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An Intensional Approach to Questions

PAVEL MATERNA

Dedicated to Prof. Dr. Otakar Zich, DrSc. at the occassion of his 70th birthday

Questions are shown to be either intensions (the values of which in the actual world one wants to know), or "constructions" that construct objects, or "meta-constructions" that construct constructions.

The concepts of presuppositions (of questions), of redundant and of non-redundant answers to questions, of indirect and of partial answers, as well as some other concepts are defined. An application to defining the concept of a mass-problem is made in the Appendix.

INTRODUCTION

Making the present proposals of how to conceive questions I do not intend to make simultaneously confrontations with various existing theories of questions. Some remarks only:

There may be various approaches to questions depending on what aspect of them we are interested in. Thus a psychological or a phenomenological theory of questions may be built up or the informational aspect may be stressed. Also, what is called "logical theory of questions" or "logic of questions" ([2], [5], N. Belnap, etc.) is not a homogeneous discipline; finally, the linguistic approaches include Chomskian ideas and, lately, the Montaguian ideas of applying logic to linguistics (see, e.g., [3]), as well as the traditional linguistic analyses.

I shall focus my attention on the logico-semantical analysis of questions. Similarly as Hausser in [3] I reject the "indirect approach to questions", although he comparison between direct and indirect questions is important and can be made within my approach. Various classifications of questions as known from the literature (see, e.g., [5]) will be also possible but they will not be primarily based on syntactical considerations. My starting point will not be an artificial formal language; it will rather be an attempt to make precise some fundamental semantical intuitions.

All the semantical theories of questions known to me share at least one rather principal shortage: they are unable to distinguish between empirical and analytical questions. The semantics of most of them is based on extensionalist presuppositions. Hausser's attempt to take intensions into considerations is based on an extension of Montague's system. My conception is, essentially, an application of Tichý's system ([11], [12], [14]) – here: the **T**-system – which I take to be most consequent intensionalist system (see also [6], [7]).

In Ch. I. I briefly reproduce the main principles of the **T**-system, as this system is contained in the first chapters of [12], which is as yet not accessible. In Ch. II. I introduce the concepts which may be to my opinion a good starting point for building up a rather universal theory of questions. The author considers Ch. III. ("Comments") to be especially important. Finally, Appendix extends the author's conception to defining the mass problems.

CHAPTER I. THE T-SYSTEM

1. Types

The logical foundations of the **T**-system consist in a modified version of Church's simple theory of types (see [1]). Because of this modification I shall reproduce the theory of types as it is formulated in the **T**-system. Besides some unessential details (like notational changes) no idea of my own is contained in this chapter.

Def 1 A *base* is a family of mutually disjoint non-empty collections.

Def 2 Let $\boldsymbol{B}$ be a base. Then

i) any member of $\boldsymbol{B}$ is a *type over* $\boldsymbol{B}$;

ii) if η, $\xi_1, \ldots, \xi_n$ are *types over* $\boldsymbol{B}$, then $\eta(\xi_1, \ldots, \xi_n)$ is a *type over* $\boldsymbol{B}$, where $\eta(\xi_1, \ldots, \xi_n)$ is the collection of functions which associate with every n-tuple of members of $\xi_1, \ldots, \xi_n$ at most one member of η.

iii) The *types over* $\boldsymbol{B}$ are just those introduced in i), ii).

Thus a type over $\boldsymbol{B}$ always is a non-empty collection: either one of the members of $\boldsymbol{B}$, or a collection of functions. Over any base there arises an infinite hierarchy of types.

Where there is no risk of misunderstanding we shall omit the phrase "over $\boldsymbol{B}$" assuming a base to be given in every such case.

Def 3 Any member of a type η is an *object of the type* η, or, briefly, an *η-object.*

Example. Let $\boldsymbol{B}$ consist of the collections $\{A, B, C\}$, say α, $\{T, F\}$, say, β, the collection of natural numbers $\{0, 1, 2, \ldots\}$, say, γ. The type $\alpha(\beta)$ consists of $4^2 = 16$ functions

$f_1, \ldots, f_{16}$. One of these $\alpha(\beta)$-objects, say, f_4, is

T	A
F	–

where "–" means that the function in question is undefined at F. The type $\gamma(\beta)$ consists of denumerably many functions, one of which is the $\gamma(\beta)$-object

T	1
F	541 .

The type $\alpha(\beta)\,(\beta, \beta, \alpha)$ consists of 17^{12} functions one of which is the $\alpha(\beta)\,(\beta, \beta, \alpha)$-object

T T A	f_{15}
T T B	f_{15}
T T C	f_1
T F A	–
T F B	f_3
T F C	f_2
F T A	–
F T B	f_1
F T C	f_1
F F A	f_1
F F B	–
F F C	–

2. Constructions

We assume that for every type η (over some base) there are infinitely many abstract entities available which represent any η-object. Such entities we call *variables*, or, for the given type η, *η-variables*. An η-object is an *instance of a variable* of type η (of an η-variable). There are, furthermore, infinitely many total functions called *valuations* that associate with each variable of a type exactly one object of the same type. Thus if α, β, γ are as in the above example, f_4 is an instance of an $\alpha(\beta)$-variable, say, x_3, as well as of infinitely many further variables of the type $\alpha(\beta)$. We can say also that f_4 is a **v**-*instance* of x_3 for every valuation **v** such that associates the function f_4 with the variable x_3.

Let us observe now the ways in which some object can be given. The most simple way is that one where the object is given directly.

 In our example, A is given directly as an α-object, as well as

$$\begin{matrix} T & A \\ F & - \end{matrix}$$

is given directly as an $\alpha(\beta)$-object.

Secondly, an object may be given by means of a valuation as an instance of a variable.

Thirdly, an object may be given indirectly, by means of some operations. In this way, e.g., 5 is given as the sum of 2 and 3 or as $\sqrt{(25)}$.

Instead of saying that an object is given in such and such way one can talk about kinds of a *construction* of an object. Thus we may say that A constructs itself, that for such valuations $\mathbf{v}$ that assign f_4 to x_{11}, x_{11} $\mathbf{v}$-*constructs* f_4, that $2 + 3$ constructs 5, that $2 + x$ $\mathbf{v}$-constructs 5 for all the valuations $\mathbf{v}$ such that they assign 3 to x, etc. A special case arises when no object is given, as e.g., when a construction $5 : 0$ is given: we say that $5 : 0$ is an *improper construction*. Similarly, $5 : x$ is said to be $\mathbf{v}$-*improper* for all the valuations $\mathbf{v}$ that assign 0 to x.

The foregoing intuitions can be summarized and generalized in the following definition:

Def 4 Let $\eta, \xi_1, \ldots, \xi_n$ be types (over a base).

i) Any η-object is an *η-construction*. Any η-variable is an *η-construction*. Let A be an η-object. Then A v-*constructs* A for any valuation $\mathbf{v}$ (= A *constructs* A). Let x be an η-variable. Then x $\mathbf{v}$-*constructs* the $\mathbf{v}$-instance of x.

ii) Let $\boldsymbol{A}, \boldsymbol{B}_1, \ldots, \boldsymbol{B}_n$ be $\eta(\xi_1, \ldots, \xi_n)$ —, $\xi_1, \ldots, \xi_n$-*constructions*, respectively. Then $\boldsymbol{A}(\boldsymbol{B}_1, \ldots, \boldsymbol{B}_n)$ is an *η-construction* called *application* of $\boldsymbol{A}$ *to* $\boldsymbol{B}_1, \ldots \ldots, \boldsymbol{B}_n$.If, for a valuation $\mathbf{v}$, at least one of $\boldsymbol{A}, \boldsymbol{B}_1, \ldots, \boldsymbol{B}_n$ is $\mathbf{v}$-improper, then $\boldsymbol{A}(\boldsymbol{B}_1, \ldots, \boldsymbol{B}_n)$ is $\mathbf{v}$-improper. Otherwise, let $A, B_1, \ldots, B_n$ be the objects $\mathbf{v}$-constructed by $\boldsymbol{A}, \boldsymbol{B}_1, \ldots, \boldsymbol{B}_n$, respectively; if A is not defined at $\langle B_1, \ldots, B_n\rangle$, then $\boldsymbol{A}(\boldsymbol{B}_1, \ldots, \boldsymbol{B}_n)$ is $\mathbf{v}$-improper. In the remaining case, $\boldsymbol{A}(\boldsymbol{B}_1, \ldots, \boldsymbol{B}_n)$ $\mathbf{v}$-constructs the value of A at $\langle B_1, \ldots, B_n\rangle$.

iii) Let $x_1, \ldots, x_n$ be ξ_1_, $\ldots, \xi_n$-variables, respectively; if $\boldsymbol{A}$ is an *η-construction*, then $\lambda x_1, \ldots, x_n(\boldsymbol{A})$ is an *$\eta(\xi_1, \ldots, \xi_n)$-construction* called *$x_1, \ldots \ldots, x_n$-abstraction* of $\boldsymbol{A}$.

