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SOME FUNDAMENTAL NOTIONS OF LARGE VARIABLE SYSTEMS

MILAN RŮŽIČKA

The paper deals with basic notions from theory of large dynamic systems variable in time. There is presented an attempt of exact definitions (or at least specifications) of such a system, its internal, boundary, input and output elements, input and output, environment and response and impuls etc. The concept of subsystem is introduced with a few illustrations covering its possible application.

INTRODUCTION

Mankind creative activity has in modern time some particular and specific figures. Among others, there is a permanently growing demand towards an integral and systematic approach to reality, which is formed by this work and transformed to human needs. Engineering way of solving problems requires language modelling of large dynamic systems and related selection of convenient language systems as their language models. That is why a reasoning concerning notion "model" and character of a relation setting-up this concept may be very actual.

In this paper I like to show some logical and semantic aspects of this approach and I try to define (or at least specify) some fundamental notions regarding large dynamic systems theory.

SOME FUNDAMENTAL NOTIONS

First I specify concept of "system". I have a few following requirements for this and other definitions. They are supposed to be consistent (or at least not contradictory) with corresponding terms formed by other authors. Further, definitions will be formulated by precise language and means of set theory. I also like to have all forecoming notions fully adequate with the former considerations regarding engineering and technical operations with large systems variable in time.

I propose generalized notion of "system" in this way:

D 1. A set $\mathcal{S}$ is *system* in a time interval Δt (symbolically: $\langle \mathcal{S}, \Delta t \rangle \in \mathcal{S}ys\ell$) iff:

a) $\mathcal{S}$ is in time interval (period) Δt identical with an ordered pair of the type $\langle \boldsymbol{U}, \mathcal{R} \rangle$,

b) $\boldsymbol{U}$ is a set of objects – "elements of $\mathcal{S}$",

c) $\mathcal{R}$ is a set of objects of the type ${}^{(s)}R_k^{(j)}$; $1 \leqq s$, $1 \leqq j \leqq n$, $1 \leqq k \leqq i_j$ (where s, j, n, k, i_j are natural variables),

d) for every object from $\mathcal{R}$ of the type ${}^{(1)}R_k^{(j)}$ holds: *)

$$ {}^{(1)}R_k^{(j)} \subset \boldsymbol{U}^{\langle j \rangle} \times \Delta t , $$

e) for every object from $\mathcal{R}$ of the type ${}^{(s_{j_k})}R_k^{(j)}$ holds:

$$ {}^{(s_{j_k})}R_k^{(j)} \subset \left\{ {}^{(s_1)}R_{k_1}^{(\delta_1)} \right\} \times \left\{ {}^{(s_2)}R_{k_2}^{(\delta_2)} \right\} \times \ldots \times \left\{ {}^{(s_{j_k}-1)}R_{k_j}^{(\delta_j)} \right\} \times \Delta t $$

$$ {}^{(s_1)}R_{k_1}^{(\delta_1)}, \ldots, {}^{(s_{j_k}-1)}R_{k_j}^{(\delta_j)} \in \mathcal{R} ; $$

$$ 1 \leqq j , \quad \delta_1, \ldots, \delta_j \leqq n ; \quad 1 \leqq k , \quad k_1, k_2, \ldots, k_j \leqq i_j , $$

$$ 0 \leqq s_1, s_2, \ldots, s_{j_k - 1} < s_{j_k} \quad \text{for} \quad s_i = 0 \Rightarrow {}^{(s_i)}R_{k_i}^{(\delta_i)} \subseteq \boldsymbol{U} , $$

f) $\Delta t \subset T$ is ordered set of real numbers.

This definition well satisfies requirement regarding type purity of classes. Left upper symbol is type denomination. From the viewpoint of the class theory:

symbol of the type ${}^{(1)}R^{(1)}$	is a name of a class of elements from the universe of this system,
symbol of the type ${}^{(1)}R^{(j)}$	a name of j-argument relation among elements from the universe of this system,
symbol of the type ${}^{(2)}R^{(1)}$	a name of class of class (properties of properties) of elements from the system universe,
symbol of the type ${}^{(2)}R^{(j)}$	a name of j-argument relation among classes or relations of elements from the system universe,
symbol of the type ${}^{(s_{j_k})}R^{(j)}$	a name of j-argument relation among classes or relations utmost $(s_{j_k - 1})$ order, when at least one argument of this relation is of $(s_{j_k - 1})$-th order.

When introducing further notions I shall not talk in particular about systems with classes and relations of distinct orders, for symbols simplicity reasons, but I do my best to achieve specifications holding true for any system.

***) $U^{\langle j \rangle}$ denotes cartesian product of j-factors.**

A certain system is connected to a certain period of time. I assume that a set of elements with their properties and relations, which is system in a specific time, in some other time interval may not be a system. So I admit time variability of systems.

Let us introduce notion of "*element* of system":

D 2. Object **x** is at a moment t_i an *element* of system $\mathscr{S}$ (symbolically: $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \in \mathscr{E}\ell$), iff:

a) there is a time interval Δt so that $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst$, there are sets $\boldsymbol{U}, \mathscr{R}$ so that $\mathscr{S} = \langle \boldsymbol{U}, \mathscr{R} \rangle$,

b) **x** belongs to an ordered j-tuple from set of the type ${}^{(1)}R_k^{(j)} \in \mathscr{R}$ where this j-tuple, when extended to $(j + 1)$-tuple by an adjoined element t_i, belongs to the set ${}^{(1)}R_k^{(j)} \times \Delta t$.

In this proposed interpretation a given object at a given moment is element of a system existing in a time period, if it is an element from the universe of the system at that particular moment and belongs to some relation or has a property from the system and this moment is from "existentional period" of the system.

