst}$.

Step 5: Find min from the result of the step 4 and from the value $\mu_{\mathcal{U}^0_T}(x, t_i)$, $\mathcal{S} = \langle \mathcal{U}_T, \mathcal{R}\rangle$.

This min is the value of $\mu_{(\mathcal{E}\ell)^0}$ for the triple $\langle x, \mathcal{S}, t_i\rangle$.

Development diagram:

Remark. When formulating the function $\mu_{(\mathscr{E}\ell)^0}$ we take care of the degree of x relevancy to the system universe, of that of t_i to Δt and coincidentally that x is at t_i in the field of a relation belonging to $\mathscr{R}$ at a certain degree.

The element x may occur in the first relation in various contexts which we consider all and we choose max value of membership grade of x at t_i to these contexts. There may be more of these relations envolving an element x at a moment t_i. Hence we have to select the max value of memberships grades.

E. 3.2. Example. (continuation of Example E. 3.1.)

$$\mu_{(\mathscr{E}\ell)^0}(x, \mathscr{S}, t_i) = 0{\cdot}75 \quad \mu_{(\mathscr{E}\ell)^0}(y, \mathscr{S}, t_j) = 0{\cdot}82$$

Let us notice that our conception of fuzzy system represents certain generalization in a comparison with common notion of fuzzy abstract automata, where "sharp" knowledge of input events and given state are assumed, but only fuzzy knowledge of proceeding occurrence. Despite the fact that we do not define in this paper notions of "fuzzy input event" and "fuzzy state" our conception does admit definitions of this kind. It is obvious that in discussed technical and medical practice with large and variable systems such a conception would be convenient.

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