Let $\mathbf{v}(x_1, \ldots, x_n/X_1, \ldots, X_n)$ be the valuation that differs from the valuation $\mathbf{v}$ at most by assigning the objects $X_1, \ldots, X_n$ to the variables $x_1, \ldots \ldots, x_n$, respectively. Then the function Y $\mathbf{v}$-constructed by $\lambda x_1, \ldots, x_n(\boldsymbol{A})$ is the following function: where $X_1, \ldots, X_n$ are ξ_1-, $\ldots, \xi_n$-objects, respectively, if $\boldsymbol{A}$ is $\mathbf{v}(x_1, \ldots, x_n/X_1, \ldots, X_n)$-improper, Y is undefined at $\langle X_1, \ldots, X_n\rangle$. Otherwise, the value of Y at $\langle X_1, \ldots, X_n\rangle$ is the object $\mathbf{v}(x_1, \ldots, x_n/X_1, \ldots, X_n)$-constructed by $\boldsymbol{A}$.

iv) The *constructions* are just *η-constructions* for some type η, as defined in i) – iii).

Convention. Let objects and variables be called *atoms*. In the following text we shall denote

variables by $a, b, c, \ldots,$

objects by $A, B, C, \ldots,$

constructions by $\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots.$

Def 5 Let x be a variable, $\mathbf{A}, \mathbf{B}_1, \ldots, \mathbf{B}_n$ some constructions.

i) If $\mathbf{A}$ is an atom, x has *free occurrence* in $\mathbf{A}$ iff $\mathbf{A}$ is x.

ii) x has *free occurrence in* $\mathbf{A}(\mathbf{B}_1, \ldots, \mathbf{B}_n)$ wherever x has *free occurrence in* $\mathbf{A}, \mathbf{B}_1, \ldots, \mathbf{B}_n$.

iii) x has *free occurrence in* $\lambda x_1, \ldots, x_n(\mathbf{A})$ wherever x has *free occurrence in* $\mathbf{A}$ and x is not identical with any of $x_1, \ldots, x_n$.

iv) Where x occurs in $\mathbf{A}$, we call this *occurrence bound in* $\mathbf{A}$ iff it is not a *free occurrence of* x in $\mathbf{A}$.

Example. Base: $\{A, B\}$, say α, $\{C, D, E\}$, say, β, $\{F, G\}$, say, γ. Let H be the following $\alpha(\beta, \gamma)(\beta, \beta)$-object:

$C\ C$	K_1
$C\ D$	–
$C\ E$	K_1
$D\ C$	K_2
$D\ D$	K_4
$D\ E$	K_1
$E\ C$	–
$E\ D$	K_2
$E\ E$	K_2

where K_1 is

$C\ F$	A
$C\ G$	A
$D\ F$	A
$D\ G$	A
$E\ F$	A
$E\ G$	A,

K_2 is

C	F	A
C	G	A
D	F	A
D	G	A
E	F	A
E	G	B,

K_4 is

C	F	A
C	G	–
D	F	A
D	G	A
E	F	B
E	G	A.

Then $H(D, D)$ constructs K_4, $H(D, D)(C, G)$ is **v**-improper for every **v**, $H(E, C)$ is **v**-improper for every **v**, $H(E, D)(E, x)$ $\mathbf{v}_1$-constructs A if $\mathbf{v}_1$ assigns F to x, and $\mathbf{v}_2$-constructs B, if $\mathbf{v}_2$ assigns G to x.

$\lambda x(H(x, E))$ is an $\alpha(\beta, \gamma)(\beta)$-construction which constructs the function, say, L:

C	K_1
D	K_1
E	K_2 ;

$\lambda x(H(x, y))$ **v**-constructs the function, say, M:

C	–
D	K_4
E	K_2

if **v** assigns D to y. If **v**′ assigns E to y, then $\lambda x H(x, y)$ **v**′-constructs L.

Remark 1. Notice that among all the kinds of construction only application can be **v**-improper for some **v**.

Remark 2. Constructions are exactly what has been defined in Def. 4. Thus what is meant by a construction is not the artificial expression necessary for fixing (naming) this construction. Analogously, by the mathematical term *5* we mean the number itself, not the digit in question.

Def 6 i) $\boldsymbol{C}$ is a *subconstruction of* $\boldsymbol{C}$.

ii) If $\boldsymbol{C}$ is $\boldsymbol{A}(\boldsymbol{B}_1, \ldots, \boldsymbol{B}_n)$, then $\boldsymbol{A}, \boldsymbol{B}_1, \ldots, \boldsymbol{B}_n$ are *subconstructions of* $\boldsymbol{C}$. If $\boldsymbol{C}$ is $\lambda x_1, \ldots, x_n(\boldsymbol{A})$, then $\boldsymbol{A}$ is a *subconstruction of* $\boldsymbol{C}$.

iii) If $\boldsymbol{X}$ is a *subconstruction of* $\boldsymbol{Y}$ and $\boldsymbol{Y}$ is a *subconstruction of* $\boldsymbol{Z}$, then $\boldsymbol{X}$ is a *subconstruction of* $\boldsymbol{Z}$.

3. Languages

A. Intensions and Extensions

In the **T**-system languages are conceived as collections of expressions that express constructions and denote (name) those objects that are constructed by these constructions. We show now briefly the way in which the base "of a language" can be reconstructed.

Two members of such a base are obviously the *universe* (of discourse), the elements of which are called *individuals*, and the collection of *truth-values*, the elements of which are T (truth) and F (falsity). The types of these two collections are ι (*iota*)and o (*omikron*), respectively. The individuals (i.e., ι-objects) are "elementary objects" which happen to have some properties. They are given *a priori* (the same holds about each element of any member of the base in question).

Remark. This conception of individuals might be characterized as anti-essentialistic, and Hintikka would call it "ontological nudism" (see [4]). As for arguments motivating this conception and showing the untenability of essentialists' view-points see [11].

To make the logical analysis of a language deep enough it is necessary to add to the above members of a base the collection of time moments, the elements of which might be identified with either rational or even with real numbers. Let the type of this collection be τ. Also, it may be useful to add a collection of numbers, say, natural numbers. Let the type of the collection of natural numbers be ν_0; if the collection of rational numbers is added, let its type be ν_1.

Such an analysis of a language which would be based on the collections $\iota, o, \tau, \nu_0, \nu_1$, would be a typically extensionalistic analysis. The **T**-system, however, has been built up mainly as a tool of a consequently intensionalistic analysis of language. The absurd consequences of extensionalism have been guessed already by G. Frege. Frege's (and Church's) solution is, however, unsatisfactory. According to Frege and Church expressions denote individuals, classes, truth-values etc., i.e., what is called "extensions" whereas they express "meaning" ("Sinn") or – by Church – intensions, concepts. Yet while the relation of denoting is exactly definable and has been explored in such fundamental semantic works like Tarski's "Wahrheitsbegriff...", nothing definite can be said about the relation of expressing (a meaning). Likewise the concept of intension has not been introduced clearly enough, so that Quine et alii could be in a sense justificated for their criticism concerning such "obscure entities" as intensions are often supposed to be.

Nevertheless, the possibility of exactly defining intensions has been guessed by Carnap and many of his followers, lately by Hintikka, Cresswell, Montague, Kripke and others. Thus what can be called "*possible-worlds-semantics*" ("*p-w-semantics*") has come into existence.

The **T**-system is also a p-w-semantics. It differs, however, by many essential features from the other p-w-systems. We do not intend to make here a systematic comparison of the **T**-system with other comparable systems. It will perhaps suffice when we stress that the **T**-system enables to reject Quinean criticism of intensions based on the criticism of "possible individuals" (there are no such things in the **T**-system), to use and exact and sophisticated logical apparatus in order to analyze languages without neglecting the distinction between classes and properties, truth-values and propositions etc., to extend the intensionalistic conception so as to cover all the expressions of a of a language (not only "modal" or "oblique" contexts), etc.

From the above it is perhaps clear that the remaining member of a base for a language is the collection of possible worlds, or the "*logical space*". The type of this collection is ω.