Consider further notion of "*internal* element of system":

D 3. Object **x** is at a moment t_i an *internal* element of system (symbolically: $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{I}nel$), iff:

a) $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{E}\ell$,

b) there are sets $\boldsymbol{U}, \mathscr{R}$ so that $\mathscr{S} = \langle \boldsymbol{U}, \mathscr{R} \rangle$, there exists an interval Δt so that $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst$,

c) for every element **y** forming together with **x** any ordered $(j + 1)$-tuple the last member of which is t_i and belonging to cartesian product of a set of the type ${}^{(1)}R_k^{(j)} \in \mathscr{R}$ and interval Δt holds $\langle \mathbf{y}, \mathscr{S}, t_i \rangle \in \mathscr{E}\ell$.

Remark. As to this definition, a certain object at a given moment is internal element of a system, if it is at this moment in relations belonging to this system with only elements from this system.

The term of "*boundary* element of system" let us define in this way:

D 4. Object **x** is at a moment t_i a *boundary* element of system $\mathscr{S}$ (symbolically: $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{B}nel$) iff:

a) $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{E}\ell$,

b) there exist sets $\boldsymbol{U}, \mathscr{R}$ so that $\mathscr{S} = \langle \boldsymbol{U}, \mathscr{R} \rangle$, there is interval Δt so that $\langle \mathscr{S}, \Delta t \rangle \in \in \mathscr{S}yst$,

c) there is at least one element **y** which forms together with **x** some ordered $(m + 1)$-tuple, the last member of which is t_i and holds $\langle \mathbf{y}, \mathscr{S}, t_i \rangle \notin \mathscr{E}\ell$.

In proposed interpretation: given object is at a given moment boundary element of a system, if it is at this moment in a relation with such an object which does not belong to this system in this moment.

D 5. Object **x** is called *input* element of a system $\mathscr{S}$ at a moment t_i (symbolically: $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{I}np el$) iff:

a) $\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{B}nel$,

b) there are time intervals Δt, $\Delta t'$ so that $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst$ and time interval $\Delta t'$ is shorter than Δt (symb.: $|\Delta t'| < |\Delta t|$),

c) there is at least one object **u** such that $\langle \mathbf{u}, \mathscr{S}, t_i - |\Delta t'| > \notin \mathscr{E}\ell$ where $|\Delta t'|$ is the length of $\Delta t'$,

d) there are properties U, V so that for every moment $t_j \in \Delta t$ where $(t_j - |\Delta t'|) \in \in \Delta t$ holds:
if **u** has a property U at moment $t_j - |\Delta t'|$ then **x** has property V in moment t_j.

In suggested specification: a given boundary element of a system is at a given moment its input element, if there is an object, which was an element of this system before some time period (of the system existence), where further there are properties U, V which can be taken on by **x** and **u** resp. so that possession of U by object **u** leads after considered time interval to that of V by **x** – input element of the system.

Simply said: input element of system changes some of its properties being effected by some property transformation of an object standing out of the system.

We need now "*output* element of system".

D 6. Object **y** is *output* element of system $\mathscr{S}$ at a moment t_i (symbolically: $\langle \mathbf{y}, \mathscr{S}, t_i \rangle \in \mathscr{O}utpel$), iff:

a) $\langle \mathbf{y}, \mathscr{S}, t_i \rangle \in \mathscr{B}nel$,

b) there are time intervals Δt, $\Delta t'$ so that $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst$ and time interval $\Delta t'$ is shorter than Δt,

c) there is at least one object **u**′ so that $\langle \mathbf{u}', \mathscr{S}, t_i + |\Delta t'| \rangle \notin \mathscr{E}\ell$ where $|\Delta t'|$ is the length of $\Delta t'$,

d) there exist properties W, U' so that for every moment $t_j \in \Delta t$ where $(t_j + |\Delta t'|) \in \in \Delta t$ holds:
if **y** has at moment t_i property W, then **u**′ has property U' at moment $t_j + |\Delta t'|$.

In proposed definition: given boundary element of a system is at given moment its output element, if there is an object, which did not belong to the system during some existentional period of the system and there are properties W, U' offered to **y**, **u**′ resp. in that mode, that if **y** takes on W then necessarily **u**′ gets U'.

Shortly: output element changes some of its properties and creates thus a property transformation of an object out of the system.

D 7. A set $\mathbf{X}$ is called *input* of a system $\mathscr{S}$ at a moment t_i (symbolically: $\langle \mathbf{X}, \mathscr{S}, t_i \rangle \in \mathscr{I}\!\mathit{np}$), iff for every element $\mathbf{x}$ holds:
$\mathbf{x}$ is at a moment t_i an element of $\mathbf{X}$, if there is an interval Δt so that

$$\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}\!\mathit{yst} \quad \text{and} \quad \langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{I}\!\mathit{npel}\,, \quad t_i \in \Delta t$$

symbolically:

$$\langle \mathbf{x}, t_i \rangle \in \mathbf{X} \times \Delta t\,.$$

In plain English: input of a given system at a given moment is set of all its input elements at that moment.

D 8. A set $\mathbf{Y}$ is said to be *output* of a system $\mathscr{S}$ at a moment t_i (symbolically: $\langle \mathbf{Y}, \mathscr{S}, t_i \rangle \in \mathscr{O}\!\mathit{utp}$), if for each object $\mathbf{y}$ holds:
If there exists interval Δt so that

$$\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}\!\mathit{yst}$$

and

$$\langle \mathbf{y}, \mathscr{S}, t_i \rangle \in \mathscr{O}\!\mathit{utpel}\,, \quad t_i \in \Delta t$$

only just then $\mathbf{y}$ is at moment t_i an element of $\mathbf{Y}$, symbolically:

$$\langle \mathbf{y}, t_i \rangle \in \mathbf{Y} \times \Delta t\,.$$

Shortly: output of a given system at a given moment is set of all its output elements at that moment.

