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On Modelling of Large Variable Systems of Higher Degree by Means of Language Systems

ROMAN BEK, MILAN RŮŽIČKA

There will be presented a few non-traditional definitions of several basic notions from a theory of complicated time variable systems modelling. In particular, a special concern is devoted to a relation between accurate language systems and real (ontic) ones.

1. PRELIMINARIES

The formulations in this article will be based on the set theory concept and principal terms of this theory will be used accordingly. Especially the following ones:

- Cartesian product (symbolically: $M_1 \times M_2$ for sets M_1, M_2),
- Cartesian power (symbolically: $M^{<n>}$ the n-th power of a set M),
- ordered set (symbolically: $\langle a_1, \ldots, a_n \rangle$, ordered set consisting of elements $a_1, \ldots, a_n$),
- elementshood of a set (symbolically: $a \in M$),
- intersection of sets (symbolically: $M_1 \cap M_2$ for sets M_1, M_2),
- subset (symbolically: $M_1 \subset M_2$ for sets M_1, M_2).

Further we assume the same distribution of ontic reality as accepted in the logical theory of types: individuals of the universe of a language reasoning we consider as entities of the zero degree, their sets and relations as entities of the first degree, sets of these sets and relations and relations among these sets and relations as entities of the second degree, ..., sets and relations of entities of the $(s - 1)$-th degree as entities of the s-th degree.

The language of language systems we shall discuss is built-up on the predicate-logical basis. Its predicate constants and variables resp. will be denoted by corresponding type symbols again within the requirements of the types theory. For instance: a predicate constant of the type ${}^{(s)}R$ denotes an ontic entity of the s-th degree. This language will also use time terms, constants and variables. These terms will join individual terms of the language. Statement: $\mathbf{R}^{(j)}(a_1, \ldots, a_j, t_i)$ will for example mean: "Objects $a_1, \ldots, a_j$ are at a moment t_i in relation $\mathbf{R}^{(j)}$".

We shall further suppose existence of a time structure containing a set of moments τ. Let this set be ordered by a time relation "forecoming", i.e. the precedence relation. Individual time intervals are ordered subsets of the set τ. Suppose, to respective moments from the set τ have been assigned real numbers by a convenient metric function. To the precedence relation corresponds then a relation "$<$" between these numbers.

2. DEFINITION OF THE NOTION "SYSTEM OF THE n-th DEGREE".

D1. An ordered pair $\langle U, \mathscr{R} \rangle$ forms at a time interval Δt system of the n-th degree (symbolically: $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst^{(n)}$) iff:

U is a set of some objects (= individuals = entities of zero degree),
$\mathscr{R}$ is a set of all entities of the form ${}^{(s)}R_k^{(j)}$, where for $s = 1$: ${}^{(s)}R_k^{(j)} \subset U^{<j>} \times \Delta t$ for every $s > 1$: there is at least one from $s_1, \ldots, s - 1$, where $s - 1$ exists, hence

$$ {}^{(s)}R_k^{(j)} \subset U^{<j_0>} \times \left\{ {}^{(s_1)}R_{k_{11}}^{(j_{11})}, \ldots, {}^{(s_1)}R_{k_{1u}}^{(j_{1u})} \right\} \times \left\{ {}^{(s_2)}R_{k_{21}}^{(j_{21})}, \ldots, {}^{(s_2)}R_{k_{2v}}^{(j_{2v})} \right\} \times $$
$$ \times \ldots \times \left\{ {}^{(s-1)}R_{k_{l1}}^{(j_{l1})}, \ldots, {}^{(s-1)}R_{k_{lz}}^{(j_{lz})} \right\} \times \Delta t , $$

$s_1, s_2, \ldots, s^{-1}$ may or may not be equal in pairs, all s_i from them which are not equal to $s - 1$ are less than $s - 1$.

$$ j = j_0 + j_{11} + \ldots + j_{1u} + j_{z1} + \ldots + j_{zv} + \ldots + j_{l1} + \ldots + j_{lz} , $$

the greatest s (from upper left index of these entities ${}^{(s)}R_k^{(j)} \in \mathscr{R}$) is equal to n. The set U is said to be "the universe of the system".

Remark. ${}^{(s)}R_k^{(j)} \in \mathscr{R}$ is $(j + 1)$-th terms relation (on its last place is always time term) the s-th degree among individuals from the universe of a system and/or other relations from $\mathscr{R}$ of the lower degree than s. Among them there must be however at least one relation of the $(s - 1)$-th degree. Denotation k in the lower index marks an order of relation ${}^{(s)}R_k^{(j)}$ among other $(j + 1)$-terms relations of the s-th degree belonging to $\mathscr{R}$.

Due to the logical principle of extensionality as it is known, we consider j-terms relations and sets of ordered j-tuples objects among those the given relations are defined, as identical ones. For $j = 1$ the relation results to a set of objects having corresponding property.

Proposed definition generalizes the notion "system", as it is commonly used. System is usually defined as a pair from the set of objects (the universe of system) and from a set of relations among these objects. In the suggested definition, system is specified with respect to a given time interval (within its limits given objects with corresponding relations are considered). Further we admit that in the set of relations there may be not only relations among individuals from the universe of system, but also relations among relations of individuals, relations of relations among relations of individual etc. We hope that by doing so we may satisfy needs of the system theory operating with large variable systems and considering there entities of various degrees.

Examples

P 1. A system of the first degree is on an interval $\Delta t = \{t_1, t_2, t_3\}$ an ordered pair $\langle U, \mathcal{R} \rangle$, where:

$$U = \{a_1, a_2, a_3, a_4\},$$
$$\mathcal{R} = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_2^{(1)}, {}^{(1)}R_1^{(2)}\},$$

and

$${}^{(1)}R_1^{(1)} = \{\langle a_1, t_1 \rangle, \langle a_1, t_2 \rangle, \langle a_1, t_3 \rangle, \langle a_2, t_3 \rangle\}.$$
$${}^{(1)}R_2^{(1)} = \{\langle a_2, t_2 \rangle, \langle a_2, t_3 \rangle, \langle a_3, t_1 \rangle, \langle a_3, t_2 \rangle, \langle a_3, t_3 \rangle, \langle a_1, t_3 \rangle\},$$
$${}^{(1)}R_1^{(2)} = \{\langle a_1, a_2, t_1 \rangle, \langle a_3, a_2, t_2 \rangle, \langle a_1, a_4, t_2 \rangle, \langle a_1, a_4, t_3 \rangle\}.$$

Object a_1 hence belongs to a set ${}^{(1)}R_1^{(1)}$ (= has property ${}^{(1)}R_1^{(1)}$) during the whole period Δt, whereas an object a_2 belongs here (= taking on this property) later on at a moment t_3. Obviously: ${}^{(1)}R_1^{(1)} \subset U^{<1>} \times \Delta t$, similarly ${}^{(1)}R_2^{(1)}$. Objects a_1, a_2 are connected by a relation ${}^{(1)}R_1^{(2)}$ at a moment t_1, objects a_3, a_2 at a moment t_2, objects a_1, a_4 at moments t_2, t_3. Obviously: ${}^{(1)}R_1^{(2)} \subset U^{<2>} \times \Delta t$.

P 2. Let an ordered pair $\langle U_{O_1}, \mathcal{R}_{O_1} \rangle = \mathcal{S}_{O_1}$ be ontic system of the third degree on interval $\Delta t_1 = \langle t_1, t_2, t_3, t_4, t_5 \rangle$, where the universe

$$U_{O_1} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\},$$

set of relations

$$\mathcal{R}_{O_1} = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_2^{(1)}, {}^{(1)}R_3^{(1)}, {}^{(1)}R_1^{(2)}, {}^{(1)}R_2^{(2)}, {}^{(1)}R_3^{(2)}, {}^{(2)}R_1^{(1)}, {}^{(3)}R_1^{(4)}\}.$$

identification of relations:

$${}^{(1)}R_1^{(1)} = \{\langle a_1, t_1 \rangle, \langle a_1, t_2 \rangle, \langle a_1, t_3 \rangle, \langle a_1, t_4 \rangle, \langle a_1, t_5 \rangle, \langle a_2, t_4 \rangle, \langle a_2, t_5 \rangle\}$$

(obviously: ${}^{(1)}R_1^{(1)} \subset U_{O_1}^{<1>} \times \Delta t_1$),

$${}^{(1)}R_2^{(1)} = \{\langle a_2, t_1 \rangle, \langle a_2, t_3 \rangle, \langle a_2, t_5 \rangle, \langle a_4, t_1 \rangle, \langle a_4, t_2 \rangle, \langle a_2, t_5 \rangle\},$$
$${}^{(1)}R_3^{(1)} = \{\langle a_6, t_1 \rangle, \langle a_6, t_2 \rangle, \langle a_7, t_3 \rangle, \langle a_8, t_3 \rangle, \langle a_9, t_4 \rangle, \langle a_{10}, t_5 \rangle\},$$
$${}^{(1)}R_1^{(2)} = \{\langle a_1, a_1, t_2 \rangle, \langle a_1, a_2, t_1 \rangle, \langle a_2, a_1, t_1 \rangle, \langle a_3, a_5, t_3 \rangle, \langle a_5, a_3, t_3 \rangle, \langle a_7, a_9, t_4 \rangle, \langle a_9, a_7, t_4 \rangle\}$$

(obviously: ${}^{(1)}R_1^{(2)} \subset \boldsymbol{U}_{O_1}^{<2>} \times \Delta t_1$),

$$ {}^{(1)}R_2^{(2)} = \{\langle a_1, a_5, t_3\rangle, \langle a_1, a_8, t_1\rangle, \langle a_1, a_{10}, t_1\rangle, \langle a_1, a_9, t_3\rangle, \langle a_2, a_6, t_4\rangle, \langle a_5, a_9, t_3\rangle, \langle a_8, a_{10}, t_1\rangle\} , $$

$$ {}^{(2)}R_1^{(1)} = \{\langle {}^{(1)}R_1^{(1)}, t_1\rangle, \langle {}^{(1)}R_1^{(1)}, t_2\rangle, \langle {}^{(1)}R_1^{(1)}, t_3\rangle, \langle {}^{(1)}R_1^{(1)}, t_4\rangle, \langle {}^{(1)}R_1^{(1)}, t_5\rangle\} $$