Remark. Intuitively, the character of the possible worlds is explained in the **T**-system as follows: Any language is connected with what Tichý calls "*intensional basis*". This is a collection of elementary traits which the objects constructible over the given universe may have or not have. To know which is the actual distribution of these traits over the objects in question is the same as to be omniscient. Yet all the thinkable (possible) distributions constitute an *a priori* given collection, namely the logical space (of the given language, of course). What is not *a priori* and what, therefore, has to be empirically established, is just which of these distributions is the real, actualized one. In other words, we do not know which of the possible worlds is the actual one.

One of the fundamental claims made by Tichý is that what we primarily talk about are intensions rather than extensions (see especially [14]); therefore, we now shall clarify the concept of *intension*.

To start with, let us analyze a simple English sentence;

(S) *The French President is older than the American President.*

According to the traditional extensionalistic view-points, the expressions "the French President" and "the American President" would denote (in 1978) d'Estaing and Carter, respectively, and the expression "(is) older (than)" would denote a binary relation between individuals. It can be shown, however, that using this kind of analysis one cannot take into account the obviously empirical character of (S). Indeed:

Let R be the relation of being older: since binary relations(-in-extension) are simply collections of ordered pairs, the pair ⟨d'Estaing, Carter⟩ either necessarily belongs to R, or it necessarily does not belong to R. In the first case (S) will be a necessarily true, in the second case a necessarily false sentence. Moreover, one should suppose that a necessary condition of understanding (S) is to know which objects are named by the particular phrases contained in (S), i.e., to know which object is named by "the French President" etc. This natural assumption is, however, incompatible with the above traditional analysis, because (S) will be understood by any user of English independently of whether he knows that the French President is d'Estaing and the American President is Carter.

Thus our analysis of (S) has to differ from the extensionalistic one at least so as to enable

a) to make evident the empirical character of (S),

b) to let the particular phrases contained in (S) name such entities that are known to any user of the language if he understands (S).

Note that from b) one easily derives the requirement that (S) should name some other entity than a truth-value, because knowing the truth-value of (S) is not a necessary condition for understanding (S).

To start with b), understanding, e.g., the term "the American President" is the same as knowing the concept of being the American President (rather than knowing who actually is the American President). Therefore, we now have to answer the following question: What sort of object is the concept of (being) the American President?

Tichý's answer is sufficiently intuitive. Modifying this answer by taking into account the time factor we claim that the concept of the American President is a function that associates with any possible world a partial function that associates with any time moment at most one individual.

Thus the concept of the American President associates with the actual world a function which associates, e.g., with all the time moments during January 1978 the individual J. Carter and which is at any time moment before 1776 undefined. This function associates with some other possible wo ld a function which is undefined at any time moment before 1830 (in such a possible world the United States of America came into existence not before 1830) and which associates with the time moments in the January 1978 the individual G. Ford.

What is important, is that if one knows the concept (e.g., of the American President) one is theoretically capable to "compute" its value for the given time moment for the given possible word, e.g., for the world he lives in (i.e., for the actual world, or, in other words, dependently on which of the possible distributions of the elements of the intensional basis in question over the given objects is the actualized one).

The functions of this kind will be called *individual concepts* and their type is – as has been verbally stated $-$ $\iota(\tau)(\omega)$. Now we can say that the phrases like "the French President", "the highest mountain in the world", "the discoverer of radioactivity", "Pegasus" etc. name individual concepts.

Take such phrases as "(to be) red", "(to be) clever", "(to be) 170 cm tall", "(to be) a whale", "(to be) older than the French President" etc. A rigid extensionalist would say that such phrases denote classes of individuals (the class of red things, the class of whales etc.). A class of individuals can be identified with the function the domain of which is the universe and which takes as its values T or F according to whether the given individual does or does not belong to the class. (We do not exclude the case that the function is not defined at some individual.) Thus the type of any class of individuals is $o(\iota)$. In general, let η be some type. Then $o(\eta)$ is the type of any class of the η-objects.

If our extensionalist were right in claiming that the above phrases name classes of individuals, it would mean that being, e.g., a whale or being red is fully determined by which individual is meant. Yet we have already adduced some criticism of this view-point. No individual is necessarily red, necessarily a whale, necessarily 170 cm tall etc. Individuals happen to be red, whales, 170 cm tall etc. Thus the above phrases cannot name classes of individuals: instead, they name properties of individuals.

Properties of individuals (in general: *of η-objects*) are functions whose domain is the logical space and whose values are functions that associate with any time moment at most one class of individuals (in general: of η-objects). Their type is, therefore, $o(\iota)(\tau)(\omega)$ (in general: $o(\eta)(\tau)(\omega)$; as we shall see, it can be also $o(\iota)(\omega)$, similarly as the type of individual concepts may be simply $\iota(\omega)$).

The property of, e.g., being 170 cm tall, is a function whose value at the actual world is the function that associates with any time moment t class of those individuals which $-$ at the moment t $-$ are actually 170 cm tall. From this point of view understanding the above phrases is tantamount to knowing some criteria for determining whether an individual at the moment t has or has not the given property, rather than being acquainted with concrete classes of individuals.

Since classes may be conceived of as unary relations-in-extension, nothing essentially new will be said about phrases like "older", "(to be) between ... and ...", "to like more ... than ..." etc. They obviously do not name relations-in-extension between individuals (in general: between ξ_1-, ..., ξ_n- objects), the type of which is $o(\iota, \ldots, \iota)$ (in general: $o(\xi_1, \ldots, \xi_n)$). They again denote functions from possible worlds: these functions use to be called *relations-in-intension* and their type is $o(\iota, \ldots, \iota)(\tau)(\omega)$ or $o(\iota, \ldots, \iota)(\omega)$ (in general: $o(\xi_1, \ldots, \xi_n)(\tau)(\omega)$ or $o(\xi_1, \ldots, \xi_n)$ . . (ω)).

Now, what does name the whole clause (S) ? Certainly we shall not answer: a truth-value, as an extensionalist (including Church) would do. (By the way, most of the adherers of the other p-w-systems would deny this answer for some kinds of context only.) A semantic analysis of a sentence cannot take into account the actual truth-value: this would mean that one mixes up understanding with verifying.

The object named by (S) is an example of what are called *propositions*. These are functions associating with every possible world a function which associates with any time moment at most one truth-value. Thus (S) denotes a function which at the actual world takes T at the time moments, e.g., during the January 1978, which is undefined at all the time moments when there is either no American or no French President, etc. This proposition is at the time moments during January 1978 false in some other possible worlds.

Individual concepts, properties, relations-in-intension, propositions are examples of *intensions*. We now adduce a general definition:

Def 7 Let η be a type (over some base "of a language"). Any $\eta(\omega)$-object will be called *intension*.

(This is a harmless – in our paper – simplification of an inductive definition by Tichý.)

Remark. One would probably expect that intensions should be defined as $\eta(\tau)(\omega)$-objects. This is, however, unnecessary: the following examples of English phrases reveal that there are such $\eta(\omega)$-objects where, for every type ξ, $\eta \neq \xi(\tau)$, which one intuitively would like to classify as intensions: "The American President at noon January 2nd, 1978" (type $\iota(\omega)$), "to be red at four o'clock April 1st, 1967" (type $o(\iota)(\omega)$), "The date of the beginning of the Second World War is September 1st, 1939" (type $o(\omega)$).

All the objects that are not intensions will be called *extensions*. Phrases that are meant as labels of some individuals (proper names, e.g., "Prague") denote directly those individuals. Similarly, names of numbers denote extensions. Names of classes ("prime numbers") and of relations-in-extension (mathematical relations) denote extensions. Phrases naming truth-values (in a sense this concerns mathematical and logical theorems) name *eo ipso* extensions, too. Especially we can show that the functions known as (the denotata of) logical connectives and quantifiers are extensions.

Take, e.g., the phrase "not" and assume that it corresponds to the logical connective called negation. Since negation has truth-functional character, it is clear that the function, say, $\sim$, named by the above phrase is defined as follows:

$$\begin{matrix} T & F \\ F & T \end{matrix}$$

and that its type is $o(o)$. Similarly, the binary logical connectives have the type $o(o, o)$.

Remark. Since not only total functions are taken into account in the **T**-system, no third "truth-value" is needed. Such a system is, truth-functionally, equivalent to the Bochvarian three-valued logic.

The objects named by the phrases like "every", "some" etc. are called quantifiers. The universal quantifiers, Π^{η}, are objects of the types $o(o(\eta))$. They associate a class of the type $o(\eta)$ with T, if this class contains all elements of η, and with F otherwise. The existential quantifiers, Σ^{η}, are objects of the same types, which associate a class of η-objects with T, if this class is non-empty, and with F otherwise. Instead of $\Pi^{\eta}(\lambda x(\boldsymbol{A}))$, $\Sigma^{\eta}(\lambda x(\boldsymbol{A}))$ we usually write $\forall x\boldsymbol{A}$, $\exists x\boldsymbol{A}$, respectively. (x is an η-variable.)