All just above mentioned definitions are formulated in accordance with common usage of these terms in automata theory.

From the technical viewpoint we can understand by "automata input" a set of all data entry associated with the automata. These entries are in certain relations to automata environment.

For instance, let a given automata have an input $\mathbf{X}$ which is a set of n-input elements of the system. These elements $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ can take on some respective "input" properties $V_1, V_2, \ldots, V_n$. In such a case we can directly characterize all possible values of total input $\mathbf{X}$ by means of 2^n n-argument variations consisting of elements of the type $\{V_i, \overline{V_i}\}$, $1 \leqq i \leqq n$

$$\begin{array}{l}
\overline{V}_1, \overline{V}_2, \ldots, \overline{V}_{n-1}, \overline{V}_n \\
\overline{V}_1, \overline{V}_2, \ldots, \overline{V}_{n-1}, V_n \\
\overline{V}_1, \overline{V}_2, \ldots, V_{n-1}, \overline{V}_n \\
\overline{V}_1, \overline{V}_2, \ldots, V_{n-1}, V_n \\
\quad \vdots \qquad\qquad\quad \vdots \quad\;\; \vdots \\
V_1, V_2, \ldots, V_{n-1}, V_n
\end{array}$$

Individual input elements of the automata in this case have mentioned properties as their binary values: for every input element $\mathbf{x}_i$ holds that "its" property V_i at moment t_i either has or does not. Symbolically:

$$\langle \mathbf{x}_i, t_i \rangle \in V_i \times T \quad \text{or} \quad \langle \mathbf{x}_i, t_i \rangle \in \overline{V}_i \times T .$$

Analogue remark obviously holds also for output and output elements. Particularly, for system-automata we can consider that for individual effectors-output elements, by means of which the automata effects directly its enviroment.

For example, let such an automata have ouput **Y** which is a set of m output elements $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_m$ taking on some respective "output" properties. Then we can characterize possible values of total output **Y** by 2^m m-argument relations-variations consisting of elements from a set of the type $\{W_i, \overline{W}_i\}$, $1 \leqq i \leqq m$

$$\begin{array}{l} \overline{W}_1, \overline{W}_2, \ldots, \overline{W}_{m-1}, \overline{W}_m \\ \overline{W}_1, \overline{W}_2, \ldots, \overline{W}_{m-1}, W_m \\ \overline{W}_1, \overline{W}_2, \ldots, W_{m-1}, W_m \\ \quad\vdots \qquad\qquad\quad \vdots \quad\;\; \vdots \\ W_1, W_2, \ldots, W_{m-1}, \overline{W}_m \\ W_1, W_2, \ldots, W_{m-1}, W_m \end{array}$$

Particular output elements of the automata take on described properties as their binary values: each output element $\mathbf{y}_i$ at a given moment t_i either has "its" output property or does not.

Symbolically:

$$\langle \mathbf{y}_i, t_i \rangle \in W_i \times T \quad \text{or} \quad \langle \mathbf{y}_i, t_i \rangle \in \overline{Y}_i \times T .$$

I consider generally defined system as "developing in time". In respective formulations of individual notions there occurs factor Δt representing existence duration of a system. That is why I require respective connection between properties of input element **x** and those of output element **y** as well as relation of input element **x** and an adequate distance inside the existentional period Δt of the system. I do not say that this relation may not exist also out of this existentional period of the system.

Let us further introduce "*environment* of system".

D 9. A set **M** is called *environment* of system $\mathcal{S}$ at a moment t_i (symbolically: $\langle \mathbf{M}, \mathcal{S}, t_i \rangle \in \mathcal{E}nv$), iff any pair of objects **x**, **u** satisfies following conditions:

a) $$\langle \mathbf{x}, \mathcal{S}, t_i \rangle \in \mathcal{E}\ell$$

b) $$\langle \mathbf{u}, \mathcal{S}, t_i \rangle \notin \mathcal{E}\ell$$

where there is such a time interval Δt that $t_i \in \Delta t$ and $\langle \mathcal{S}, \Delta t \rangle \in \mathcal{S}yst$,

c) **x**, **u** are in an ordered $(m + 1)$-tuple, the last element of which is element t_i, then only holds:

$$\langle \boldsymbol{u}, t_i \rangle \in \boldsymbol{M} \times \Delta t$$

Shortly: environment of a system at a given moment is a set of all objects located in that time of of the system and each of these objects has in that time a certain relation with an element of the system.

D 10. A set **S** is said to be *impuls* of system $\mathscr{S}$ at a moment t_i, (symbolically: $\langle \mathbf{S}, \mathscr{S}, t_i \rangle \in \mathscr{Imp}$), iff for every element **x** and each property V holds: whether

$$\langle \mathbf{x}, \mathscr{S}, t_i \rangle \in \mathscr{Input}$$

$$\langle \mathbf{x}, t_i \rangle \quad \in V \times T$$

when there is time interval Δt so that $t_i \in \Delta t$, $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{Syst}$ then

$$\langle V, t_i \rangle \in \mathscr{S} \times \Delta t .$$

Briefly: impuls of a system at a considered moment is set of all properties of input elements of this system at that moment.

Let us define now notion of "*response* of system":

D 11. A set **S**$'$ is defined to be *response* of system $\mathscr{S}$ at a moment t_i (symbolically: $\langle \mathbf{S}', \mathscr{S}, t_i \rangle \in \mathscr{Resp}$), iff for every object **y** and each property W holds:
whether

$$\langle \mathbf{y}, \mathscr{S}, t_i \rangle \in \mathscr{Output}$$

$$\langle \mathbf{y}, t_i \rangle \quad \in W \times T$$

where there exists time interval Δt so that $t_i \in \Delta t$, $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{Syst}$ then

$$\langle W, t_i \rangle \in \mathbf{S}' \times \Delta t .$$

In proposed definition: response of a given system at a particular moment is set of all its output elements properties of the system at that moment.