(relation ${}^{(1)}R_1^{(1)}$ has for the whole period of time Δt_1 property ${}^{(2)}R_1^{(1)}$; obviously: ${}^{(2)}R_1^{(1)} \subset \{{}^{(1)}R_1^{(1)}\} \times \Delta t_1$),

$$ {}^{(3)}R_1^{(4)} = \{\langle {}^{(1)}R_2^{(1)}, {}^{(2)}R_1^{(1)}, {}^{(1)}R_3^{(1)}, a_5, t_3\rangle, \langle {}^{(1)}R_2^{(1)}, {}^{(2)}R_1^{(1)}, {}^{(1)}R_3^{(1)}, a_5, t_5\rangle\} $$

(relation ${}^{(1)}R_2^{(1)}$ is together with relation ${}^{(2)}R_1^{(1)}$, ${}^{(1)}R_3^{(1)}$ and object a_5 at moment t_3 in relation ${}^{(3)}R_1^{(4)}$, relation ${}^{(1)}R_2^{(1)}$ is together with relation ${}^{(2)}R_1^{(1)}$, ${}^{(1)}R_3^{(1)}$ and object a_5 at moment t_5 in relation ${}^{(3)}R_1^{(4)}$; obviously: ${}^{(3)}R_1^{(4)} \subset \boldsymbol{U}_{O_1}^{<1>} \times \{{}^{(1)}R_2^{(1)}, {}^{(1)}R_3^{(1)}\} \times \{{}^{(2)}R_1^{(1)}\} \times \Delta t_1$).

P 3. Ontic system $\mathscr{S}_{O_2}$ of the second degree is on an interval $\Delta t_2 = \langle t_1, t_2, t_4, t_5\rangle$ and ordered pair $\langle \boldsymbol{U}_{O_2}, \mathscr{R}_{O_2}\rangle$, where the universe

$$ \boldsymbol{U}_{O_2} = \{a_1, a_2, a_7, a_8, a_{10}\} $$

set of relations

$$ \mathscr{R}_{O_2} = \{{}^{(1)}R_1'^{(1)}, {}^{(1)}R_2'^{(1)}, {}^{(1)}R_1'^{(2)}, {}^{(1)}R_2'^{(2)}, {}^{(2)}R_1'^{(1)}\} $$

identification of relations

$$ {}^{(1)}R_1'^{(1)} = \{\langle a_1, t_1\rangle, \langle a_2, t_4\rangle\} , $$

$$ {}^{(1)}R_2'^{(1)} = \{\langle a_2, t_5\rangle\} , $$

$$ {}^{(1)}R_1'^{(2)} = \{\langle a_7, a_9, t_4\rangle, \langle a_9, a_7, t_4\rangle\} , $$

$$ {}^{(2)}R_1'^{(1)} = \{\langle {}^{(1)}R_1'^{(1)}, t_1\rangle, {}^{(1)}R_1'^{(1)}, t_4\rangle\} . $$

3. DEFINITION OF NOTION "SUBSYSTEM"

D 2. System $\mathscr{S}_2$ is at a time interval Δt_2 a subsystem of system $\mathscr{S}_1$ (symbolically: $\langle \mathscr{S}_2, \mathscr{S}_1, \Delta t_2\rangle \in \mathscr{S}ubsyst$) iff there exists Δt_1 so that $\Delta t_2 \subseteq \Delta t_1$,

$$ \langle \mathscr{S}_1, \Delta t_1\rangle \in \mathscr{S}yst^{(s_1)} , \quad \langle \mathscr{S}_2, \Delta t_2\rangle \in \mathscr{S}yst^{(s_2)} , $$

there are sets $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\mathscr{R}_1$, $\mathscr{R}_2$ so that

$$ \mathscr{S}_1 = \langle \boldsymbol{U}_1, \mathscr{R}_1\rangle , \quad \mathscr{S}_2 = \langle \boldsymbol{U}_2, \mathscr{R}_2\rangle , $$

there exists a mapping $\mathscr{Z}_t$ transforming from set of all moments into the same set so that to every moment $t \in \Delta t_1$ located in the field of given relation ${}^{(s_i)}R_k^{(j)} \in \mathscr{R}_1$ ($s_i \leqq s_1$) uniquelly assigning as its image again an element from field of this relation.

There is a mapping $\mathscr{Z}_0$ transforming from the universe U_1 into itself so that to every element $x \in U_1$ from the field of a given relation ${}^{(s_i)}R_k^{(j)} \in \mathscr{R}_1$ uniquelly is assigning as its image again an element from the field of this relation. For every s_i, $1 \leqq s_i \leqq s_1$ there further exists a couple of mappings $\mathscr{Z}_{s_i-1}$, $\mathscr{Z}_{s_i}$: $\mathscr{Z}_{s_i-1}$ transforms from set of all elements from the field of given relation ${}^{(s_i)}R_k^{(j)} \in \mathscr{R}_1$ (apart from elements from set Δt_1 and U_1) into the same set using this way: to every object it assignes uniquelly as its image again some object of the same degree from the field of this relation. This mapping regards all objects from fields of all relations belonging to s_i. For $s_i = 1$, $\mathscr{Z}_{s_i-1}$ is identical with mapping $\mathscr{Z}_O$. $\mathscr{Z}_{s_i}$ maps from set of all relations s_i-th degree belonging to $\mathscr{R}_1$, into the set of all its subrelations like this: to every relation of the type ${}^{(s_i)}R_k^{(j)} \in \mathscr{R}_1$ uniquelly assignes as its image some of its subrelation – including empty subrelation. For each of these mapping, following condition holds: to no object which is being mapped are by two distinct mappings associated two different images. If there is assigned to α_n as its image α_m by some mapping, then it must not appear again as image of some object by any mapping and the given object and its image must be identical (i.e. it must be image of itself).

Mentioned mappings $\mathscr{Z}_{s1}(\mathscr{Z}_{s1-1}(\mathscr{Z}_{s1-1}(\ldots(\mathscr{Z}_1(\mathscr{Z}_0)) \ldots)))$ associate with individual relations of the type ${}^{(s_i)}R_k^{(j)} \in \mathscr{R}_1$ uniquelly relations of the type ${}^{(s_i)}R_l^{(j)} \in \mathscr{R}_2$. The universe U_2 is set of images of mapping $\mathscr{Z}_0$, Δt_2 is ordered set of images of $\mathscr{Z}_t$.

Remark. Mentioned definition of notion "subsystem" is more general from a few viewpoints than usual specification. First there is an introduction and transformation of moments from given time structure (given by mapping $\mathscr{Z}_t$). Result of mapping is a time interval of the existence of given system ($\mathscr{S}_2$) as a subsystem of other system ($\mathscr{S}_1$). This mapping has been enclosed into the definition for a better correspondence with intuitive imaginations related with construction of subsystem in epistemic process over given system under investigation. We often cannot distinguish two different moments located near to each other because of very limited technical equipment when studying a variable system. In practice we make these moments identical. We consider such a (defined) system as a subsystem of studied system.

Example. Let a system $\mathscr{S}_1$ be defined on an time interval $\Delta t_1 = \langle t_1, t_2, t_3, t_4, t_5, t_6 \rangle$. Let mapping $\mathscr{Z}_t$ associate:

$$t_1 \to t_1 ,$$

$$t_2 \to t_2 ,$$

$$t_3 \to t_1 ,$$

$$t_4 \to t_4 ,$$

$$t_5 \to t_2 ,$$

$$t_6 \to t_6 .$$

Interval $\Delta t_2 = \langle t_1, t_2, t_4, t_6 \rangle$ can be hence a period of existence of some system $\mathscr{S}_2$ as a subsystem $\mathscr{S}_1$ (if the other conditions of definitions are met).

Secondly, there is an analogous transformation of elements from the universe U_1 of system $\mathscr{S}_1$ given by mapping $\mathscr{Z}_0$. There are similar reasons as in the previous case. Frequently we cannot identify two distinct elements from the universe of studied system and therefore we make them identical. Sometimes we do such a simplification even in a case we can distinguish elements under consideration.

Example. Let the universe U_1 of a system $\mathscr{S}_1$ be formed by a set of elements $\{a_1, a_2, a_3, a_4, a_5\}$. Let mapping $\mathscr{Z}_0$ assign:

$$a_1 \to a_3\,,$$

$$a_2 \to a_2\,,$$

$$a_3 \to a_3\,,$$

$$a_4 \to a_3\,,$$

$$a_5 \to a_2\,.$$

The set $U_2 = \{a_2, a_3\}$ forms the universe of system $\mathscr{S}_2$ which is a subsystem $\mathscr{S}_1$, so far the other conditions of definition are satisfied.

It is also obvious that due to specification of $\mathscr{Z}_0$ always holds: if $\mathscr{S}_2$ is a subsystem of $\mathscr{S}_1$, then the universe U_2 of system $\mathscr{S}_2$ is subset of the universe U_1 of system $\mathscr{S}_1$. This statement is a part of traditional specification of notion "subsystem".

Thirdly, there are analogous transformations of those objects from relations of given system which are relations themselves (of lower degrees). For every degree of these relations there is a mapping (of a form $\mathscr{Z}_{s_i-1}$), having similar function as the transformation $\mathscr{Z}_0$.