B. Expressions and Constructions

According to Frege and Church, a meaningful expression denotes an extension and expresses its "sense" ("meaning"), where the "sense" of the expression E is the "concept" of the object which is denoted by E. Since "concept" might be explicable as intension, one can imagine that the schema of these interrelations has the following form:

$$\begin{array}{ccc} & \text{E} & \\ \text{denotes} \swarrow & & \searrow \text{expresses} \\ \text{object (extension)} & & \text{intension} \end{array}$$

In the **T**-system this form is essentially modified:

$$\begin{array}{ccc} & \text{E} & \\ \text{denotes} \swarrow & & \searrow \text{expresses} \\ \text{intension} & \xleftarrow[\text{constructs}]{} & \text{construction} \end{array}$$

(In some cases "intension" may be substituted for by "extension" – see A. about names of extensions.)

Thus if E is a sentence one gets:

Frege-Church

$$\begin{array}{ccc} & \text{sentence} & \\ \text{denotes} \swarrow & & \searrow \text{expresses} \\ \text{truth-value} & & \text{proposition} \end{array}$$

T-system

$$\begin{array}{ccc} & \text{sentence} & \\ \text{denotes} \swarrow & & \searrow \text{expresses} \\ \text{proposition} & & \text{an } o(\tau)\ (\omega)\text{-construction} \end{array}$$

On the basis of the **T**-system schema one can define two kinds of synonyms:

Def 8 Two expressions E_1 and E_2 that express constructions of the type $\eta(\xi_1, \ldots, \xi_n)$, $n \geqq 1$, are *weakly synonymous* iff E_1 denotes the same object as E_2. Any two expressions E_1 and E_2 are *strongly synonymous* iff E_1 expresses the same construction as E_2.

Example. Where A, B are names of individuals A, B, respectively, the sentence

(S_1) *A is older than B*

is weakly synonymous with the sentence

(S_2) *B is younger than A*

because (S_1) denotes the same proposition as (S_2),
while

$$\sqrt{(2)}$$

is strongly synonymous with the expression

the square root of 2.

In the **T**-system a special kind of construction is distinguished: the "linguistic constructions". Tichý's hypothesis is that every meaningful expression of a language is principally unambiguously analyzable so that the result of this analysis is a linguistic construction. Terminologically, this construction is just what Tichý calls *the* analysis of the given expression.

The reproduction of the definition of "linguistic constructions" would call for a very detailed exposition. Therefore, we shall not introduce the concept of linguistic constructions at all. Our applications of the **T**-system will, however, be not as precise as they would be if we used this concept. Instead, we shall give some examples of intuitive analyses of some expressions.

Take the sentence

(1) *The tallest man in the world is a friend of A*,

where A is a name of the individual A. For the sake of simplicity, let G be the atom which is the tallest man in the world. It is clear that G is of the type $\iota(\tau)(\omega)$. Furthermore, if H is the relation-in-intension of being a friend (of), i.e., if H is of the type $o(\iota, \iota)(\tau)(\omega)$, then we can analyze (1) as follows:

$$(1') \qquad \lambda w(\lambda t(\mathsf{H}(w)(t)(\mathsf{G}(w)(t), \mathsf{A}))),$$

where w, t, are ω-, τ-variables, respectively.

Applying Def 4 we can derive the type of (1′), which should be – according to what has been said – $o(\tau)(\omega)$. The process of this derivation can be represented by the following table:

Construction	Type	The relevant part of Def 4
$H(w)$	$o(\iota, \iota)(\tau)$	ii)
$H(w)(t)$	$o(\iota, \iota)$	ii)
$G(w)$	$\iota(\tau)$	ii)
$G(w)(t)$	ι	ii)
A	ι	i)
$H(w)(t)(G(w)(t), A)$	o	ii)
$\lambda t(H(w)(t)(G(w)(t), A))$	$o(\tau)$	iii)
$(1')$	$o(\tau)(\omega)$	iii)

Let the value of G in the world W at the time moment S be the individual B, i.e., let B be the tallest man ... in W at S. Let the value of H in W at S be a relation-in-extension, one member of which is the pair $\langle B, A\rangle$. It follows then from Def 4 that $(1')$ constructs in W at S the truth-value T. We say that (1) is true in W at S or better that the proposition named by (1) is true in W at S. To claim that (1) is true at S is the same as to claim that among the possible worlds in which the proposition denoted by (1) is true at S is the actual world. (Since the semantics cannot determine which of the possible worlds is the actual one the problem of the truth-value of (1) in the actual world cannot be, of course, solved semantically.)

Another example: (C is the name of an individual C)

(2) *C believes that the tallest man in the world is a friend of A*

The phrase "to believe" denotes an atom, say, K, that is a relation-in-intension between an individual and a proposition. Thus the type of K is $o(\iota, o(\tau)(\omega))(\tau)(\omega)$. We analyze (2) as follows (we omit some parentheses without risking an ambiguity):

$$(2') \qquad \lambda w \lambda t(K(w)(t)(C, \lambda w \lambda t(H(w)(t)(G(w)(t), A)))) .$$

The type of $(2')$ is again $o(\tau)(\omega)$, as one can see from the following table:

Construction	Type
$K(w)$	$o(\iota, o(\tau)(\omega))(\tau)$
$K(w)(t)$	$o(\iota, o(\tau)(\omega))$
C	ι
$K(w)(t)(C, (1'))$	o
$\lambda t(K(w)(t)(C, (1')))$	$o(\tau)$
$\lambda w \lambda t(K(w)(t)(C, (1')))$	$o(\tau)(\omega)$

Another example:

(3) *C believes that the tallest man in the world was a friend of A*

Taking into account "was" in the subclause we get

$$(3') \qquad \lambda w \lambda t(K(w)(t)(C, \lambda w \lambda t\, \exists t'(\wedge\, (<(t', t), H(w)(t')(G(w)(t'), A)))))$$

where t' is a τ-variable and $<$ is the familiar $o(\tau, \tau)$-object.

Indeed, giving such and similar examples cannot be considered as being a satisfactory substitute for introducing a collection of rules of analysis for expressions of a (natural) language. Making up such a collection is, however, primarily a task for a linguist (who also has to take into account that natural languages are not "pure languages" in the sense of our considerations). Thus analyzing in the following text English expressions we shall do it intuitively, assuming that there are some rules which would justify our intuitions.

CHAPTER II. QUESTIONS

We shall suppose that a language L with the corresponding base $\boldsymbol{B}_L$ is given. Thus talking about constructions, types etc. we mean constructions, types etc. with respect to $\boldsymbol{B}_L$. Examples will be given for L being a fragment of English ($\boldsymbol{B}_L$ being *a priori* given to any user of English). Since no natural language is a "pure language" (which should contain no ambiguities and other "defects") many simplifications on the level of applying theoretical principles to real phenomena have to be admitted.

Furthermore, if a concept will be introduced not precisely enough, this introduction will be labelled as Pseudodef rather than Def. Pseudodefinitions should fix some intuitions; they would become definitions if some terms they contain were reduced to phrases that are intuitively perfectly clear (as the most simple mathematical terms are).

Def 9 Let $\mathfrak{Constr}_0$ be the collection of just those constructions which contain no free occurrence of a variable. The following kinds of members of $\mathfrak{Constr}_0$ are of importance: ***a****-constructions* ("analytic"): they are non-atomic η-constructions where η is not $\xi(\omega)$ for any type ξ. ***s***$_0$*-constructions:* they are $\eta(\omega)$-constructions where for every type ξ holds $\eta \neq \xi(\tau)$.
s$_1$*-constructions:* they are $\eta(\tau)(\omega)$-constructions. ***s***$_0$- and ***s***$_1$-constructions will be called ***s****-constructions* ("synthetic"). ***a***- and ***s***-constructions will be called ***q****-constructions.*

Def 10 An object constructed by an ***s***-construction is an ***s****-question*. An ***a***-construction is an ***a****-question*. ***s***-questions and ***a***-questions are ***questions***.

Def 11 A question Q is a *trivial* question iff either Q is an ***a***-question, or Q is an ***s***-question and there is an object A such that $Q(w) = A$ for every valuation.