THE CONCEPT OF SUBSYSTEM WITH A FEW ILLUSTRATIONS

D 12. System $\mathscr{S}'$ is said to be *subsystem* of system $\mathscr{S}$ at a moment t_i, (symbolically: $\langle \mathscr{S}', \mathscr{S}, t_i \rangle \in \mathscr{Subsyst}$), iff the following conditions are met:

a) there are time intervals $\Delta t, \Delta t', \Delta t' \subseteq \Delta t$, $\langle \mathscr{S}', \Delta t' \rangle \in \mathscr{Syst}$

$$\langle \mathscr{S}, \Delta t \rangle \in \mathscr{Syst} , \quad t_i \in \Delta t' ,$$

b) there exist sets $\boldsymbol{U}, \boldsymbol{U}', \mathscr{R}, \mathscr{R}'$ so that $\mathscr{S}' = \langle \boldsymbol{U}', \mathscr{R}' \rangle$, $\mathscr{S} = \langle \boldsymbol{U}, \mathscr{R} \rangle$ and $\boldsymbol{U}' \subseteq \boldsymbol{U}$,

c) there exist pairs of transformations (mappings) where: $\langle Z_0, Z_1 \rangle$ Z_0 transforms set of all members of respective sets of the type $R_k^{(j)} \in \mathscr{R}$ onto empty set and if it is not so for some of them, then they are transformed by Z_1.

$\boldsymbol{Z}_1$ uniquely assigns individual elements of these members (i.e. particular elements of the system) their Z_1-transformations, which are again elements of the system. To each set $R_k^{(j)} \in \mathscr{R}$ with terms of the type $\langle a_1, a_2, \ldots, a_j, t_i \rangle$ is thus uniquely associated a set $R_l'^{(j)} \in \mathscr{R}'$ with terms of the type

$$\langle Z_1(a_1), Z_1(a_2), \ldots Z_1(a_j), t_i \rangle .$$

d) $\mathscr{R}'$ is a set of all sets $R_l'^{(j)}$ which are the result of the mapping $\langle Z_0, Z_1 \rangle$ of all sets $R_k^{(j)} \in \mathscr{R}$ of the system $\mathscr{S}$.

e) $\boldsymbol{U}'$ is a set of elements of the type $Z_1(a)$ which are the result of Z_1-transformation of all elements of the system $\mathscr{S}$.

In proposed definition: $\mathscr{S}'$ is called "subsystem of system $\mathscr{S}$ at a moment t_i". if $\mathscr{S}'$ exists as a system within existentional time limits of system $\mathscr{S}$, if its universe $\boldsymbol{U}'$ is subset of universe $\boldsymbol{U}$, set of properties or relations $\mathscr{R}'$ is obtained either by omitting some sets from $\mathscr{R}$ of system $\mathscr{S}$ and sets from $\mathscr{R}'$ are properties or relations of Z_1--images of elements of corresponding properties or relations from $\mathscr{R}$ respectively.

Let me stress that proposed notion of subsystem is defined more generally than usually. The set $\mathscr{R}'$ does not have to be namely a subset as, as a rule, required. I have formed this generalization because of further coordination between concept of mapping with notion of "transformation creating homomorphy of systems".

If Z_1 is identical mapping then the concept of subsystem comes to traditional one with common demand $\mathscr{R}' \subseteq \mathscr{R}$.

Mapping $\langle Z_0, Z_1 \rangle$ can be chosen from various standpoints. Accordingly we can later divide given system into a sequence of respective subsystems. Mapping in practice is selected due to significance of properties and relations of system which is being divided.

Choice of mapping can be for instance directed by selected relation "to be less substantial than" which is ordering class of properties and relations. We choose a property or a relation as a lower ("lowest") one. Such a subsystem defined in this way encloses only those properties and relations from $\mathscr{R}$ which are beyond this lower limit (boundary) of "to be substantial".

For example let there be a very simple system (time factor not considered)

$$\mathscr{S}_1 = \langle \boldsymbol{U}_1, \mathscr{R}_1 \rangle$$
$$\boldsymbol{U}_1 = \{a_1, a_2, a_3\}$$
$$R_1 = \{F, G, R_1, R_2\}$$
$$F = \{a_1, a_3\} \; ; \quad G = \{a_2, a_3\}$$
$$R_1 = \{\langle a_1, a_2 \rangle, \langle a_2, a_3 \rangle, \langle a_3, a_2 \rangle\}$$
$$R_2 = \{\langle a_1, a_2 \; a_3 \rangle, \langle a_2, a_3, a_2 \rangle\}$$

Let a chosen mapping assign:

to set $F = \{a_1, a_3\}$ as its image $F' = \{a_3\}$

to set $G = \{a_2, a_3\}$ as its image $G' = \{a_2, a_3\}$

to set R_1 as its image $R_1' = \{\langle a_2, a_2\rangle, \langle a_3, a_2\rangle\}$

to set R_2 as its image $R_2' = \{\langle a_2, a_3, a_2\rangle\}$

then $\boldsymbol{U}_1' = \{a_2, a_3\}$, $\mathscr{R}_1' = \{F', G', R_1', R_2'\}$ is subsystem $\mathscr{S}_1' = \langle \boldsymbol{U}_1', \mathscr{R}_1'\rangle$.