Example. Let a system $\mathscr{S}_1$ have the universe U_1 as in the previous example and his set of relations be

$$\mathscr{R}_1 = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_2^{(1)}, {}^{(2)}R_1^{(1)}, {}^{(2)}R_1^{(2)}\}\,.$$

(Let relations ${}^{(1)}R_1^{(1)}, {}^{(1)}R_2^{(1)}$ belong to fields of relations ${}^{(2)}R_1^{(1)}, {}^{(2)}R_1^{(2)}$). Let mapping $\mathscr{Z}_{2-1}$ assign:

$${}^{(1)}R_1^{(1)} \to {}^{(1)}R_1^{(1)}\,,$$

$${}^{(1)}R_2^{(1)} \to {}^{(1)}R_1^{(1)}\,.$$

Let the unique image ${}^{(1)}R_1^{(1)}$ belong to set $\mathscr{R}_2$ of system $\mathscr{S}_2$. If $\mathscr{S}_2$ satisfies all other conditions of definition, then $\mathscr{S}_2$ is subsystem of $\mathscr{S}_1$.

Further there is a mapping $\mathscr{Z}_{s_i}$ defined for every degree (greater than zero) of relation from set $\mathscr{R}_1$ of system $\mathscr{S}_1$. In epistemic process over the system $\mathscr{S}_1$ we can meet (or only specify) merely parts of relations, some relations even we vanish completely.

Example. Let relations of system $\mathscr{S}_1$ from recently mentioned three examples be:

$$^{(1)}R_1^{(1)} = \{\langle a_1, t_2\rangle, \langle a_1, t_3\rangle, \langle a_2, t_3\rangle, \langle a_3, t_4\rangle\} ,$$

$$^{(1)}R_2^{(1)} = \{\langle a_1, t_4\rangle, \langle a_3, t_4\rangle, \langle a_3, t_5\rangle, \langle a_4, t_2\rangle, \langle a_4, t_5\rangle, \langle a_5, t_6\rangle\} ,$$

$$^{(2)}R_1^{(1)} = \{\langle {}^{(1)}R_1^{(1)}, t_1\rangle, \langle {}^{(1)}R_1^{(1)}, t_3\rangle, \langle {}^{(1)}R_1^{(2)}, t_5\rangle, \langle {}^{(1)}R_1^{(2)}, t_6\rangle\} ,$$

$$^{(2)}R_1^{(2)} = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(1)}, t_3\rangle, \langle {}^{(1)}R_2^{(1)}, {}^{(1)}R_1^{(1)}, t_6\rangle\} .$$

Let mapping $\mathscr{Z}_1$ assign: to relation $^{(1)}R_1^{(1)}$ its image $^{(1)}R_1'^{(1)} = \{\langle a_1, t_2\rangle, \langle a_3, t_4\rangle\}$ (relation $^{(1)}R_2^{(1)}$ has been already eliminated by mapping $\mathscr{Z}_{2-1}$ to $^{(1)}R_1^{(1)}$). Let mapping $\mathscr{Z}_2$ assign: to relation $^{(2)}R_1^{(1)}$ its image $^{(2)}R_1'^{(1)} = \{\langle {}^{(1)}R_1^{(1)}, t_1\rangle, \langle {}^{(1)}R_1^{(2)}, t_5\rangle\}$ to relation $^{(2)}R_1^{(2)}$ its image $^{(2)}R_1'^{(2)} = \emptyset$ (empty subrelation, i.e. empty set of ordered triples). (Mapping $\mathscr{Z}_{2-1}$ is obviously not identical with mapping $\mathscr{Z}_1$!) It is clear that by simplification of specification for mapping $\mathscr{Z}_{s_i}$ in this way: $\mathscr{Z}_{s_i}$ assigns to every relation of the s_i-th degree, belonging to $\mathscr{R}_1$ of system $\mathscr{S}_1$, the same relation or empty subrelation. Together with simplification $\mathscr{Z}_0$ to identical transformation, for set $\mathscr{R}_2$ of subsystem $\mathscr{S}_2$ will hold: $\mathscr{R}_2 \subset \mathscr{R}_1$. This statement forms the second part of traditional definition of notion "subsystem".

Example. Let system $\mathscr{S}_2$ be given on interval $\Delta t_2 = \langle t_1, t_2, t_3, t_4, t_6\rangle$, $\mathscr{S}_2 = \langle U_2, \mathscr{R}_2\rangle$, $U_2 = \{a_2, a_3\}$, $\mathscr{R}_2 = \{{}^{(1)}R_1'^{(1)}, {}^{(2)}R_1'^{(1)}\}$, where

$$^{(1)}R_1'^{(1)} = \{\langle a_3, t_2\rangle, \langle a_3, t_4\rangle\} ,$$

$$^{(2)}R_1'^{(1)} = \{\langle {}^{(1)}R_1'^{(1)}, t_1\rangle, \langle {}^{(1)}R_1'^{(1)}, t_2\rangle\} .$$

Since interval Δt_2 was obtained from interval Δt_1 by the use of mentioned mapping $\mathscr{Z}_t$ from previous examples, the universe U_2 was obtained from the universe U_1 by means of the mentioned mapping $\mathscr{Z}_0$ of those examples, set of relations $\mathscr{R}_2$ was obtained from set of relations $\mathscr{R}_1$ successively by the use of mappings $\mathscr{Z}_2$, $\mathscr{Z}_{2-1}$, $\mathscr{Z}_1$, $\mathscr{Z}_0$, $\mathscr{Z}_t$ from those examples, $\mathscr{S}_2$ is subsystem of system $\mathscr{S}_1$ on interval Δt_2.

Example. P 4. Ontic system $\mathscr{S}_{O_2}$ from example P 3 is subsystem of ontic system $\mathscr{S}_{O_1}$ from example P 2 on interval Δt_2 because of existence of mapping:

$$\begin{aligned} \mathscr{Z}_t : t_1 &\to t_1 , \\ t_2 &\to t_1 , \\ t_3 &\to t_4 , \\ t_4 &\to t_4 , \\ t_5 &\to t_5 ; \end{aligned}$$

$\mathscr{Z}_0$ let be identical mapping

$$\mathscr{Z}_1 : {}^{(1)}R_1^{(1)} \to {}^{(1)}R_1'^{(1)} ,$$
$$ {}^{(1)}R_2^{(1)} \to {}^{(1)}R_2'^{(1)} ,$$
$$ {}^{(1)}R_3^{(1)} \to \emptyset ,$$
$$ {}^{(1)}R_1^{(2)} \to {}^{(1)}R_1'^{(2)} ,$$
$$ {}^{(1)}R_2^{(2)} \to {}^{(1)}R_2'^{(2)} ,$$
$$ {}^{(1)}R_3^{(2)} \to \emptyset ,$$
$$ {}^{(1)}R_1^{(3)} \to \emptyset .$$

Let $\mathscr{Z}_{2-1}$ be identical mapping,
$\mathscr{Z}_2 : {}^{(2)}R_1^{(1)} \to {}^{(2)}R_1'^{(1)}$,
$\mathscr{Z}_{3-1}$ be identical mapping,
$\mathscr{Z}_3 : {}^{(3)}R_1^{(4)} \to \emptyset$.

4. DEFINITION OF NOTION "*L*-SYSTEM"

D 3. System $\mathscr{S}_L = \langle \boldsymbol{U}_L, \mathscr{R}_L \rangle$, $\langle \mathscr{S}_L, \Delta t \rangle \in \mathscr{S}yst^{(1)}$ is said to be $\boldsymbol{L}$-system on interval Δt iff $\boldsymbol{U}_L$ is set of non-logical constants of language $\boldsymbol{L}$, $\mathscr{R}_L$ is set of sets of the type $R_{L_k}^{(j)}$ correctly formed statements of language $\boldsymbol{L}$ holding in time interval Δt, where each from sets $R_{L_k}^{(j)} \in \mathscr{S}_L$ consists of statements of the type $R_{L_{ki}}^{(j)}$ where $R_{L_{ki}}^{(j)} \subset \boldsymbol{U}_L^{<j>} \times \times \Delta t^{<\tau>} \times \mathscr{L}^{<l>}$,

and j is number of occurences of non-logical constants (apart from time constant in this statement,
k_i is ordering number of this statement in set $\mathbf{R}_{L_k}^{(j)}$,
$\boldsymbol{\Delta t}$ is ordered set of names of all moments belonging to Δt
τ is number of occurences of symbols from $\boldsymbol{\Delta t}$ in this statement,
$\mathscr{L}$ is set of variables, logical connectives, logical operators and brackets in alphabets of language $\boldsymbol{L}$,
l is number of occurrences of symbols from set $\mathscr{L}$ in this statements.

Example. Let statement

$$\mathbf{R}_{L_{ki}}^{(3)} = \forall \mathbf{x}\, \exists \mathbf{y}((\mathbf{F}(\mathbf{x}, \mathbf{t}_1) \wedge \mathbf{R}(\mathbf{x}, \mathbf{y}, \mathbf{t}_1)) \to \mathbf{F}(\mathbf{y}, \mathbf{t}_1))$$

be correctly formed one of language $\boldsymbol{L}$, whose alphabet contains symbols: **F**, **R**, **x**, **y**, $\mathbf{t}_1$, $\wedge$, $\to$, $\forall$, $\exists$, (,) and holding on interval Δt. Let $\boldsymbol{U}_L$ be set of all non-logical constants of $\boldsymbol{L}$, hence $\mathbf{F}, \mathbf{R} \in U_L$, let $\mathbf{t}_1$ be name of moment $t_1 \in \Delta t$, let symbols **x**, **y**, $\wedge$, $\to$, $\forall$, $\exists$, (,) $\in \mathscr{L}$; then

$$R_{L_{ki}}^{(3)} \subset \boldsymbol{U}_L^{<3>} \times \boldsymbol{\Delta t}^{<3>} \times \mathscr{L}^{<20>} ,$$

holds because non-logical constants **F**, **R** occur in this statement on three places, time constant $\mathbf{t}_1$ occurs in this statement on three places, variables **x**, **y**, logical connectives $\wedge$, $\rightarrow$, operators $\forall$, $\exists$ and brackets (,) occur in this statement on twenty places.

Remark. "*L*-system" is specification of "system". It may be a language system, whose correctly formed sentences only describe empirical facts and among them there is not logical relation of deduction. However it may also be an axiomatic system containing some of its sentences as axioms (holding on a given interval Δt) and all of them correctly logically derived statements (hence valid also interval Δt).