Thus the construction 5 is not a trivial question because it is no question at all, but the construction $+(3, 2)$ is a trivial question. The object constructed by $\lambda w(+(3, 2))$ (i.e., the function which assigns to every possible world the number 5) is also a trivial question. Another example is the proposition constructed by

$$\lambda w \lambda t(\mathit{Older}(w)(t)(A, B) \rightarrow \mathit{Younger}(w)(t)(B, A))$$

(where *Older* and *Younger* are the relations-in-intension of being older and being younger, respectively, and the connective of implication is written in a traditional manner).

Remark. Instead of "$\sim(\boldsymbol{A})$", "$\wedge(\boldsymbol{A}, \boldsymbol{B})$", etc., "$=(\boldsymbol{A}, \boldsymbol{B})$" we shall write "$\sim\boldsymbol{A}$", "$(\boldsymbol{A} \wedge \boldsymbol{B})$", etc., "$(\boldsymbol{A} = \boldsymbol{B})$".

Pseudodef 1. A ***q**-expression* (of the language L) is an expression (or L) which expresses a **q**-construction.

Remark. Pseudodef 1 would become a definition, if the relation of expressing (a construction) were given by a collection of rules of analysis (see the concluding considerations in Ch. I.).

Pseudodef 2 An *interrogative transformation* (IT) is a function which associates with every **q**-expression E a set of expressions satisfying the following conditions:

i) If E is an **a**- or **s**-expression and the relevant **q**-construction expressed by E is an o- or $o(\omega)$- or $o(\tau)(\omega)$-construction, then IT associates with E any grammatically correct expression that differs from E by containing the question-mark and – as the case may be – by having a changed word-order and/or an inserted auxiliary verb. i) holds approximatively for English; it is insufficient, e.g., for Polish.

ii) If E is a **q**-expression of any kind and the type of the **q**-construction expressed by E is other than in i), then IT associates with E any grammatically correct expression that differs from E by containing the question-mark, by having inserted at least one "wh-expression" like "which are", "who is" "what is", and – as the case may be – by having in the remaining part a changed word-order.

The following table shows some possibilities of meaning-preserving substituting some phrases for by other phrases (the italics marks the beginning of the relevant **q**-expression):

"original" phrase	possible variant
which are *the reasons of the fact that*	why
which are *the cause of the fact that*	why
which is *the time when*	when
which is *the place where*	where
which is *the way how*	how
what is *the number of*	how many ... there are

Convention. The expression E to which an IT is applied will be called the *IT^E-input*, the resulting set of expressions will be called the *IT^E-output*.

Pseudodef 3 An *interrogative* E-*sentence* is any member of the IT^E-output. An *interrogative sentence* is an interrogative E-sentence for some ***q***-expression E.

Remark. The expression E will be called *the core* of the given interrogative E-sentence.

An interrogative sentence A differs from its core B only in that A demonstrates the interrogative attitude of the (potential) speaker to the question which is either identical with the ***q***-construction expressed by B (the case of ***a***-questions) or constructed by the ***q***-construction expressed by B (the case of ***s***-questions). This is to say that *the interrogative mood is something what does not bear a semantic character*. The *pragmatic* character of this mood in this connection is stipulated in [9], [10]. Indeed, questions are entities to which one can adopt an interrogative (*inter alia*) attitude: thus according to Def 10 one can adopt the interrogative attitude (which does not exclude the possibility of adopting another attitude) to constructions (***a***-questions) and to intensions (***s***-questions). The circumstance (demonstrated by the interrogative mood) that one adopts just the interrogative attitude is relevant to the "internal pragmatics" (see [9]) rather than to semantics.

Pseudodef 4 Let A be an interrogative E-sentence.

i) If E is an ***a***-expression we say that A ***a****-asks* the question which is identical with the ***a***-construction expressed by E.

ii) If E is an ***s***-expression we say that A ***s****-asks* the question which is constructed by the ***s***-construction expressed by E.

iii) A *asks* the question Q iff A ***a***-asks Q ot A ***s***-asks Q.

Def 12 Q is a *yes-no-question* iff Q is an ***a***-construction of type o or Q is a proposition. Q is a *which-question* iff Q is a question and Q is not a yes-no-question.

178 **Pseudodef 5** Let A be an interrogative sentence which asks the question Q. Then

i) A is *trivial* iff Q is a trivial question;

ii) A is a *yes-no-interrogative sentence* (a *which-interrogative sentence*) iff Q is a yes-no-question (a which-question).

Example. 1) The interrogative sentence

(1) *Is* $\sqrt{(64)}$ *greater than* 7?

is trivial: the core of (1) is the sentence

$\sqrt{(64)}$ *is greater than* 7 ,

which expresses the ***a***-construction

(1′) $>(\sqrt{(64)}, 7)$.

Since (1′) is an o-construction, (1) is also a yes-no-interrogative sentence.

2) The interrogative sentence

(2) *Which are the colors?*

is trivial: the word "color" names the class of colors, i.e., an $o(o(\iota)(\tau)(\omega)$-object – thus the core of (2) expresses the ***a***-construction $\lambda x(Col(x))$, where x is an $o(\iota)(\tau)(\omega)$-variable. It is also clear that (2) is a which-interrogative sentence.

3) The interrogative sentence

(3) *Which is the even prime number?*

is trivial. The construction expressed by the core of (3) is

(3′) $\imath x(Pr(x) \wedge Ev(x))$,

where x is a v_0-variable, Pr is the class of prime numbers and Ev the class of even numbers. (3′) is an ***a***-question.

4) The interrogative sentence

(4) *What is subtracting?*

might be conceived of as expressing a *mass problem* (see Appendix) or as a trivial interrogative sentence. In this last case the construction expressed by the core of (4) is

(4′) $\lambda xy(-(x, y))$,

where x, y are, say, v_1-variables.

5) The interrogative sentence

(5) *Is the American President older than the French President?*

is not trivial. The question asked by (5) is the proposition constructed by the s_1-construction

$$(5') \qquad \lambda w\, \lambda t(Older(w)\,(t)\,(Amp(w)\,(t),\, Frp(w)\,(t)))\,,$$

where w, t are as usually and the abbreviations make clear which atoms they stand for.

6) The interrogative sentence (a famous one!)

(6) *Is the French King bald?*

is not trivial. The question asked by (6) is the proposition constructed by the s_1-construction

$$(6') \qquad \lambda w\, \lambda t(Ba(w)\,(t)\,(Frk(w)\,(t)))\,.$$

7) The interrogative sentence

(7) *Who is the Pope 1. 1. 1978 at noon?*

is not trivial. The question asked by (7) is the individual concept constructed by the s_0-construction

$$(7') \qquad \lambda w(\imath x(x = Po(w)\,(S)))\,,$$

where x is an ι-variable, Po is the Pope, i.e., an object, and S is the time moment determined by the date in (7).

Remark. Similarly as in (3′), $\imath x(\boldsymbol{A})$, where x is an η-variable and $\boldsymbol{A}$ is an o-construction, stands for $I(\lambda x(\boldsymbol{A}))$, where I is an $\eta(o(\eta))$-object: it is the function which associates the unit classes of η-objects with the only member of the given class and is undefined at the other classes.

8) The interrogative sentence

(8) *Is A older than B, or is he younger than B?*

is a non-trivial which-interrogative sentence. The core of (8) is the expression

the only true proposition from: that A is older than B, and that A is younger than B, which expresses the following s_1-construction:

$$(8') \qquad \lambda w\, \lambda t(\imath x(Tr(w)\,(t)\,(x) \wedge (x = \lambda w\, \lambda t(Older(w)\,(t)\,(A, B)) \vee x = \lambda w\, \lambda t(Younger(w)\,(t)\,(A, B)))))$$

where x is an $o(\tau)(\omega)$-variable and Tr is an $o(o(\tau)(\omega))(\tau)(\omega)$-object: it is the function which associates every possible world with a function which at any time moment t associates every proposition true at t with T and every other proposition with F. In other words, it is the property of propositions "being true".

Remark. We shall see in Ch. III. that no analysis (in the sense of Ch. I. B.) of mathematical alternative interrogative sentences is possible in our system. This regrettable fact will be, of course, explained.

Before we define answers to questions we shall define the important concept of *presuppositions* of questions. Leaving aside the "pragmatic presuppositions" (see, e.g., [5]) we construct our definition so as to make it very universal. Thus one can see that various kinds of presuppositions of questions being known from the literature can be understood as special instances of our concept, all satisfying Def 13. Also, it is not without interest that our definition does not presuppose – as, e.g., Kubiński does – that the concept of answer has been already defined.

Def 13 i) Let Q be an $\boldsymbol{a}$-question of type η. The *presupposition of* Q is the construction

$$\exists x(x = Q),$$

where x is an η-variable.

ii) Let Q' be an s_0-question.
A *presupposition of* Q' is any proposition that takes the value T in the world W if Q' takes a value in W.

iii) Let Q'' be an s_1-question.
A *presupposition of* Q'' is any proposition that takes the value T in the world W at the time moment S if Q'' takes a value in W at S.