Another simple example can be system $\mathscr{S}_2$ existing in a certain time period:

$$\mathscr{S}_2 = \langle\{a_1, a_2, a_3, a_4, a_5\}, \ \{R_1^{(1)}, R_2^{(1)}, R_3^{(1)}, R_1^{(2)}, R_2^{(2)}, R_1^{(3)}, R_2^{(3)}\}\rangle,$$

where

$$R_1^{(1)} = \{a_1, a_3, a_5\}, \quad R_2^{(1)} = \{a_2, a_4\}, \quad R_3^{(1)} = \{a_1, a_4\}$$

$$R_1^{(2)} = \{\langle a_1, a_4\rangle, \langle a_1, a_3\rangle\}, \quad R_2^{(2)} = \{\langle a_3, a_5\rangle, \langle a_2, a_1\rangle\}$$

$$R_1^{(3)} = \{\langle a_2, a_1, a_5\rangle, \langle a_1, a_3, a_5\rangle, \langle a_3, a_4, a_5\rangle\}$$

$$R_2^{(3)} = \{\langle a_4, a_5, a_1\rangle, \langle a_2, a_2, a_1\rangle\}$$

Let us choose a mapping on the base of relation "to be less substantial than" ordering set of relations and properties of system $\mathscr{S}_2$ as follows:

$$\langle R_2^{(2)}, R_2^{(1)}, R_1^{(1)}, R_2^{(3)}, R_3^{(1)}, R_1^{(2)}, R_1^{(3)}\rangle$$

Let property $R_1^{(1)}$ be a limit (boundary) for a selection from this set. In this way there is defined subsystem $\mathscr{S}_2'$ existing within existentional limits of system $\mathscr{S}_2$:

$$\mathscr{S}_2' = \langle\{a_1, a_2, a_3, a_4, a_5\}, \{R_3^{(1)}, R_1^{(2)}, R_1^{(3)}, R_2^{(3)}\}\rangle$$

For the same relation "to be less substantial than", but different boundary choice there will be defined a different subsystem. If property $R_3^{(1)}$ is considered as another boundary, then subsystem $\mathscr{S}_2''$ will be defined and existing within existentional limits of system $\mathscr{S}_2$ as follows:

$$\mathscr{S}_2'' = \langle\{a_1, a_2, a_3, a_4, a_5\}, \{R_1^{(2)}, R_1^{(3)}\}\rangle$$

It is obvious that when using various types of ordering and choice of lower boundary of substantionality, we can obtain distinct subsystems as to their relative significance.

As another illustration let me introduce "production" system $\mathscr{S}_v$ whose universe $\boldsymbol{U}_v$ is a set of machine tools and transport devices (time factor still omitted).

The universe $\boldsymbol{U}_v$ will be:

$$\boldsymbol{U}_v = \{s_1, s_2, s_3, \ldots, s_{40}\}$$

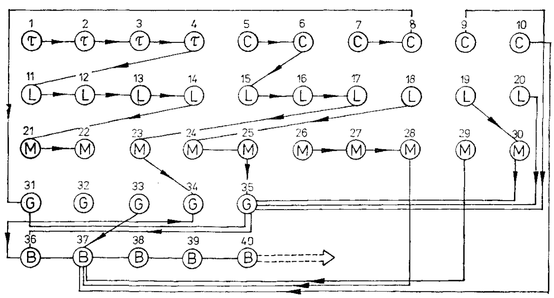
the respective elements:

$s_1, s_2, \ldots, s_{10}$	are travelling cranes	$(s_1, s_2, \ldots, s_{10}) \in C$
$s_{11}, s_{12}, \ldots, s_{20}$	are lathes	$(s_{11}, s_{12}, \ldots, s_{20}) \in L$
$s_{21}, s_{22}, \ldots, s_{30}$	are milling machines	$(s_{21}, s_{22}, \ldots, s_{30}) \in M$
$s_{31}, s_{32}, \ldots, s_{35}$	are grinding machines	$(s_{31}, s_{32}, \ldots, s_{35}) \in G$
$s_{36}, s_{37}, \ldots, s_{40}$	are moving belts	$(s_{36}, s_{37}, \ldots, s_{40}) \in B$

Corresponding pairs of machines or devices are in a relation "follow each other in production operation" as follows:

$$\mathscr{R}_p = \{\langle s_1, s_2\rangle, \langle s_2, s_3\rangle, \langle s_3, s_4\rangle, \langle s_5, s_6\rangle, \langle s_7, s_8\rangle, \langle s_9, s_{11}\rangle, \langle s_{11}, s_{12}\rangle, \langle s_{12}, s_{13}\rangle, \langle s_{13}, s_{14}\rangle, \langle s_6, s_{15}\rangle, \langle s_{15}, s_{16}\rangle, \langle s_{16}, s_{17}\rangle, \langle s_{14}, s_{21}\rangle, \langle s_{21}, s_{22}\rangle, \langle s_{17}, s_{23}\rangle, \langle s_{19}, s_{24}\rangle, \langle s_{24}, s_{25}\rangle, \langle s_{26}, s_{27}\rangle, \langle s_{27}, s_{28}\rangle, \langle s_{19}, s_{30}\rangle, \langle s_8, s_{31}\rangle, \langle s_{22}, s_{32}\rangle, \langle s_{32}, s_{33}\rangle, \langle s_{23}, s_{34}\rangle, \langle s_9, s_{35}\rangle, \langle s_{20}, s_{35}\rangle, \langle s_{25}, s_{35}\rangle, \langle s_{27}, s_{35}\rangle, \langle s_{30}, s_{35}\rangle, \langle s_{31}, s_{35}\rangle, \langle s_{34}, s_{36}\rangle, \langle s_{35}, s_{36}\rangle, \langle s_{36}, s_{37}\rangle, \langle s_{28}, s_{37}\rangle, \langle s_{29}, s_{37}\rangle, \langle s_{33}, s_{37}\rangle, \langle s_{10}, s_{37}\rangle, \langle s_{37}, s_{38}\rangle, \langle s_{38}, s_{39}\rangle, \langle s_{39}, s_{40}\rangle\} .$$