We consider merely *L*-systems of the first degree, since in set $\mathscr{R}_L$, other members of $\mathscr{R}_L$ do not occur in any of its members as elements of corresponding relation We understand relations involved in set $\mathscr{R}_L$ (as elements of its sets) as relations among symbols of given language *L*, hence as syntactic formations.

Fact that set $\mathbf{R}_L$ does not contain directly these relations, but their sets, is motivated by aim of this reasoning (*L*-systems will be further associated with ontic systems as their language models). More accurate explanation comes out from the following consideration.

Examples

P 5. *L*-system of the first degree $\mathscr{S}_{L_1}$ is on interval $\Delta t_1 = \langle t_1, t_2, t_3, t_4, t_5 \rangle$ an ordered pair $\langle U_{L_1}, \mathscr{R}_{L_1} \rangle$, where

$$U_{L1} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_7, \mathbf{a}_8, \mathbf{a}_9, \mathbf{a}_{10}, {}^{(1)}\mathbf{R}_1^{(1)}, {}^{(1)}\mathbf{R}_2^{(1)}, {}^{(1)}\mathbf{R}_1^{(2)}, {}^{(1)}\mathbf{R}_2^{(2)}, {}^{(2)}\mathbf{R}_1^{(1)}\},$$

$$\mathscr{R}_{L_1} = \{\mathbf{R}_{L_1}^{(2)}, \mathbf{R}_{L_2}^{(2)}, \mathbf{R}_{L_3}^{(2)}, \mathbf{R}_{L_4}^{(2)}, \mathbf{R}_{L_5}^{(3)}, \mathbf{R}_{L_6}^{(3)}, \mathbf{R}_{L_7}^{(3)}, \mathbf{R}_{L_8}^{(3)}, \mathbf{R}_{L_9}^{(2)}\},$$

and

$$\mathbf{R}_{L_1}^{(2)} = \{\mathbf{R}_{L_{11}}^{(2)}\}.$$

$$\mathbf{R}_{L_{11}}^{(2)} = \forall \mathbf{x}\, \forall \mathbf{t}_i ({}^{(1)}\mathbf{R}_1^{(1)}(\mathbf{x}, \mathbf{t}_i) \rightarrow {}^{(1)}\mathbf{R}_2^{(1)}(\mathbf{x}, \mathbf{t}_{i+1}));$$

$$\mathbf{R}_{L_2}^{(2)} = \{\mathbf{R}_{L_{21}}^{(2)}, \mathbf{R}_{L_{22}}^{(2)}\},$$

$$\mathbf{R}_{L_{21}}^{(2)} = {}^{(1)}\mathbf{R}_1^{(1)}(\mathbf{a}_1, \mathbf{t}_1),$$

$$\mathbf{R}_{L_{22}}^{(2)} = {}^{(1)}\mathbf{R}_1^{(1)}(\mathbf{a}_2, \mathbf{t}_4);$$

$$\mathbf{R}_{L_3}^{(2)} = \{\mathbf{R}_{L_{31}}^{(2)}, \mathbf{R}_{L_{32}}^{(2)}\}$$

$$\mathbf{R}_{L_{31}}^{(2)} = {}^{(1)}\mathbf{R}_2^{(1)}(\mathbf{a}_1, \mathbf{t}_2),$$

$$\mathbf{R}_{L_{32}}^{(2)} = {}^{(1)}\mathbf{R}_2^{(1)}(\mathbf{a}_2, \mathbf{t}_5);$$

$$\mathbf{R}_{L_4}^{(2)} = \{\mathbf{R}_{L_{41}}^{(2)}\},$$

$$\mathbf{R}_{L_{41}}^{(2)} = \forall \mathbf{x}\, \forall \mathbf{y}\, \forall \mathbf{t}_i ({}^{(1)}\mathbf{R}_1^{(2)}(\mathbf{x}, \mathbf{y}, \mathbf{t}_i) \rightarrow {}^{(1)}\mathbf{R}_1^{(2)}(\mathbf{y}, \mathbf{x}, \mathbf{t}_i))$$

("${}^{(1)}\mathbf{R}_1^{(2)}$ is symmetric relation at every moment")

$$\mathbf{R}_{L_5}^{(3)} = \{\mathbf{R}_{L_{51}}^{(3)}\},$$

$$\mathbf{R}^{(3)}_{L_{51}} = \forall \mathbf{x}\, \forall \mathbf{y}\, \forall \mathbf{z}\, \forall \mathbf{t}_i((^{(1)}\mathbf{R}^{(2)}_2(\mathbf{x}, \mathbf{y}, \mathbf{t}_i) \wedge {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{x}, \mathbf{z}, \mathbf{t}_i)) \to {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{x}, \mathbf{z}, \mathbf{t}_i))$$

("$^{(1)}\mathbf{R}^{(2)}_2$ is transitiv relation at every moment")

$$\mathbf{R}^{(3)}_{L_6} = \{\mathbf{R}^{(3)}_{L_{61}}, \mathbf{R}^{(3)}_{L_{62}}\},$$

$$\mathbf{R}^{(3)}_{L_{61}} = {}^{(1)}\mathbf{R}^{(2)}_1(\mathbf{a}_7, \mathbf{a}_9, \mathbf{t}_4),$$

$$\mathbf{R}^{(3)}_{L_{62}} = {}^{(1)}\mathbf{R}^{(2)}_L(\mathbf{a}_9, \mathbf{a}_7, \mathbf{t}_4);$$

$$\mathbf{R}^{(3)}_{L_7} = \{\mathbf{R}^{(3)}_{L_{71}}, \mathbf{R}^{(3)}_{L_{72}}, \mathbf{R}^{(3)}_{L_{73}}\},$$

$$\mathbf{R}^{(3)}_{L_{71}} = {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{a}_1, \mathbf{a}_8, \mathbf{t}_1),$$

$$\mathbf{R}^{(3)}_{L_{72}} = {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{a}_8, \mathbf{a}_{10}, \mathbf{t}_1),$$

$$\mathbf{R}^{(3)}_{L_{73}} = {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{a}_1, \mathbf{a}_{10}, \mathbf{t}_1),$$

$$\mathbf{R}^{(3)}_{L_8} = \{\mathbf{R}^{(3)}_{L_{81}}\},$$

$$\mathbf{R}^{(3)}_{L_{81}} = \exists \mathbf{x}\, \exists \mathbf{t}_i(^{(1)}\mathbf{R}^{(1)}_1(\mathbf{x}, \mathbf{t}_i) \to {}^{(2)}\mathbf{R}^{(1)}_1(^{(1)}\mathbf{R}^{(1)}_1, \mathbf{t}_i));$$

$$\mathbf{R}^{(2)}_{L_9} = \{\mathbf{R}^{(2)}_{L_{91}}, \mathbf{R}^{(2)}_{L_{92}}\},$$

$$\mathbf{R}^{(2)}_{L_{91}} = {}^{(2)}\mathbf{R}^{(1)}_1(^{(1}\,\mathbf{R}^{(1)}_1, \mathbf{t}_1),$$

$$\mathbf{R}^{(2)}_{L_{92}} = {}^{(2)}\mathbf{R}^{(1)}_1(^{(1)}\mathbf{R}^{(1)}_1, \mathbf{t}_4).$$

It is obvious that in $\mathscr{S}_{L_1}$ appear statements:

$$\mathbf{R}^{(2)}_{L_{11}}, \mathbf{R}^{(2)}_{L_{41}}, \mathbf{R}^{(3)}_{L_{51}}, \mathbf{R}^{(3)}_{L_{81}}, \mathbf{R}^{(2)}_{L_{21}}, \mathbf{R}^{(2)}_{L_{22}}, \mathbf{R}^{(3)}_{L_{61}}, \mathbf{R}^{(3)}_{L_{71}}, \mathbf{R}^{(3)}_{L_{72}},$$

as axioms, other statements are deduced statements through relation of deduction:

$$\mathbf{R}^{(2)}_{L_{11}}, \mathbf{R}^{(2)}_{L_{21}} \vdash \mathbf{R}^{(2)}_{L_{31}},$$

$$\mathbf{R}^{(2)}_{L_{11}}, \mathbf{R}^{(2)}_{L_{22}} \vdash \mathbf{R}^{(2)}_{L_{32}},$$

$$\mathbf{R}^{(2)}_{L_{41}}, \mathbf{R}^{(3)}_{L_{61}} \vdash \mathbf{R}^{(3)}_{L_{62}},$$

$$\mathbf{R}^{(3)}_{L_{51}}, \mathbf{R}^{(3)}_{L_{71}}, \mathbf{R}^{(3)}_{L_{72}} \vdash \mathbf{R}^{(3)}_{L_{73}},$$

$$\mathbf{R}^{(3)}_{L_{81}}, \mathbf{R}^{(2)}_{L_{21}} \vdash \mathbf{R}^{(2)}_{L_{91}},$$

$$\mathbf{R}^{(3)}_{L_{81}}, \mathbf{R}^{(2)}_{L_{22}} \vdash \mathbf{R}^{(2)}_{L_{92}}.$$