Examples. 9) Take the interrogative sentence

(9) *Is* 5 : 0 *greater* 5?

According to Def 13 the presupposition of (9) is the construction

$$\exists x(x = > (:(5, 0), 5)), \tag{9'}$$

where x is an o-variable.

Note that (9′) does not construct T. The necessary condition for constructing T is for (9′) – as well as for any presupposition of an $\boldsymbol{a}$-question – that every subconstruction of it such that no variable has a free occurrence in it is proper. This condition is not satisfied for the subconstruction

$$:(5, 0).$$

10) The interrogative sentence (6) asks the s_1-question constructed by (6'). Among the presuppositions of this question there is the proposition constructed by

$$(6'') \qquad \lambda w\, \lambda t(\exists x(x = Frk(w)\,(t)))\,.$$

Since this proposition takes in the actual world, say, in July 1978, always the value F, it is clear that the proposition which is the s_1-question constructed by (6') takes in the actual world at the above time moments no value at all.

11) Take the interrogative sentence (8). A presupposition of the question constructed by (8') is – when we use "$\boldsymbol{Trwtx}$" as abbreviation for the maximal subconstruction of (8') beginning with "$Tr(w)$ (t) (x)" – the proposition constructed by

$$(8'') \qquad \lambda w\, \lambda t(\exists x(\boldsymbol{Trwtx}))\,.$$

Take a world W and a time moment S where A and B are twins. It is clear that the proposition constructed by (8'') takes F in W at S. At the same time, the function constructed by (8') takes in W at S no value. Notice that nobody can offer a right answer to our question in W at S.

12) The interrogative sentence

(12) *Does A know that B is older than C?*

asks the question constructed by the s_1-construction

$$(12') \qquad \lambda w\, \lambda t(Kn(w)\,(t)\,(A, \lambda w\, \lambda t(Older(w)\,(t)\,(B, C))))\,.$$

The construction

$$(12'') \qquad \lambda w\, \lambda t(Older(w)\,(t)\,(B, C))$$

constructs a presupposition of the question constructed by (12'): if the proposition constructed by (12'') does not take T in the world W at S, then the question constructed by (12') takes in W at S no value.

Taking over Hausser's term ([3]) we define the concept of a redundant answer to a question.

Def 14 i) Let Q be an a-question. A *redundant answer* (RA) *to* Q is any construction

$$Q = \boldsymbol{A}\,,$$

where $\boldsymbol{A} \in \mathfrak{Constr}_0$, the type of $\boldsymbol{A}$ = the type of Q, and $\boldsymbol{A}$ differs from Q not only by bound occurrences of variables.

ii) Let Q be an s_0-question. Furthermore, where $\boldsymbol{C}$ is a construction of type $\eta(\omega)$, let $\boldsymbol{C}'$ be defined as follows: $\lambda w(\boldsymbol{C}')$ is the construction $\boldsymbol{C}$. Then an *RA to* Q is any proposition constructed by

$$\lambda w(\boldsymbol{C}'_Q = \boldsymbol{A})\,.$$

where $\mathbf{C}_Q$ is the construction constructing Q, $\mathbf{A} \in \mathfrak{Constr}_0$ and $\mathbf{A}$ is of the same type as $\mathbf{C}'_Q$.

iii) Let Q be an s_1-question. Furthermore, where $\mathbf{C}$ is a construction of type $\eta(\tau)(\omega)$, let $\mathbf{C}''$ be defined as follows: $\lambda w\, \lambda t(\mathbf{C}'')$ is the construction $\mathbf{C}$. Then an *RA to Q* is any proposition constructed by

$$\lambda w\, \lambda t(\mathbf{C}''_Q = \mathbf{A}),$$

where the conditions are analogous as in ii).

Def 15 Let P_Q be a presupposition of the question Q. Let $\mathbf{C}_{P_Q}$ be the construction constructing P_Q, $\mathbf{C}'$ and $\mathbf{C}''$ be as in Def 14.

i) If Q is an $\boldsymbol{a}$-question, then *the refutation* of Q is the construction

$$\sim \exists x(x = Q).$$

ii) If Q is an s_0-question, then the proposition constructed by

$$\lambda w(\sim \mathbf{C}'_{P_Q})$$

is *a refutation of Q*.

iii) If Q is an $\boldsymbol{s}_1$-question, then the proposition constructed by

$$\lambda w\, \lambda t(\sim \mathbf{C}''_{P_Q})$$

is *a refutation of Q*.

Def 16 An $\boldsymbol{a}$-construction is *true* iff it constructs T. A proposition of type $o(\omega)$ is *true* iff it takes T in the actual world.
A proposition of type $o(\tau)(\omega)$ is *true at* (the time moment) S iff it takes T in the actual world at S.

Def 17 Let Q be an $\boldsymbol{a}$-question or an s_0-question. A *right RA to Q* is such an *RA* to Q that is true. Let Q' be an s_1-question. The right *RA* to Q' at S is such an *RA* to Q' that is true at S.

Two very simple statements might be easily proved:

Statement 1. For every question Q there is an *RA* to Q.

Statement 2. Let Q, Q' be as in Def 17. There is a right *RA* to Q iff every presupposition of Q is true. There is a right *RA* to Q' at S iff every presupposition of Q' is true at S.

Corollary. There is no right *RA* to Q (no right *RA* to Q' at S) iff at least one refutation of Q (of Q') is true (true at S).

Examples. Two RA's to the question (1′) are:

a) $> (\sqrt{(64)}, 7) = T$ and

b) $> (\sqrt{(64)}, 7) = F$.

The presupposition of (1′) is true. The right RA to (1′) is a).

The right RA to the question asked by (2) is the construction

$$\lambda x(Col(x)) = \lambda x(x = White \vee x = Red \vee \ldots \vee x \equiv Black)\,.$$

The right RA to (3′) is the construction

$$\imath x(Pr(x) \wedge Ev(x)) = 2\,.$$

Let $\mathbf{D}^-$ be some definition of subtracting Let $\mathbf{D\bar{x}}$ be the construction which is the result of substituting the $v_1(v_1, v_1)$-variable x for the atom $-$ (subtracting) in $\mathbf{D}^-$. Then a right RA to (4′) is the construction

$$\lambda xy(-(x, y)) = \imath z(\mathbf{D\bar{z}})\,,$$

where z is an $v_1(v_1, v_1)$-variable.

Let S be the time moment June 1st, 1978, midnight. No refutation of the question constructed by (5′) is true at S, therefore there is a right RA at S to this question, namely the proposition constructed by

(5″) $$\lambda w\, \lambda t((Older(w)(t)(Amp(w)(t), Frp(w)(t))) = F)$$

(which is, be the way, the same proposition as that one constructed by

$$\lambda w\, \lambda t(\sim Older(w)(t)(Amp(w)(t), Frp(w)(t))).)\,.$$

Let S be as in the preceding example. A refutation of the question asked by (6) is true at S: indeed, there is no French King at S.Therefore, there is no right RA at S to this question.

Take now the question asked by the interrogative sentence

(13) *Who are the three tallest men in the world?*

Let $\mathbf{C}_{13}$ be the construction expressed by the core of (13), i.e., the construction constructing the question asked by (13). Notice that an RA to this question will be any proposition constructed by

$$\lambda w\, \lambda t(\mathbf{C}''_{13} = \lambda x(x = A \vee x = B \vee x = C))\,,$$

where A, B, C are individuals, whereas propositions named by sentences like

The three tallest men in the world are the French President, the Governor of ..., the most famous piano player are not *RA*'s to this question in the sense of our definitions (***NB*** the condition $\boldsymbol{A} \in \in \mathfrak{Constr}_0$ in Def 14!).

Let us abbreviate the subconstruction of (8′) that begins with $\imath x$ as "$\mathbf{C}_8$". Let A be in the actual world (at some moment S) older than B. Then the right *RA* at S to the question asked by (8) is the proposition constructed by

$$(8'') \qquad \lambda w\, \lambda t(\mathbf{C}_8 = \lambda w\, \lambda t(Older(w)(t)(A, B))) .$$

Def 17 i) Let the construction

$$Q = \boldsymbol{A} ,$$

say, $\mathbf{C}$, be an *RA* to an $\boldsymbol{a}$-question Q. A *non-redundant answer to Q with respect to* $\mathbf{C}$ (NRA_C) will be the object constructed by $\boldsymbol{A}$.

ii) Let the proposition constructed by the construction

$$\lambda w(\mathbf{C}'_Q = \boldsymbol{A}) ,$$

say, $\mathbf{C}$, be an *RA* to an s_0-question Q. NRA_C *to* Q will be the object constructed by $\boldsymbol{A}$.

iii) Let the proposition constructed by the construction

$$\lambda w\, \lambda t(\mathbf{C}''_Q = \boldsymbol{A}) ,$$

say, $\mathbf{C}$ be an *RA* to an s_1-question Q. An NRA_C *to* Q will be the object constructed by $\boldsymbol{A}$.