The whole production system we can plot in this way:



Input elements are assumed to be: $s_1, s_5, s_7, s_9, s_{10}, s_{18}, s_{19}, s_{20}, s_{26}, s_{29}$. The system $\mathscr{S}_v$ can be characterized within its function period as a pair:

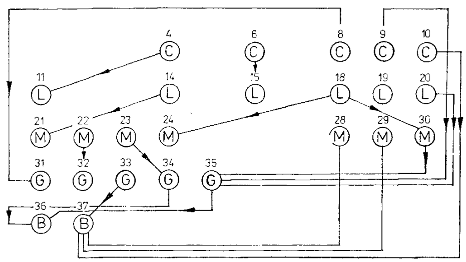
$$\mathscr{S}_v = \langle\{s_1, s_2, \ldots, s_{40}\}, \{C, L, M, G, B, \mathscr{R}_p\}\rangle .$$

The system $\mathscr{S}_v$ can be divided into subsystems from various viewpoints. For instance, let a mapping be given by distribution of relation $\mathscr{R}_p$ into those members-

-pairs of the class $\mathscr{R}_p$, whose elements differ as to their properties from the class $\{C, L, M, G, B\}$. The elements from this class with the same properties let be identified. Thus we get subsystem:

$$\mathscr{S}'_v = \langle\{s_4, s_6, s_8, s_9, s_{10}, s_{11}, s_{14}, s_{15}, s_{18}, s_{19}, s_{20}, s_{21}, s_{22}, s_{23}, s_{24}, s_{28}, s_{29}, s_{30}, s_{31}, s_{32}, s_{33}, s_{34}, s_{35}, s_{36}, s_{37}\}, \{C, L, M, G, B, R'_p\}\rangle ,$$

$$\mathscr{R}'_p = \{\langle s_4, s_{11}\rangle, \langle s_6, s_{15}\rangle, \langle s_8, s_{31}\rangle, \langle s_9, s_{35}\rangle, \langle s_{20}, s_{37}\rangle, \langle s_{14}, s_{21}\rangle, \langle s_{18}, s_{24}\rangle, \langle s_{19}, s_{30}\rangle, \langle s_{20}, s_{35}\rangle, \langle s_{23}, s_{34}\rangle, \langle s_{22}, s_{32}\rangle, \langle s_{28}, s_{37}\rangle, \langle s_{29}, s_{37}\rangle, \langle s_{30}, s_{35}\rangle, \langle s_{33}, s_{37}\rangle, \langle s_{34}, s_{36}\rangle, \langle s_{35}, s_{36}\rangle\} .$$



At this distribution we are interested in only those proceeding sequences of work operations, which take place between different kinds of machines or devices.

When approaching the problem from another standpoint, we can form partition of the class $\{C, L, M, G, B\}$ by partly ordering relation which formes there subclasses:

$$\{C\}, \{L, M, G\}, \{B\} .$$

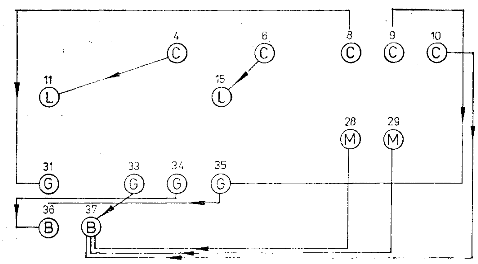
Let us divide the relation $\mathscr{R}_p$ so that we take out from it merely those pairs of elements as substantial once, which belong to distinct subclasses under consideration.

Thus we obtain subsystem:

$$\mathscr{S}''_v = \langle\{s_4, s_6, s_8, s_9, s_{10}, s_{11}, s_{15}, s_{28}, s_{29}, s_{31}, s_{33}, s_{34}, s_{35}, s_{36}, s_{37}\} , \{C, L, M, G, B, \mathscr{R}''_p\}\rangle ,$$

where

$$\mathscr{R}''_p = \{\langle s_4, s_{11}\rangle, \langle s_6, s_{15}\rangle, \langle s_8, s_{31}\rangle, \langle s_9, s_{35}\rangle, \langle s_{10}, s_{37}\rangle, \langle s_{28}, s_{37}\rangle , \langle s_{29}, s_{37}\rangle, \langle s_{33}, s_{37}\rangle, \langle s_{34}, s_{36}\rangle, \langle s_{35}, s_{36}\rangle\} .$$



At this distribution and partition of $\mathscr{S}_v$ only those sequences of work operations following each other are interesting for us, which occur among groups of travelling cranes, machine tools and moving belts, but regardless of the situation inside these groups.

Let us choose finally such a mapping which transforms the class

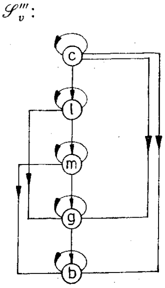
$$\{C, L, M, G, B, \mathscr{R}_p\} \quad \text{onto} \quad \{C, L, M, G, B, \mathscr{R}_p'''\}$$

where $\mathscr{R}_p'''$ is subclass of $\mathscr{R}_p$ enclosing only those ordered pairs of elements $s_1, s_2, \ldots$ $\ldots, s_{40}$ which remain after following identification of elements inside the sets C, L, M, G, B,

$$\begin{aligned} s_1 &= s_2 = \ldots = s_{10} = c\,, \\ s_{11} &= s_{12} = \ldots = s_{20} = l\,, \\ s_{21} &= s_{22} = \ldots = s_{30} = m\,, \\ s_{31} &= s_{32} = \ldots = s_{35} = g\,, \\ s_{36} &= s_{37} = \ldots = s_{40} = b\,, \end{aligned}$$

$$\mathscr{R}_v''' = \{\langle c, c\rangle, \langle c, l\rangle, \langle c, g\rangle, \langle c, b\rangle, \langle l, l\rangle, \langle l, m\rangle, \langle l, g\rangle, \langle m, m\rangle, \langle m, g\rangle, \langle m, b\rangle, \langle g, g\rangle, \langle g, b\rangle, \langle b, b\rangle\}\,.$$

obtained subsystem denote by $\mathscr{S}_v'''$:

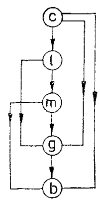


At this distribution we are interested in only relations among respective kinds of machines or devices (included relation with itself). Hence it is in fact a sequence of direct work relation among individual workshops.

When further simplifying the relation $\mathscr{R}_p$:

$$\mathscr{R}_p''' = \{\langle c, l\rangle, \langle c, g\rangle, \langle c, b\rangle, \langle l, m\rangle, \langle l, g\rangle, \langle m, g\rangle, \langle m, b\rangle, \langle g, b\rangle\}$$

we get subsystem $\mathscr{S}_v''''$, graphically:



enclosing only work relations among workshops, but not those inside particular workshops.

Time variable systems can be divided into subsystems also from time viewpoint. This distribution plays often an important role.

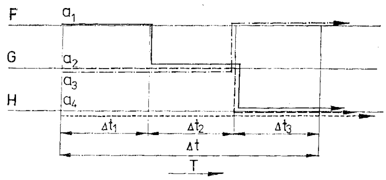
Simple example: Let there be defined a system $\mathscr{S}_T$ in time interval $\Delta t \subset T$ as follows:

$$\mathscr{S}_T = \langle \boldsymbol{U}_T, \mathscr{R}_T\rangle\,, \quad \boldsymbol{U}_T = \{a_1, a_2, a_3, a_4\}\,, \quad \mathscr{R}_T = \{F, G, H, R_1, R_2\}$$

and let there exist following subintervals of interval Δt:

$$\langle a_1, t_{i_1}\rangle \in F \times \Delta t \quad \text{for every} \quad t_{i_1} \in \Delta t_1$$
$$\langle a_1, t_{i_2}\rangle \in G \times \Delta t \quad \text{for every} \quad t_{i_2} \in \Delta t_2$$
$$\langle a_1, t_{i_3}\rangle \in H \times \Delta t \quad \text{for every} \quad t_{i_3} \in \Delta t_3$$
$$\langle a_2, t_{i_1}\rangle \in G \times \Delta t \quad \text{for every} \quad t_{i_1} \in \Delta t_1$$
$$\langle a_2, t_{i_2}\rangle \in G \times \Delta t \quad \text{for every} \quad t_{i_2} \in \Delta t_2$$
$$\langle a_2, t_{i_3}\rangle \in H \times \Delta t \quad \text{for every} \quad t_{i_3} \in \Delta t_3$$
$$\langle a_3, t_{i_1}\rangle \in G \times \Delta t \quad \text{for every} \quad t_{i_1} \in \Delta t_1$$
$$\langle a_3, t_{i_2}\rangle \in G \times \Delta t \quad \text{for every} \quad t_{i_2} \in \Delta t_2$$
$$\langle a_3, t_{i_3}\rangle \in F \times \Delta t \quad \text{for every} \quad t_{i_3} \in \Delta t_3$$
$$\langle a_4, t_i\rangle \in H \times \Delta t \quad \text{for every} \quad t_i \in \Delta t$$

Time changes of objects properties on the interval Δt can be illustrated by graph:



$$\langle a_1, a_2, a_3, t_{i_1}\rangle \in R_1 \times \Delta t \quad \text{for every} \quad t_{i_1} \in \Delta t_1$$

$$\langle a_1, a_2, a_3, t_{i_2}\rangle \in R_2 \times \Delta t \quad \text{for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_2, a_3, a_4, t_{i_1}\rangle \in R_1 \times \Delta t \quad \text{for every} \quad t_{i_1} \in \Delta t_1$$

$$\langle a_2, a_3, a_4, t_{i_2}\rangle \in R_1 \times \Delta t \quad \text{for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_2, a_3, a_1, t_{i_3}\rangle \in R_1 \times \Delta t \quad \text{for every} \quad t_{i_3} \in \Delta t_3$$

Chosen transformation let assign to an element F of set $\mathscr{R}_T$ its image such that

$$\langle a_1, t_{i_1}\rangle \in F \times \Delta t \quad \text{just only for every} \quad t_{i_1} \in \Delta t_1$$

and let further this mapping associate with an element G of set $\mathscr{R}_T$ its image such that

$$\langle a_2, t_{i_1}\rangle \in G \times \Delta t \quad \text{just only for every} \quad t_{i_1} \in \Delta t_1$$

$$\langle a_3, t_{i_1}\rangle \in G \times \Delta t \quad \text{just only for every} \quad t_{i_1} \in \Delta t_1$$

to every element H of set $\mathscr{R}_T$ its image so that

$$\langle a_4, t_{i_1}\rangle \in H \times \Delta t \quad \text{just only for every} \quad t_{i_1} \in \Delta t_1$$

and to each element R_1 of set R_T its image so that

$$\langle a_1, a_2, a_3, t_{i_1}\rangle \in R_1 \times \Delta t \quad \text{just only for every} \quad t_{i_1} \in \Delta t_1$$

$$\langle a_2, a_3, a_4, t_{i_1}\rangle \in R_1 \times \Delta t \quad \text{just only for every} \quad t_{i_1} \in \Delta t_1$$

and finally to element R_2 of set $\mathscr{R}_T$ as its image empty set. The subsystem defined in this way is

$$\mathscr{S}'_T = \langle \boldsymbol{U}'_T, \mathscr{R}'_T\rangle\,, \quad \boldsymbol{U}'_T = \{a_1, a_2, a_3, a_4\}\,, \quad \mathscr{R}'_T = \{F, G, H, R_1\}$$

where

F involves at every moment $t_{i_1} \in \Delta t_1$ as its elements an object a_1 and no other element from $\boldsymbol{U}'_T$,