P 6. ***L*****-system** of the first degree $\mathscr{S}_{L_2}$ is on interval $\Delta t_2 = \langle t_1, t_2, t_3, t_4, t_5 \rangle$ ordered pair $\langle \boldsymbol{U}_{L_2}, \mathscr{R}_{L_2} \rangle$, where:

$$\boldsymbol{U}_{L_2} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_7, \mathbf{a}_8, \mathbf{a}_{10}, {}^{(1)}\mathbf{R}^{(1)}_1, {}^{(1)}\mathbf{R}^{(1)}_2, {}^{(1)}\mathbf{R}^{(2)}_1, {}^{(1)}\mathbf{R}^{(2)}_2, {}^{(2)}\mathbf{R}^{(1)}_1\},$$

$$\mathscr{R}_{L_2} = \{\mathbf{R}'^{(2)}_{L_2}, \mathbf{R}'^{(2)}_{L_3}, \mathbf{R}'^{(3)}_{L_6}, \mathbf{R}'^{(3)}_{L_7}, \mathbf{R}'^{(2)}_{L_9}\},$$

 and

$$\begin{aligned}
\mathbf{R}'^{(2)}_{L_2} &= \{\mathbf{R}'^{(2)}_{L_{21}}, \mathbf{R}'^{(2)}_{L_{22}}\}, \\
&\quad \mathbf{R}'^{(2)}_{L_{21}} = {}^{(1)}\mathbf{R}^{(1)}_1(\mathbf{a}_1, \mathbf{t}_1), \quad \mathbf{R}'^{(2)}_{L_{22}} = {}^{(1)}\mathbf{R}^{(1)}_1(\mathbf{a}_2, \mathbf{t}_4); \\
\mathbf{R}'^{(2)}_{L_3} &= \{\mathbf{R}'^{(2)}_{L_{31}}\}, \\
&\quad \mathbf{R}'^{(2)}_{L_{31}} = {}^{(1)}\mathbf{R}^{(1)}_2(\mathbf{a}_2, \mathbf{t}_5); \\
\mathbf{R}'^{(3)}_{L_6} &= \{\mathbf{R}'^{(3)}_{L_{61}}, \mathbf{R}'^{(3)}_{L_{62}}\}, \\
&\quad \mathbf{R}'^{(3)}_{L_{61}} = {}^{(1)}\mathbf{R}^{(2)}_1(\mathbf{a}_7, \mathbf{a}_9, \mathbf{t}_4), \quad \mathbf{R}'^{(3)}_{L_{62}} = (\mathbf{a}_9, \mathbf{a}_7, \mathbf{t}_4); \\
\mathbf{R}'^{(3)}_{L_7} &= \{\mathbf{R}'^{(3)}_{L_{71}}, \mathbf{R}'^{(3)}_{L_{72}}, \mathbf{R}'^{(3)}_{L_{73}}\}, \\
&\quad \mathbf{R}'^{(3)}_{L_{71}} = {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{a}_1, \mathbf{a}_8, \mathbf{t}_9), \quad \mathbf{R}'^{(3)}_{L_{72}} = {}^{(1)}\mathbf{R}^{(2)}_{(2)}(\mathbf{a}_8, \mathbf{a}_{10}, \mathbf{t}_4), \\
&\quad \mathbf{R}'^{(3)}_{L_{73}} = {}^{(1)}\mathbf{R}^{(2)}_2(\mathbf{a}_1, \mathbf{a}_{10}, \mathbf{t}_1); \\
\mathbf{R}'^{(2)}_{L_9} &= \{\mathbf{R}'^{(2)}_{L_{91}}, \mathbf{R}'^{(2)}_{L_{92}}\}, \\
&\quad \mathbf{R}'^{(2)}_{L_{91}} = {}^{(2)}\mathbf{R}^{(1)}_1({}^{(1)}\mathbf{R}^{(1)}_1, \mathbf{t}_1), \\
&\quad \mathbf{R}'^{(2)}_{L_{92}} = {}^{(2)}\mathbf{R}^{(1)}_1({}^{(1)}\mathbf{R}^{(1)}_1, \mathbf{t}_4).
\end{aligned}$$

L-systems can also contain their subsystems, which are clearly again ***L***-systems. Notion "subsystem of ***L***-system" is again specified by definition D 2.

P 7. ***L***-system $\mathcal{S}_{L_2}$ (from example P 5.) is subsystem of ***L***-system $\mathcal{S}_{L_1}$ (from example P 4.) on interval $\Delta t_2 = \langle t_1, t_2, t_3, t_4, t_5 \rangle$ as a consequence of existence of the following mappings:

$\mathscr{Z}_t$: identical mapping

$\mathscr{Z}_0$: identical mapping

$$\begin{aligned}
\mathscr{Z}_{L_1} : \mathbf{R}^{(2)}_{L_1} &\to \emptyset, \\
\mathbf{R}^{(2)}_{L_2} &\to \mathbf{R}'^{(2)}_{L_2}, \\
\mathbf{R}^{(2)}_{L_3} &\to \mathbf{R}'^{(2)}_{L_3}, \\
\mathbf{R}^{(2)}_{L_4} &\to \emptyset, \\
\mathbf{R}^{(3)}_{L_5} &\to \emptyset, \\
\mathbf{R}^{(3)}_{L_6} &\to \mathbf{R}'^{(3)}_{L_6}, \\
\mathbf{R}^{(3)}_{L_7} &\to \mathbf{R}'^{(3)}_{L_7}, \\
\mathbf{R}^{(3)}_{L_8} &\to \emptyset, \\
\mathbf{R}^{(2)}_{L_9} &\to \mathbf{R}'^{(2)}_{L_9}.
\end{aligned}$$

(***L***-system $\mathcal{S}_{L_2}$ was created obviously from ***L***-system $\mathcal{S}_{L_1}$ by eliminating general sentences, existentional sentence $\mathbf{R}^{(1)}_{L_{81}}$. There remain statements describing individual concrete events).

5. DEFINITION OF NOTIONS "ISOMORPHY OF SYSTEMS", "GENERALIZED HOMOMORPHY OF SYSTEMS"

D 4. System $\mathcal{S}_2$ is at a time interval Δt isomorphic with system $\mathcal{S}_1$ (symbolically: $\langle \mathcal{S}_2, \mathcal{S}_1, \Delta t\rangle \in \mathcal{I}\mathit{ssyst}$),
iff there exist intervals Δt_1, Δt_2 so that $\Delta t = \Delta t_1 \cap \Delta t_2$,

$$\langle \mathcal{S}_1, \Delta t_1\rangle \in \mathcal{S}\mathit{yst}^{(s_1)}, \quad \langle \mathcal{S}_2, \Delta t_2\rangle \in \mathcal{S}\mathit{yst}^{(s_2)},$$

there exist sets $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\mathcal{R}_1$, $\mathcal{R}_2$ so that $\mathcal{S}_1 = \langle \boldsymbol{U}_1, \mathcal{R}_1\rangle$, $\mathcal{S}_2 = \langle \boldsymbol{U}_2, \mathcal{R}_2\rangle$.
to every relation of the type ${}^{(s)}R_k^{(j)} \in \mathcal{R}_1$ there exists at every moment $t_i \in \Delta t$ just one relation of the type ${}^{(s')}R_l'^{(j)} \in \mathcal{R}_2$ so that ${}^{(s)}R_k^{(j)}$ is isomorphic with respect to ${}^{(s')}R_l'^{(j)}$ and at the same time corresponding one-to-one mapping (setting up isomorphy) from set $\mathcal{R}_1$ preserves in set $\mathcal{R}_2$ relation of type superiority, moreover set $\mathcal{R}_2$ does not contain any more relation, which would not be an image of some relation from $\mathcal{R}_1$ at this mapping.

Definition of notion "relation of type superiority":

D. 5. Relation ${}^{(s_1)}R_k^{(j)}$ is in relation of type superiority with respect to relation ${}^{(s_2)}R_l^{(j)}$ iff $s_1 > s_2$ (i.e. degree of relation ${}^{(s_1)}R_k^{(j)}$ is greater than that of relation ${}^{(s_2)}R_l^{(j)}$).

D 6. System $\mathcal{S}_2$ is on time interval Δt generally homomorphic with respect to system $\mathcal{S}_1$, symbolically $\langle \mathcal{S}_2, \mathcal{S}_1, \Delta t\} \in \mathcal{G}\mathit{enhomsyst}$,
iff there exist intervals Δt_1, Δt_2 so that $\Delta t = \Delta t_1 \cap \Delta t_2$,

$$\langle \mathcal{S}_1\ \Delta t_1\rangle \in \mathcal{S}\mathit{yst}^{(s_1)}, \quad \langle \mathcal{S}_2, \Delta t_2\rangle \in \mathcal{S}\mathit{yst}^{(s_2)},$$

there exist set $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, $\mathcal{R}_1$, $\mathcal{R}_2$ so that $\mathcal{S}_1 = \langle \boldsymbol{U}_1, \mathcal{R}_1\rangle$, $\mathcal{S}_2 = \langle \boldsymbol{U}_2, \mathcal{R}_2\rangle$,
there exists single-valued mapping $\mathcal{Z}$ so that to every relation of the type ${}^{(s)}R_k^{(j)} \in \mathcal{R}_1$, it assigns at every moment $t_i \in \Delta t$ not more than one relation of the type ${}^{(s')}R_l'^{(j)} \in \mathcal{R}_2$ where $\mathcal{Z}$ is homomorphism of ${}^{(s)}\mathcal{R}_k^{(j)}$ into ${}^{(s')}R_l'^{(j)}$, $\mathcal{Z}$ preserved in set $\mathcal{R}_2$ among its relations the same correspondence of type superiority which was in the set $\mathcal{R}_1$ among patterns of these relations, moreover set $\mathcal{R}_2$ does not contain any other relation which would not associate with some relation from $\mathcal{R}_1$ due to this mapping.

Remark. Discussed relation among systems are called "generally homomorphic", since its definition is a certain generalization of well-known definition of two-relation homomorphy.

By comparing definitions D 2, D 4, D 6 we can obtain following fact: if system $\mathcal{S}_2$ is a subsystem of system $\mathcal{S}_1$ on interval Δt and system $\mathcal{S}_3$ is isomorphic with system $\mathcal{S}_2$ at Δt, then system $\mathcal{S}_3$ is generally homomorphic with system $\mathcal{S}_1$ at Δt.