Def 18 i) Let Q be an $\boldsymbol{a}$-question. *The right NRA to Q* is the object constructed by Q.

ii) Let Q be an s_0-question. *The right NRA to Q* is the value of Q in the actual world.

iii) Let Q be an s_1-question. *The right NRA to Q at* S is the object which is the value of Q in the actual world at S.

Statement 3. Let E be an *RA* and G an NRA_E to the question Q. Then E is a right *RA* (at S) to Q iff G is the right *NRA* (at S) to Q.

(This is easily provable by means of confrontation of Def 14, Def 17, and Def 18.)

Examples. The right *NRA* to the question (1′) is T. The right *NRA* to the question (3′) is 2.

An *NRA* to the question asked by (8) is the proposition that A is older than B. Whether this proposition is a right *NRA* at S to this question depends on the actual state of affairs at S.

Remark. Answers, as well as questions, are in our conception not expressions of a language (being either constructions or intensions). Asking a question (in the commonly used sense, not in the sense of Pseudodef 4) means adopting an interrogative attitude to an entity. In the same way, answering a question means using the language for naming the answers in our sense. Therefore, something like "*answering expression*" – analogously as in Pseudodef 3 – might be defined. Thus for example the expressions of English naming the truth-values are "yes" (naming T) and "no" (naming F). Since an *NRA* to a yes-no-question is T or F, answering in English a yes-no-question means to say "yes" or "no" (or to formulate a refutation of the question).

Def 19 Let S_1, S_2 be interrogative sentence asking

i) the $\boldsymbol{a}$-questions Q_1, Q_2, respectively; S_1 *is equivalent to* S_2 iff Q_1 differs from Q_2 at most by Q_2 being the result of a collisionless overnaming of the (bound) variables occurring in Q_1;

ii) the $\boldsymbol{s}$-questions Q_1', Q_2', respectively; S_1 *is equivalent to* S_2 iff $Q_1' = Q_2'$.

Remarks. Since expressions of a language do not contain (bound) occurrences of variables, i) reduces to the requirement that S_1 differs from S_2 by at most some semantically irrelevant grammatical features.

One not very intuitive consequence of i) is that, e.g., the interrogative sentence

Which is the sum of three and five?

is not equivalent to the interrogative sentence

Which is the sum of five and three?

This may seem to be surprising, of course, but no absurdity is present. Matters of this kind are connected with the special character of $\boldsymbol{a}$-constructions; a little more will be said in Ch. III.

Def 20 Let Q be a non-trivial $\boldsymbol{s}$-question.

i) If B is an *RA* to Q, then any other proposition that takes T in at least one possible world and implies B (i.e., is such that B takes T in every world in which this proposition takes T) will be called *an indirect answer to Q*.

ii) If B is an *RA* to Q, then any other proposition that takes F in at least one possible world and is implied by B will be called a *partial answer to Q*.

Examples. Take the question asked by (5). An indirect answer to it will be, e.g., the proposition that some individual, say, C is younger that the French President and at the same time older than the American President: it is clear that this proposition implies that the American President is not older than the French President, so that an *RA* to the given question is implied. A partial answer to the above question is, e.g., the proposition that the American President is not younger than the French President: this proposition is implied by one of the RAs to the given question.

To take a which-question, consider the question asked by the interrogative sentence

How many students on the Oxford University are active sportsmen?

Recall that the core of this interrogative sentence is the expression "the number of the students of the Oxford University which are active sportsmen". A partial answer to this question is, e.g., the proposition named by the sentence

The number of the students on the Oxford University which are active sportsmen is not less than 250.

Indeed, this proposition is implied by any *RA* to it according to which the number of the students etc. is *N*, where *N* is at least 250.

Remark. Unlike the *NRA*'s, the indirect and the partial answers always are propositions.

CHAPTER III. COMMENTS

1. The Main Idea

The main idea stimulating the present conception of questions consists in explicating the following intuition:

If we ask a question, then we wish to know an entity which is co-determined by the question.

In our conception, if the question is a construction (the case of ***a***-questions), then the object we wish to know is the result of a procedure which is given by this construction. On the other hand, if the question is an intension, then the object we are interested in is the value of this intension in the actual world, or, as the case may be, in the actual world at a given time moment.

Thus the core of any yes-no-s-interrogative sentence expresses a construction which constructs a proposition. In such a case, using the interrogative sentence we do not wish to know the proposition named by the core of this sentence – we do know this proposition when understanding the sentence. We wish to know the truth-value being taken by this proposition in the actual world (at the moment S). Similarly asking who is the American President (at the moment S) we already know the individual concept named by the expression "the American President"; we are interested

in knowing which individual is the value of this concept in the actual world (at S).
Or, using an alternative interrogative sentence

$$A_1,\ or\ A_2,\ \ldots\ or\ A_n?$$

we already know the concept of such a proposition which is true and is one of the propositions named by $A_1, \ldots, A_n$; what we wish to know is the value of this concept in the actual world (at S), i.e., the concrete proposition satisfying in the actual world (at S) the criterion given by this concept.

The above idea explains the way in which we have defined RAs and NRA's to questions.

Notice also that the logical analyses made according to this idea are not subject to the restrictions which are necessary within the 1st order systems (see, e.g., [8]).

2. Some Troubles

It may be hoped that showing (instead of hiding) some troubles connected with our conception will help to find such remedies which will bring about further development of this conception.

a) Firstly, one special case, not too harmful but requiring our attention: take the alternative question asked by the interrogative sentence

(Alt) *Is A younger than B, or is B older than A?*

Intuitively, the best reaction to such a question would be refuting it. Yet in our conception no reason for a true refutation can be found. Indeed, our question is constructed by

$$\lambda w\, \lambda t(\iota x(Tr(w)(t)(x) \wedge (x = \lambda w\, \lambda t(\mathit{Younger}(w)(t)(A, B)) \vee \\ \vee\ x = \lambda w\, \lambda t(\mathit{Older}(w)(t)(B, A)))))$$

Let us assume that A and B are of different age (in the actual world at S). Then both the interesting presuppositions of our question Q hold (in the actual world at S): the first presupposition is constructed by

$$\lambda w\, \lambda t(\exists x(Tr(w)(t)(x) \wedge (x = \lambda w\, \lambda t(\mathit{Younger}(w)(t)(A, B)) \vee \\ \vee\ x = \lambda w\, \lambda t(\mathit{Older}(w)(t)(B, A)))))$$

and under the above assumption, is true (at S); the second presupposition is constructed by

$$\lambda w\, \lambda t(\forall y\, \forall z(((Tr(w)(t)(y) \wedge (y = \lambda w\, \lambda t(\mathit{Younger}(w)(t)(A, B)) \vee \\ \vee\ y = \lambda w\, \lambda t(\mathit{Older}(w)(t)(B, A)))) \wedge \\ \wedge\ Tr(w)(t)(z) \wedge (z \equiv \lambda w\, \lambda t(\mathit{Younger}(w)(t)(A, B)) \vee \\ \vee\ z = \lambda w\, \lambda t(\mathit{Older}(w)(t)(B, A))))) \rightarrow y = z))$$

188 Since, however, the proposition constructed by

$$(C_1) \qquad \lambda w\, \lambda t(\mathit{Younger}(w)\,(t)\,(A, B))$$

is identical with the proposition constructed by

$$(C_2) \qquad \lambda w\, \lambda t(\mathit{Older}(w)\,(t)\,(B, A))$$

(i.e., the sentence "A is younger than B" is weakly synonymous with the sentence "B is older than A"), this second presupposition holds, too. Thus there is a right *RA* and a right *NRA* to Q; e.g., the right *NRA* will be the proposition constructed by (C_1), or which is the same, by (C_2); the way of specifying this proposition is not determined within our conception: the name of the *NRA* may be

A is younger than B

as well

B is older than A.

We repeat that this solution is not what one intuitively would expect: answering Q in the above way we can be suspected of joking. Nevertheless, the troubles arising from this disharmony with our intuitions (or with our prejudices?) are not too great.

b) The main troubles are however, connected with ***a***-questions and mathematical interrogative sentences.