G involves at every moment $t_{i_1} \in \Delta t_1$ as its elements objects a_2, a_3 and no others from $\boldsymbol{U}'_T$,

H involves at every moment $t_{i_1} \in \Delta t_1$ as its element an object a_4 and no other element from U'_T,

R_1 involves at every moment $t_{i_1} \in \Delta t_1$ as its elements triples $\langle a_1, a_2, a_3 \rangle$, $\langle a_2, a_3, a_4 \rangle$ and no other triples from U'_T.

Let us choose another transformation associating respective elements $F, G, H, R_1, R_2 \in \mathscr{R}_T$ their images so that F is associated with empty set

$$\langle a_1, t_{i_2} \rangle \in G \times \Delta t \quad \text{just only for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_2, t_{i_2} \rangle \in G \times \Delta t \quad \text{just only for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_3, t_{i_2} \rangle \in G \times \Delta t \quad \text{just only for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_4, t_{i_2} \rangle \in H \times \Delta t \quad \text{just only for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_2, a_3, a_4, t_{i_2} \rangle \in R_1 \times \Delta t \quad \text{just only for every} \quad t_{i_2} \in \Delta t_2$$

$$\langle a_1, a_2, a_3, t_{i_2} \rangle \in R_2 \times \Delta t \quad \text{just only for every} \quad t_{i_2} \in \Delta t_2$$

So there is defined subsystem

$$\mathscr{S}''_T = \langle U''_T, \mathscr{R}''_T \rangle\,, \quad U''_T = \{a_1, a_2, a_3, a_4\}\,, \quad \mathscr{R}''_T = \{G, H, R_1, R_2\}$$

G encloses at every moment $t_{i_2} \in \Delta t_2$ as its element objects a_1, a_2 and no other elements from U''_T,

H encloses at every moment $t_{i_2} \in \Delta t_2$ as its element object a_1 and no other elements from U''_T,

R_1 encloses at every moment $t_{i_2} \in \Delta t_2$ as its element triple $\langle a_2, a_3, a_4 \rangle$ and no other triples from U''_T,

R_2 encloses at every moment $t_{i_2} \in \Delta t_2$ as its element triple $\langle a_1, a_2, a_3 \rangle$ and no other triples from U''_T.

Let us finally choose a mapping assigning to elements $F, G, H, R_1, R_2 \in \mathscr{R}_T$ their respective images so that

$$\langle a_3, t_{i_3} \rangle \in F \times \Delta t \quad \text{just only for every} \quad t_{i_3} \in \Delta t_3$$

$$\langle a_1, t_{i_3} \rangle \in H \times \Delta t \quad \text{just only for every} \quad t_{i_3} \in \Delta t_3$$

$$\langle a_2, t_{i_3} \rangle \in H \times \Delta t \quad \text{just only for every} \quad t_{i_3} \in \Delta t_3$$

$$\langle a_4, t_{i_3} \rangle \in H \times \Delta t \quad \text{just only for every} \quad t_{i_3} \in \Delta t_3$$

the image of G is empty set,

$$\langle a_2, a_3, a_1, t_{i_3} \rangle \in R_1 \times \Delta t \quad \text{just only for every} \quad t_{i_3} \in \Delta t_3$$

the image of R_2 is empty set.

In this way there is defined subsystem:

$$\mathcal{S}'''_T = \langle \boldsymbol{U}'''_T, \mathcal{R}'''_T \rangle\,, \quad \boldsymbol{U}'''_T = \{a_1, a_2, a_3, a_4\}\,, \quad \mathcal{R}'''_T = \{F, H, R_1\}$$

F has at every moment $t_{i_3} \in \Delta t_3$ as its element object a_3 and no other element from $\boldsymbol{U}'''_T$,

H has at every moment $t_{i_3} \in \Delta t_3$ as its elements objects a_1, a_2, a_4 and no other elements from $\boldsymbol{U}'''_T$,

R_1 has at every moment $t_{i_3} \in \Delta t_3$ as its element triple $\langle a_2, a_3, a_1 \rangle$ and no other triple of elements from $\boldsymbol{U}'''_T$.

It is obvious that just mentioned triple of mappings has enabled partition of system $\mathcal{S}_T$ into three subsystems $\mathcal{S}'_T, \mathcal{S}''_T, \mathcal{S}'''_T$, which can be classified as "development stages" of system $\mathcal{S}_T$. Original system has thus its "history", which we can describe precisely in time. Interval Δt can be divided, generally speaking, into n ordered subintervals and with increasing n even "ontology" of $\mathcal{S}_T$ development becomes greater.

Mentioned mapping can be chosen so that even time changes of "integral" system $\mathcal{S}_T$ as to its origin or termination substantial properties and relations would be envolved in particular time periods.

Given specifications D1 – D12 may enable exact description of large, in time developing, systems. These systems are called (perhaps not quite precisely) "dynamic systems".

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Dr. Milan Růžička, CSc., katedra matematiky strojní fakulty ČVUT (Department of Mathematics, Faculty of Mechanical Engineering — Czech Technical University), Karlovo nám. 13, 121 35 Praha 2. Czechoslovakia.