6. DEFINITION OF NOTIONS "MODEL$_1$", "MODEL$_2$", "MODEL$_3$"

D 7. System $\mathcal{S}_2$ is model$_1$ (or model$_2$ or model$_3$ resp.) of system $\mathcal{S}_1$ on time interval Δt,
symbolically $\langle \mathcal{S}_2, \mathcal{S}_1, \Delta t\rangle \in \mathit{mod}_1$ (or mod_2 or mod_3 resp.)
iff there exist intervals Δt_1, Δt_2 so that $\Delta t = \Delta t_1 \cap \Delta t_2$,

$$\langle \mathcal{S}_1, \Delta t_1\rangle \in \mathcal{S}\mathit{yst}^{(s_1)}\,, \quad \langle \mathcal{S}_2, \Delta t_2\rangle \in \mathcal{S}\mathit{yst}^{(s_2)}\,,$$

moreover holds $\langle \mathcal{S}_2, \mathcal{S}_1, \Delta t\rangle \in \mathcal{I}\mathit{ssyst}$, $\langle \mathcal{S}_2, \mathcal{S}_1, \Delta t\rangle \in \mathcal{G}\mathit{enhomsyst}$ resp., there are such systems $\mathcal{S}'_1$, $\mathcal{S}'_2$ resp. that $\langle \mathcal{S}'_1, \mathcal{S}_1, \Delta t\rangle \in \mathcal{S}\mathit{ubsyst}$, $\langle \mathcal{S}'_2, \mathcal{S}_2, \Delta t\rangle \in \mathcal{S}\mathit{ubsyst}$ and simultaneously: $\langle \mathcal{S}'_2, \mathcal{S}'_1, \Delta t\rangle \in \mathcal{I}\mathit{ssyst}$.

7. DISCUSSION OVER THE DEFINED NOTIONS

We shall use the defined notions for discussion of notions "language model of ontic system" and "semantic model of language system".

D 8. *L*-system $\mathcal{S}_L$ is on time interval Δt language model of ontic system $\mathcal{S}_O$ iff

$$\langle \mathcal{S}_L, \mathcal{S}_O, \Delta t\rangle \in \mathit{mod}_1 \text{ (or } \mathit{mod}_2 \text{ or } \mathit{mod}_3 \text{ resp.)}$$

moreover there exists mapping setting-up isomorphy $\mathcal{S}_L$ to $\mathcal{S}_O$ (or isomorphy $\mathcal{S}_L$ to some subsystem $\mathcal{S}_O$ resp., or isomorphy of some subsystem $\mathcal{S}_L$ to a subsystem $\mathcal{S}_O$ resp.), on interval Δt and this mapping satisfies following requirements: to individual non-logical predicate constants of the type ${}^{(s)}\mathbf{R}_k^{(j)}$ from the universe of system $\mathcal{S}_L$ (or its subsystem resp.) individual relations of the type ${}^{(s)}R_l^{(j)}$ of system $\mathcal{S}_O$ (or its subsystem resp.) are mutually assigned, to individual non-logical constants of the type $\mathbf{a}_m$ from the universe of system $\mathcal{S}_L$ (or to its subsystem resp.) individual objects a_m from the universe $\mathcal{S}_O$ (or its subsystem resp.) are uniquelly and mutually assigned, to individual symbols of the type $\mathbf{t}_i$ (to names of time moments) from alphabet of language of system $\mathcal{S}_L$ individual time moments t_i from interval Δt of existence of system $\mathcal{S}_O$ are mutually uniquelly associated, hence: to individual statements $\mathcal{S}_L$ (or their subsystem resp.) of the type ${}^{(s)}\mathbf{R}_k^{(j)}(\mathbf{a}_1, \ldots, \mathbf{a}_j, \mathbf{t}_i)$ elements of relations of $\mathcal{S}_O$ of the type $\langle a_1, \ldots, a_j, t_i\rangle \in {}^{(s)}R_l^{(j)}$ are uniquelly associated for $s = 1$ and to particular statements $\mathcal{S}_L$ (or their subsystem resp.) of the type ${}^{(s)}\mathbf{R}_k^{(j)}$. $.({}^{(s_1)}\mathbf{R}_{k_1}^{(j_1)}, \ldots, {}^{(s_j)}\mathbf{R}_{k_j}^{(j_1+j)}, \mathbf{t}_i)$ elements of relations $\mathcal{S}_O$ of the type $\langle {}^{(s_1)}R_{k_1}^{(j_1)}, \ldots$ $\ldots, {}^{(s_p)}R_{k_p}^{(j_1+j)}, t_i\rangle \in {}^{(s)}R_l^{(j)}$ are uniquelly associated by this mapping(to individual logical connectives and operators from statements are assigned meanings in accordance with rules of interpretation known from logical semantic, similarly are associated individual and predicate variables from these statements their domains of variability).

D 9. System $\mathcal{S}$ is on time interval Δt semantic model of language system $\mathcal{S}_L$ iff $\langle \mathcal{S}, \mathcal{S}_L, \Delta t\rangle \in \mathit{mod}_1$ where mapping setting-up isomorphy $\mathcal{S}$ to $\mathcal{S}_L$ satisfies the same requirements which were formulated in D 8.

Remark. In the previous definition D 9 we did not admit in correspondence $\mathscr{S}$ and $\mathscr{S}_L$ alternatives "mod_2", "mod_3" according to traditional concept of notion "semantic model" in logical semantic. Semantic model of language system can be ontic system, but also some other language system can be applicable (due to common semantic practice).

Examples.

P 8. Systems $\mathscr{S}_{O_2}$ and $\mathscr{S}_{L_2}$ from examples P 3 and P 6 are mutually isomorphic on interval Δt_2, because relation ${}^{(1)}R_1'^{(1)}$ is isomorphic with relation $\mathbf{R}_{L_2}'^{(2)}$,

$$(\langle a_1, t_1\rangle \in {}^{(1)}R_1'^{(1)} \Leftrightarrow \mathbf{R}_{L_{21}}'^{(2)}(= {}^{(1)}\mathbf{R}_1'^{(1)}(\mathbf{a}_1, \mathbf{t}_1))\,,$$

$$(\langle a_2, t_4\rangle \in {}^{(1)}R_1'^{(1)} \Leftrightarrow \mathbf{R}_{L_{22}}'^{(2)}(= {}^{(1)}\mathbf{R}_1'^{(1)}(\mathbf{a}_2, \mathbf{t}_4))\,,$$

relation ${}^{(1)}R_2'^{(1)}$ is isomorphic with relation $\mathbf{R}_{L_3}'^{(2)}$,

$$(\langle a_2, t_5\rangle \in {}^{(1)}R_2'^{(1)} \Leftrightarrow \mathbf{R}_{L_{31}}'^{(2)}(= {}^{(1)}\mathbf{R}_2'^{(1)}(\mathbf{a}_2, \mathbf{t}_5))\,,$$

relation ${}^{(1)}R_1'^{(2)}$ is isomorphic with relation $\mathbf{R}_{L_6}'^{(3)}$,

$$(\langle a_7, a_9, t_4\rangle \in {}^{(1)}R_1'^{(2)} \Leftrightarrow \mathbf{R}_{L_{61}}'^{(3)}(= {}^{(1)}\mathbf{R}_1'^{(2)}(\mathbf{a}_7, \mathbf{a}_9, \mathbf{t}_4))\,,$$

$$(\langle a_9, a_7, t_4\rangle \in {}^{(1)}R_1'^{(2)} \Leftrightarrow \mathbf{R}_{L_{62}}'^{(3)}(= {}^{(1)}\mathbf{R}_1'^{(2)}(\mathbf{a}_9, \mathbf{a}_7, \mathbf{t}_4))\,,$$

relation ${}^{(1)}R_2'^{(2)}$ is isomorphic with relation $\mathbf{R}_{L_7}'^{(3)}$,

$$(\langle a_1, a_8, t_1\rangle \in {}^{(1)}R_2'^{(2)} \Leftrightarrow \mathbf{R}_{L_{71}}'^{(3)}(= {}^{(1)}\mathbf{R}_2'^{(2)}(\mathbf{a}_1, \mathbf{a}_8, \mathbf{t}_1))\,,$$

$$(\langle a_8, a_{10}, t_1\rangle \in {}^{(1)}R_2'^{(2)} \Leftrightarrow \mathbf{R}_{L_{72}}'^{(3)}(= {}^{(1)}\mathbf{R}_2'^{(2)}(\mathbf{a}_8, \mathbf{a}_{10}, \mathbf{t}_1))\,,$$

$$(\langle a_1, a_{10}, t_1\rangle \in {}^{(1)}R_2'^{(2)} \Leftrightarrow \mathbf{R}_{L_{73}}'^{(3)}(= {}^{(1)}\mathbf{R}_2'^{(2)}(\mathbf{a}_1, \mathbf{a}_{10}, \mathbf{t}_1))\,.$$

relation ${}^{(2)}R_1'^{(1)}$ is isomorphic with relation $\mathbf{R}_{L_9}'^{(2)}$,

$$(\langle {}^{(1)}R_1'^{(1)}, t_1\rangle \in {}^{(2)}R_1'^{(1)} \Leftrightarrow \mathbf{R}_{L_{91}}'^{(2)}(= {}^{(2)}\mathbf{R}_1'^{(1)}({}^{(1)}\mathbf{R}_1'^{(1)}, \mathbf{t}_1))\,,$$

$$(\langle {}^{(1)}R_1'^{(1)}, t_4\rangle \in {}^{(2)}R_1'^{(1)} \Leftrightarrow \mathbf{R}_{L_{92}}'^{(2)}(= {}^{(2)}\mathbf{R}_1'^{(1)}({}^{(1)}\mathbf{R}_1'^{(1)}, \mathbf{t}_4))\,.$$

Therefore: $\langle \mathscr{S}_{O_2}, \mathscr{S}_{L_2}, \Delta t_2\rangle \in mod_1$, $\langle \mathscr{S}_{L_2}, \mathscr{S}_{O_2}, \Delta t_2\rangle \in mod_1$ (correspondence of isomorphy is symmetric).