Let us begin with the problem of *alternative mathematical interrogative sentences.* Take, e.g., the sentence

(Alt') *Is* $\sqrt{}(64)$ *greater than 7, or smaller than 7?*

The attempt to analyze sentences like (Alt') in the manner which would be analogous to the analysis of the alternative interrogative *s*-sentences will break down. We shall show this as follows: Assume that asking a question Q asked by (Alt') we wish to know the only object that is true and is identical with just one of the objects named by "$\sqrt{}(64)$ is greater than 7" and "$\sqrt{}(64)$ is smaller than 7". We might be, therefore, tempted to take for the construction Q the following construction:

$$(*) \qquad \iota x(x = T \wedge (x = {>}(\sqrt{}(64), 7) \vee x = {<}(\sqrt{}(64), 7)))\,.$$

Yet, unfortunately, (*) is not what we wish to get. The range of x are the truth-values, but what we are interested in, is not a truth-value (we cannot answer Q by saying "yes" or "no"). In the case of alternative ***s***-questions the relevant object is a proposition. No ***a***-construction, however, constructs a proposition, so that this solution is impossible.

One apparent way out is thinkable: we could let the ***a***-subconstructions of (*) be substituted for by *s*-constructions like

$$\lambda w > (\sqrt{}(64), 7)\,;$$

such constructions construct trivial questions, of course. Our analysis of (Alt') will then be

$$(^{**})\qquad \lambda w\imath x(Tr'(w)(x) \wedge (x = \lambda w(>(\sqrt{(64)}, 7)) \vee x = \lambda w(<(\sqrt{(64)}, 7))))$$

Since, however, every construction of this kind constructs the concept of a trivial proposition (i.e., of such a proposition that associates every possible world with T or every possible world with F or is underfined at every possible world), the consequences of this "way out" are absolutely counterintuitive.

Remark. In (**) the atom Tr' has been used. Its type is easily derivable, its character is analogous to the character of Tr.

Thus we come to the second point concerning the ***a***-questions. It is impossible within our system to define an equivalence relation between two interrogative ***a***-sentences in such a manner that the conditions of S_1's being equivalent to S_2 were weaker than those in Def. 19. If some analogy with Def 19 ii) were used for defining equivalence of interrogative ***a***-sentences, then the interrogative sentence

Is $\sqrt{(64)}$ *greater than 7?*

would be equivalent to the sentence

Does it hold that the sum of the angles in an Euclidean triangle equals 180°*?*

Similarly, the sentence

How many edges has a tetraeder?

would be equivalent to the sentence

Which is the greatest common divisor of twelve and eighteen?

This is counterintuitive, which can be easily seen from the following consideration.

Any definition of the equivalence of interrogative sentences has to take into account the principle according to which the interrogative sentence S_1 asking Q_1 is equivalent to the interrogative sentence S_2 asking Q_2 iff, for every individual X, the proposition that X asks Q_1 is identical with the proposition that X asks Q_2.

Applying Def 19 ii) to the non-trivial interrogative sentences we obviously satisfy this principle: the proposition that, e.g., C asks whether A is younger than B is the same proposition as that C asks whether B is older than A. On the other hand, nobody will be disposed to claim that the proposition that C asks whether $\sqrt{(64)}$ is greater than 7 is the same proposition as that C asks whether it holds that the sum of the angles in Euclidean triangle equals 180°. Similarly, asking who is the latest husband of Mrs. A is the same as asking who is the man whom Mrs. A has last time married;

 but asking how many edges has a tetraeder is not the same as asking which is the greatest common divisor of twelve and eighteen.

This problem is a very fundamental one and its solution would make it possible to solve not only the other troubles with the **a**-questions and mathematical interrogative sentences but also the famous problems connected with the "belief sentences".

Now we shall sketch an orientation towards this solution.

Let an individual X ask an s-question Q^s. By asking Q^s X manifests his attitude towards an intension: X wishes to know the value of this intension in the actual world (at S).

*Let X ask an **a**-question Q^a. By asking Q^a X manifests his attitude towards a construction: X wishes to know the object constructed by Q^a.*

Therefore, there are two principally different groups of asking-relations:

I. *Relations-in-intension between an individual and an intension*, i.e., $o(\iota, \eta(\tau)(\omega))(\tau)(\omega)$-objects where η is a type.

II. *Relations-in-intension between an individual and a construction.* In our conception (better to say: in the **T**-system) there is no possibility of dealing with such relations except only verbally because the bases that are considered within this system do not contain a collection of constructions. We would have to shift our position "one level higher" and *let the constructions that construct the objects over our original base(s) become members of a new "meta-base".* (It would be then also necessary of course to define the new "*meta-constructions*".)

Thus e.g. only such indirect questions are analyzable within our system which contain the names of the asking-relations from the Ist group.

This approach to the above problems gives some hints concerning the problem of analyzing alternative and other – no more such simple as here – mathematical interrogative sentences. It seems that the core of such sentences is also analyzable only within a system based on a meta-base: indeed, using, e.g., an alternative mathematical interrogative sentence we wish to know neither a truth-value nor a proposition: what we wish to know is rather the only construction such that constructs truth and is one of the constructions determined by the particular members of the question.

Thus the following schema may be adduced:

Asking a question which is	*one wants to know*
an intension	*its value in the actual world (at S)*
*an **a**-construction*	*the object being constructed*
a meta-construction	*the construction being constructed.*

We now briefly show that our approach to questions is easily generalizable to defining mass problems. Our attention will be confined to what is commonly considered to be a mass problem; the mass problems are supposed to be mathematical constructions, so that no intension will be involved, although a generalization in this direction is thinkable. The limits of our generalization are the same as those considered in Ch. III. in connection with meta-constructions.

Def A1 Let Q be an $\boldsymbol{a}$-question, i.e., an a-construction. Let $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$ differ from Q just by containing, for every $i = 1, \ldots, n$, x_i instead of every occurrence of the object A_i in Q, where $x_1, \ldots, x_n$ are variables of appropriate types and such that they differ from every variable with a bound occurrence in Q. We call

$$Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$$

a ***p-construction***.

Def A2 Let $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$ and $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x'_1, \ldots, x'_n)$ be two p-constructions differing one from another at most by the circumstance that, for at least one i, $1 \leqq i \leqq n$, x'_i differs from x_i. We say that $Q(\mathsf{A}_1, \ldots, A_n : : x_1, \ldots, x_n)$ *constitutes the same mass problem as* $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x'_1, \ldots, x'_n)$. We abbreviate this by writting $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$ *CMP* $Q(\mathsf{A}_1, \ldots \ldots, \mathsf{A}_n/x'_1, \ldots, x'_n)$.

Obviously, the relation *CMP* between constructions is reflexive, symmetric and transitive. This justifies the following definition:

Def A3 Any abstraction class of the relation *CMP* is a *mass problem*. Let the members of a mass problem be $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$, $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x'_1, \ldots \ldots, x'_n)$, etc., where $x_1, \ldots, x_n$ are the alphabetically first n (mutually different) variables. This mass problem will be called *the mass problem with respect to* $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$. The question Q will be called – as well as any question $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/\mathsf{B}_1, \ldots, \mathsf{B}_n)$ – *an instance of the mass problem with respect to* $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$. The mass problem with respect to $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$ will be denoted by $\mathscr{P}_{Q(A_1,\ldots,A_n)}$.

Def A4 A solution to $\mathscr{P}_{Q(A_1,\ldots,A_n)}$ is an algorithm (i.e., a Turing machine, or a recursive function, or a normal algorithm, etc.) that for every valuation $\mathbf{v}(x_1, \ldots \ldots, x_n/X_1, \ldots, X_n)$ computes the object (if any) $\mathbf{v}(x_1, \ldots, x_n/X_1, \ldots, X_n)$-constructed by $Q(\mathsf{A}_1, \ldots, \mathsf{A}_n/x_1, \ldots, x_n)$.

Def A5 A mass problem is *solvable* iff there is a solution to it.

Def A6 The mass problem $\mathscr{P}_{Q(A_1,\dots,A_n)}$ is *equivalent to* the mass problem $\mathscr{P}_{Q'(A_1,\dots,A_n)}$ iff the solutions to the former compute the same function as the solution to the latter.

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Doc. PhDr. Pavel Materna, CSc., Projektový ústav dopravních a inženýrských staveb (Design Institute PÚDIS), Lidových milicí 69, 120 00 Praha 1. Czechoslovakia.

*) This Tichý's paper has not been taken into account here, because the author did not read it during his working out the present paper. The coincidence of many essential points is remarkable, of course.