Simultaneously we can consider $\mathscr{S}_{L_2}$ as language model$_1$ of ontic system $\mathscr{S}_{O_2}$ and conversely system $\mathscr{S}_{O_2}$ as semantic model of language model $\mathscr{S}_{L_2}$, if conditions of definitions D 8, D 9 are satisfied:

 to constant $^{(1)}\mathbf{R}_1'^{(1)}$ of system $\mathscr{S}_{L_2}$ relation $^{(1)}R_1'^{(1)}$ of $\mathscr{S}_{O_2}$ is uniquelly associated
to constant $^{(1)}\mathbf{R}_2'^{(1)}$ of system $\mathscr{S}_{L_2}$ relation $^{(1)}R_2'^{(1)}$ of $\mathscr{S}_{O_2}$ is uniquelly associated
to constant $^{(1)}\mathbf{R}_1'^{(2)}$ of system $\mathscr{S}_{L_2}$ relation $^{(1)}R_1'^{(2)}$ of $\mathscr{S}_{O_2}$ is uniquelly associated
to constant $^{(1)}\mathbf{R}_2'^{(2)}$ of system $\mathscr{S}_{L_2}$ relation $^{(1)}R_2'^{(2)}$ of $\mathscr{S}_{O_2}$ is uniquelly associated
to constant $^{(2)}\mathbf{R}_1'^{(1)}$ of system $\mathscr{S}_{L_2}$ relation $^{(1)}R_1'^{(1)}$ of $\mathscr{S}_{O_2}$ is uniquelly associated

to constant $\mathbf{a}_1$ of system $\mathscr{S}_{L_2}$ object a_1 from the universe of system $\mathscr{S}_{O2}$ is uniquelly associated

to constant $\mathbf{a}_2$ of system $\mathscr{S}_{L_2}$ object a_2 from the universe of system $\mathscr{S}_{O_2}$ is uniquelly associated

to constant $\mathbf{a}_7$ of system $\mathscr{S}_{L_2}$ object a_7 from the universe of system $\mathscr{S}_{O_2}$ is uniquelly associated

to constant $\mathbf{a}_9$ of system $\mathscr{S}_{L_2}$ object a_9 from the universe of system $\mathscr{S}_{O_2}$ is uniquelly associated

to constant $\mathbf{a}_{10}$ of system $\mathscr{S}_{L_2}$ object a_{10} from the universe of system $\mathscr{S}_{O_2}$ is uniquelly associated

to symbol $\mathbf{t}_1$ of system $\mathscr{S}_{L_2}$ moment t_1 from interval Δt_2 is uniquelly associated
to symbol $\mathbf{t}_4$ of system $\mathscr{S}_{L_2}$ moment t_4 from interval Δt_2 is uniquelly associated
to symbol $\mathbf{t}_5$ of system $\mathscr{S}_{L_2}$ moment t_5 from interval Δt_2 is uniquelly associated

P 9. System $\mathscr{S}_{L_2}$ is on time interval Δt_2 generally homomorphic to a system $\mathscr{S}_{O2}$ from example P 2, since there is system $\mathscr{S}_{O_2}$, which is on Δt_2 subsystem $\mathscr{S}_{O_1}$ and $\mathscr{S}_{L_2}$ is on Δt_2 isomorphic to $\mathscr{S}_{O_2}$. Easily we can find out that mapping founding generalized homomorphy $\mathscr{S}_{L_2}$ to $\mathscr{S}_{O_1}$ we obtain by composition of two mappings from P 4 and P 8:

$$
\begin{aligned}
{}^{(1)}R_1^{(1)} &\Rightarrow \mathbf{R}_{L_2}'^{(2)} ,\\
{}^{(1)}R_2^{(1)} &\Rightarrow \mathbf{R}_{L_3}'^{(2)} ,\\
{}^{(1)}R_3^{(1)} &\Rightarrow \emptyset ,\\
{}^{(1)}R_1^{(2)} &\Rightarrow \mathbf{R}_{L_6}'^{(3)} ,\\
{}^{(1)}R_2^{(2)} &\Rightarrow \mathbf{R}_{L_7}'^{(3)} ,\\
{}^{(1)}R_1^{(3)} &\Rightarrow \emptyset ,\\
{}^{(2)}R_1^{(1)} &\Rightarrow \mathbf{R}_{L_9}'^{(2)} ,\\
{}^{(3)}R_1^{(4)} &\Rightarrow \emptyset .
\end{aligned}
$$

Hence $\mathscr{S}_{L_2}$ is also model$_2$ of ontic system $\mathscr{S}_{O_1}$ on interval Δt_2 and, if conditions from D 8 are met, is also its language model$_2$ on Δt_2.

P 10. By examples P 4 and P 7 $\mathscr{S}_{O_2}$ is subsystem of $\mathscr{S}_{O_1}$ on interval Δt_2, $\mathscr{S}_{L_2}$ is subsystem of $\mathscr{S}_{L_1}$ on interval Δt_2. Therefore we can assume $\mathscr{S}_{L_1}$ as model$_3$ of ontic system $\mathscr{S}_{O_1}$ on Δt_2 and conversely also $\mathscr{S}_{O_1}$ as model$_3$ of system $\mathscr{S}_{L_1}$ on Δt_2. If conditions from D 8 are satisfied, we can consider $\mathscr{S}_{L_1}$ as language model$_3$ of system $\mathscr{S}_{O_1}$.

Proposed definition of notion "model" and its specification for notions "model$_1$", "model$_2$", "model$_3$" may look unusual and perhaps even artificial and strange. However consider that in practice of language (especially mathematical) modelling of real ontic systems we often use language and axiomatic system with general axioms. To these general axioms, we cannot associate mutually and uniquelly any elements of relations (or collections of these elements) of real systems. Thus there is no possibility to take those language systems for models$_1$ of given real systems. Despite that we do talk about "language (for instance mathematical) model of real system "also in these cases. For this reason we have introduced notion "model$_2$".

Similar reasoning leads to acceptance of specification "model$_3$". In practice we hardly detect existing large real and variable systems in their all completeness. We cannot hence form language system modelling those real systems in complete complexivity due to very strict demand from definition of notion "model$_1$". This fact is strengthened by known desires of finiteness imposed on modelling language systems when modelling real systems by modern computers (finite automata). Conversely, when we shall use axiomatic language systems with general axioms for modelling, then we cannot apply even notion "model$_2$". Therefore we have introduced notion "model$_3$".

Applicability of proposed concept will be particularly stressed when we consider possibility of prediction of future events due to logical operations in modelling language system.

Without formulating the whole concept on general level, we shall present very simple example.

Let us return to examples P 2 – P 10. Suppose that system $\mathscr{S}_{L_1}$ is given to our disposal already at moment t_4. From assumptions $\mathbf{R}^{(2)}_{L_{11}}$ and $\mathbf{R}^{(2)}_{L_{22}}$, it is possible by means of logical deduction (by substitution and detachment) to obtain statement $\mathbf{R}^{(2)}_{L_{32}}$:

$$^{(1)}\mathbf{R}^{(1)}_2(\mathbf{a}_2, \mathbf{t}_5)\,,$$

describing (at given interpretation), that at future moment t_5 object a_2 will have property $^{(1)}R^{(1)}_2$. The first proposition $\mathbf{R}^{(2)}_{L_{11}}$ is general sentence which we have accepted into system $\mathscr{S}_{L_1}$ for instance from a theory. The second assumption $\mathbf{R}^{(2)}_{L_{22}}$ is sentence which we can use to describe an event which passed by in system $\mathscr{S}_{O_1}$ at the same moment t_4 ($\langle a_2, t_4\rangle \in {}^{(1)}R^{(1)}_2$) in terms of language of system $\mathscr{S}_{L_1}$. Since language system $\mathscr{S}_{L_1}$ contains this sentence, we cannot classify that at moment t_4 as model$_1$ of system $\mathscr{S}_{O_1}$ (which presumably at t_4 we cannot empiricaly detect completely).

System $\mathscr{S}_{O_1}$ anyway encloses events with no corresponding sentences in $\mathscr{S}_{L_1}$ – for example event: $\langle a_6, t_1 \rangle \in {}^{(1)}R_3^{(1)}$. For this reason we cannot characterise correspondence $\mathscr{S}_{L_1}$ and $\mathscr{S}_{O_1}$ for interval Δt as "model$_3$". However due to existence of model of this kind we can "predict future".

At the end of this reasoning we introduce example dealing with the correspondence "model" among studied system, known only partially, method "black box" and deterministic automata.

Let us have system $\mathscr{S}_O$, whose structure we do not know and we can determine on it two limit points a_1, a_2. We shall make large series of experiments associating thus one of the elements a_1 impulsion of various kinds and we shall investigate reaction of system on the other limit point a_2. Hence we study correspondence between assigned value of input element a_1 and values of output element a_2 taken on after duration of given time period. Let experiments be created during given time interval Δt_B. We describe their results in the Tab. 1.

Tab. 1.

to element a_1 was assigned		element a_2 took on	
value	at moment	value	at moment
λ	t_1	λ	t_2
λ	t_3	λ	t_4
λ	t_5	λ	t_6
$\vdots$	$\vdots$	$\vdots$	$\vdots$
λ	t_{10}	λ	t_{11}
A	t_{11}	A	t_{12}
λ	t_{13}	λ	t_{14}
A	t_{14}	A	t_{15}
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Let results of experiments be resolved and generalized for the whole interval Δt_B as shown in Tab. 2.

We shall now define system $\mathscr{S}_B$ based on mentioned experiments and existing on interval Δt_B: $\mathscr{S}_B = \langle U_B, \mathscr{R}_B \rangle$.

$$U_B = \{a_1, a_2\}, \quad \mathscr{R}_B = \{\lambda, A, B, C\},$$

$$\lambda = \{\langle a_1, t_1\rangle, \langle a_1, t_3\rangle, \langle a_1, t_5\rangle, \langle a_1, t_{13}\rangle, \ldots, \langle a_2, t_2\rangle, \langle a_2, t_4\rangle, \langle a_2, t_6\rangle, \langle a_2, t_{14}\rangle, \ldots, \},$$

$$A = \{\langle a_1, t_{11}\rangle, \langle a_1, t_{14}\rangle, \ldots \quad \ldots \langle a_2, t_{12}\rangle, \langle a_2, t_{15}\rangle \ldots\}.$$

$$\vdots$$

Tab. 2.

input word	at moments	output word	at moments
λ	t_i	λ	t_{i+1}
λA	t_i, t_{i+1}	λA	t_{i+1}, t_{i+2}
λB		λB	
λC		λC	
$A\lambda$		$A\lambda$	
AA		AA	
AB		AC	
AC		AB	
$B\lambda$	t_i, t_{i+1}	$B\lambda$	t_{i+1}, t_{i+2}
BA		BB	
BB		BA	
BC		BC	
$C\lambda$		$C\lambda$	
CA		CB	
CB		CA	
CC		CC	

(Due to Table 1). $\mathscr{S}_B$ does not contain other relations except those which were found from experiments summarized in Table 1.

We define further axiomatic language system $\mathscr{S}_{L_B}$ holding on interval Δt_B: $\mathscr{S}_{L_B} = \langle U_{L_B}, \mathscr{R}_{L_B} \rangle$,

$$U_{L_B} = \{\mathbf{a}_1, \mathbf{a}_2, \lambda, \mathbf{A}, \mathbf{B}, \mathbf{C}\} .$$

$\mathscr{R}_{L_B}$ contains:

1. general statements

$$\mathbf{P}_1 \quad \forall \mathbf{t}_i(\lambda(\mathbf{a}_1, \mathbf{t}_i) \rightarrow \lambda(\mathbf{a}_2, \mathbf{t}_{i+1})) ,$$
$$\mathbf{P}_2 = \forall \mathbf{t}_i((\lambda(\mathbf{a}_1, \mathbf{t}_i) \wedge \mathbf{A}(\mathbf{a}_1, \mathbf{t}_{i+1})) \rightarrow (\lambda(\mathbf{a}_2, \mathbf{t}_{i+1}) \wedge \mathbf{A}(\mathbf{a}_2, \mathbf{t}_{i+2}))) ,$$
$$\vdots$$
$$\mathbf{P}_{16} = \forall \mathbf{t}_i((\mathbf{C}(\mathbf{a}_1, \mathbf{t}_i) \wedge \mathbf{C}(\mathbf{a}_1, \mathbf{t}_{i+1})) \rightarrow (\mathbf{c}(\mathbf{a}_2, \mathbf{t}_{i+1}) \wedge \mathbf{C}(\mathbf{a}_2, \mathbf{t}_{i+2}))) ;$$

(they are description of result of generalization from Tab. 2).

2. particular statements:

$$\mathbf{V}_{1_1} = \lambda(\mathbf{a}_1, \mathbf{t}_1) ,$$
$$\mathbf{V}_{1_2} = \lambda(\mathbf{a}_1, \mathbf{t}_3) ,$$
$$\vdots$$

(statements regarding entry a_1 are description of recalled situations in experiments described in Table 1, statements concerning output a_2 are logical consequence of general statements and particular statements about entry a_1 and are in accordance with description in Table 1).

Further there are here statements obtained from general ones $P_1, \ldots, P_{16}$ by substitution of names of time moments from interval Δt_B and time variables:

$$\mathbf{V}_{m_1} = (\lambda(\mathbf{a}_1, \mathbf{t}_1) \rightarrow \lambda(\mathbf{a}_2, \mathbf{t}_2)),$$
$$\mathbf{V}_{n_2} = ((\lambda(\mathbf{a}_1, \mathbf{t}_1) \wedge \mathbf{A}(\mathbf{a}_1, \mathbf{t}_2)) \rightarrow (\lambda(\mathbf{a}_2, \mathbf{t}_2) \wedge \mathbf{A}(\mathbf{a}_2, \mathbf{t}_3))),$$
$$\vdots$$

Let us form the automaton shown on Fig. 1 from technical elements.

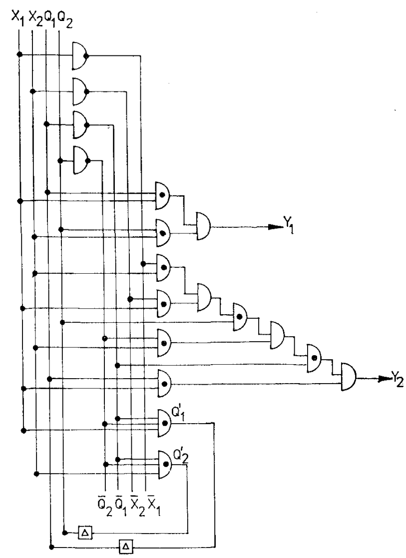

Fig. 1.

Automaton has two binary inputs X_1, X_2 and two binary outputs Y_1, Y_2. Q_1, Q_2 are binary parts of state. We can describe relations of automaton by canonic equations (by means of Boolean algebra):

$$Y_1 = Q_1X_1 + Q_2X_2,$$
$$Y_2 = \bar{Q}_1[\bar{Q}_2X_2 + Q_2(\bar{X}_1X_2 + X_1\bar{X}_2)] + Q_1X_1,$$
$$Q'_1 = \bar{Q}_1\bar{Q}_2X_1,$$
$$Q'_2 = \bar{Q}_1\bar{Q}_2X_2.$$

Let us translate (mutually uniquelly) binary symbols to letters of alphabet according to code:

input symbols	letter of input alphabet
$\bar{X}_1\bar{X}_2$	λ
$\bar{X}_1X_2$	A
$X_1\bar{X}_2$	B
X_1X_2	C

output symbols	letter of output alphabet
$\bar{Y}_1\bar{Y}_2$	λ
$\bar{Y}_1Y_2$	A
$Y_1\bar{Y}_2$	B
Y_1Y_2	C

binary parts of state	denotation of state
$\bar{Q}_1\bar{Q}_2$	S_1
$\bar{Q}_1Q_2$	S_2
$Q_1\bar{Q}_2$	S_3
Q_1Q_2	S_4

Because of this translation it is possible to consider automata as a system with one input element x_1 and one output element y_1. Both input and output element can take on values λ, A, B, C.

Not taking in to mind internal (technical) structure of automaton let us define on that subsystem $\mathcal{S}_A$:

$$\mathcal{S}_A = \langle U_A, \mathcal{R}_A \rangle ,$$

$$U_A = \{x_1, y_1\} , \quad \mathcal{R}_A = \{\lambda, A, B, C, R_1, R_2, \ldots\} .$$

Let $\mathcal{S}_A$ exist again on interval Δt_B. Relations which are elements of set of relations $\mathcal{R}_A$ are more terms relations between input, its values, corresponding moments, output, its values and corresponding moments. To element of a relation $R_i \in \mathcal{R}_A$:

$$\langle x_1, A, t_1, B, t_2, y_1, A, t_2, C, t_3 \rangle$$

there will be uniquelly assigned a statement of language system $\mathcal{S}_{L_B}$:

$$\mathbf{V}_{i_j} = ((\mathbf{A}(\mathbf{a}_1, \mathbf{t}_1) \wedge \mathbf{B}(\mathbf{a}_1, \mathbf{t}_2)) \wedge (\mathbf{A}(\mathbf{a}_2, \mathbf{t}_2) \wedge \mathbf{C}(\mathbf{a}_2, \mathbf{t}_3)))$$

which is element of a set of statements $\mathbf{V}_i \in \mathcal{R}_{L_B}$.

The automaton has been formed to assign to every relation $R_i \in \mathcal{R}_A$ uniquelly a set of statements from set $\mathcal{R}_{L_B}$ of language system $\mathcal{S}_{L_B}$. If this association is mutually unique, there exists a correspondence "model$_1$", on interval Δt_B between both systems.

On the contrary, between system $\mathcal{S}_B$ and $\mathcal{S}_{L_B}$ there is only correspondence "model$_2$" on interval Δt_B, since to general statements of system $\mathscr{S}_{L_B}$ cannot be assigned any relations of system $\mathscr{S}_B$ – system $\mathscr{S}_B$ was defined merely for summarizing particular empirically obtained knowledge on issuing data of input and taking on values of output in individual moments.

Obviously it is possible to define subsystem $\mathscr{S}'_{L_B}$ on system $\mathscr{S}_{L_B}$, isomorphic on interval Δt_B with system $\mathscr{S}_B$. Between $\mathscr{S}_A$ and $\mathscr{S}_{L_B}$ is therefore also correspondence "model$_2$" on interval Δt_B.

System $\mathscr{S}_B$ was however defined as subsystem on system $\mathscr{S}_O$. It is therefore also correspondence "model$_2$" on interval Δt_B between $\mathscr{S}_O$ and $\mathscr{S}_A$. There surely are relations in $\mathscr{S}_O$ connecting input and output words for those moments in which no corresponding experiments were done on $\mathscr{S}_B$. Because of that it is possible by experimenting with $\mathscr{S}_A$ (made as experiments with automaton, on which $\mathscr{S}_A$ was defined), to predict or postdict or explanate resp. processes which can run on in model system $\mathscr{S}_O$.

Let us further consider a possibility (which will not be discussed here in details) to assess structure of incompletely known automata $\mathscr{S}_O$ on the base of knowledge of internal (technical) structure given automata. We can easily construct considered deterministic automaton if we know its canonic equations.

Discussed example corresponds with real procedures in technical practice. On incompletely known system "black box" we define, on the base of really created experiments, subsystem. To that we search adequate language model. If this model is axiomatic system, then statements describing results of made experiments belong to its theorems. To such a language system we look for corresponding automata modelling (at least some) statements of system. Experiments with automata can be replaced by further experiments with system "black box". We assess structure of system "black box" due to knowledge regarding structure of modelling automata.

Let us notice at the end that system $\mathscr{S}_A$ enclosing relations among input, output, their values and moments is a system of the second degree (values of input or output elements resp. are considered as its properties).

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Doc. PhDr. Roman Bek, DrSc., katedra aplikované matematiky, strojní fakulta ČVUT (Department of Applied Mathematics, Faculty of Mechanical Engineering — Czech Technical University), Suchbátarova 4, 160 00 Praha 6. Czechoslovakia.

PhDr. Milan Růžička, katedra matematiky, strojní fakulta ČVUT (Department of Mathematics, Faculty of Mechanical Engineering — Czech Technical University), Karlovo nám. 13, 120 00 Praha 2. Czechoslovakia.