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Dynamic Systems and Theory of Simulation

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The paper contains the definition of dynamic system and related conceptions so that they reflect the properties of the technique of simulation. The mathematical theory of simulation has been designed in [1] in four phases; the first phase has been presented in details in [2], the second one in [3]. In the present paper basic terms of the third phase — theory of dynamic systems — are declared, namely dynamic systems, based dynamic systems, dynamic attributes and dynamic classes. Subsidiary terms as events and dynamics are also presented; they serve to classify pragmatically dynamic systems. The development of the basic definition is described.

1. INTRODUCTION

In [1] a theory of simulation has been designed, based on traditional theory of sets. The necessity of such a theory has been implied in order to eliminate some misunderstandings about the definition of terms *system* and *model* used in the theory and practice of simulation, to clear some obstacles in deciding whether some language is a simulation one or whether some simulation language is a discrete event simulation language or continuous system simulation one and to reflect the richness of several hundreds of simulation languages which have been implemented and used and which represent a plentiful source of system abstraction. The designed theory has been structured into four phases: theory of static systems, theory of static models, theory of dynamic systems and theory of dynamic and simulation models. The first phase has been elaborated very detailly in [2]: the theory of static systems has been built hierarchically from the lowest of static attributes, over the middle level of static classes, up to the highest level of static systems. Static systems have been classified in order to reflect various aspects of computer modelling and to enable proving of important theorems. In the same paper the theory of subsystems and of system enlargements has been presented for static systems, too.

In the present paper the corresponding notions of the theory of dynamic systems are defined. As dynamic systems have a lot of important properties bound with their existence in time, which cannot be introduced for static systems, we have presented only the definitions concerning the basic hierarchy of attributes, classes and systems, while the theorems and the theory of subsystems and enlargements have been omitted. Instead of them specific notions for the theory of dynamic systems are presented, as events and dynamics of dynamic systems and new conception has been defined under the term based dynamic system which permit to classify the system components into transactions and activities (permanent elements) and to distinguish the dynamic systems into continuous ones and discrete ones.

In the present paper the same conventions concerning the terminology and the formalization are used as in [2]. The only exclusion is that instead of the term of attribute set of a class or of a system a more convenient term *characteristics* has been introduced. The acronyms of introduced notions have more letters than the corresponding acronyms introduced in [2] for static systems; thus a formal unambiguity has been ensured for a case of handling static and dynamic systems parallelly.

2. UNFORMAL PROPERTIES OF DYNAMIC SYSTEMS

The known concepts of dynamic systems introduced in the literatute have no value in the theory of simulation, because they do not permit to study the structural properties of dynamic systems and they exclude either continuous systems (those which permit a change in any moment of their existence which is an interval of real numbers) or systems with discrete changes. The dynamic system in simulation is structured into a set of classes; any class contains elements with "similar" properties (called commonly attributes): the similarity consists of the same name and of the same range of the values of such a property.

The simulation languages have introduced a practice that one element cannot be present in more than one class. In the modern universal language SIMULA 67 that practice has been eliminated by a hierarchical structuring of classes. Thus the axiom that the domains of two different classes must be disjoint has been eliminated of the presented theory. But in a relation with several implementation rules of modern programming languages permitting to incorporate effectivity in run time and in memory and security in computation we can observe that dynamic systems satisfy the following rules:

2.1. A class cannot change the number of its attributes.

2.2. An element cannot leave one class and enter another one.

2.3. An element cannot leave the system and then return to it.

2.4. The relativistic effects are neglected; namely the concept of contemporary events is substantially important.

In a contrast with the mentioned restrictions the following properties have been permitted and commonly applied:

2.5. An element can enter the system after the beginning of the existence of the same system and it can leave the system before the end of its existence.

2.6. The systems can exist in various sets of moments, among others in intervals, in discrete sets and in finite unions of such sets.

In the following parts the mentioned terms are defined and formalized. We use the same symbols as they have been introduced in part 1 and 2 of [2].

3. DYNAMIC ATTRIBUTES

Dynamic quasiattribute is an ordered pair $\langle n, f\rangle$ such that n is a text and f is a function defined at a subset of the Cartesian product of a nonstandard set and a set of real numbers.

$$\mathbf{quattr}(a) \equiv \exists n\, \exists f\, \exists p(a = \langle n, f\rangle \wedge n \in \mathscr{T} \wedge \mathbf{gen}(p) \wedge \mathbf{function}(f) \wedge \mathbf{domain}(f) \subseteq \subseteq p \times \mathscr{R}).$$

If $a = \langle n, f\rangle$ is a dynamic quasiattribute, we define its *name* as n, its *function* as f, its *range* as the range of f, its *definition set* as the domain of f and its *domain* resp. its *existence* as the projections of its definition set. We say that an element x is present in a at time t or that a is defined for x at time t if $\langle x, t\rangle$ is in the definition set of a. We write $a(x, t)$ instead of $f(x, t)$.

$$\mathbf{name}(a) = n$$
$$\mathbf{func}(a) = f$$
$$\mathbf{ran}(a) = \mathbf{range}(\mathbf{func}(a))$$
$$\mathbf{def}(a) = \mathbf{domain}(\mathbf{func}(a))$$
$$\mathbf{dom}(a) = \{x \mid \exists t(\langle x, t\rangle \in \mathbf{def}(a))\}$$
$$\mathbf{ex}(a) = \{t \mid \exists x(\langle x, t\rangle \in \mathbf{def}(a))\}$$
$$\mathbf{pres}(x, t, a) \equiv \langle x, t\rangle \in \mathbf{def}(a)$$

If the range of a dynamic quasiattribute is a subset of a standard set we call that quasiattribute *standard dynamic attribute*. If the range of a dynamic quasiattribute contains **none** or elements of a nonstandard set the quasiattribute is called *dynamic pointer*. A quasiattribute is called *dynamic attribute* if it is a standard dynamic attribute or a dynamic pointer.

$$\mathbf{sattr}(a) \equiv \exists P(\mathbf{stand}(P) \wedge \mathbf{ran}(a) \subseteq P)$$

$$\boldsymbol{ptr}(a) \equiv \boldsymbol{gen}(\boldsymbol{ran}(a) - \{\text{none}\})$$
$$\boldsymbol{attr}(a) \equiv \boldsymbol{sattr}(a) \vee \boldsymbol{ptr}(a)$$

Inasmuch we do not handle other attributes than dynamic ones in the present paper, we shall use to omit the word *dynamic* in case the meaning of the text is clear.

4. DYNAMIC CLASSES

Dynamic class is an ordered triplet $\langle n, P, G\rangle$ where n is a text, P is a subset of the Cartesian product of a nonstandard set and a set of real numbers and G is a set of attributes with different names and with their definition set equal to P.

$$\boldsymbol{Class}(A) \equiv \exists n\, \exists P\, \exists G\, \exists Q(A = \langle n, P, G\rangle \wedge n \in \mathscr{T} \wedge P \subseteq Q \times \mathscr{R} \wedge \boldsymbol{gen}(Q) \wedge$$
$$\wedge (a)(a \in G \rightarrow \boldsymbol{attr}(a) \wedge \boldsymbol{def}(a) = P \wedge (b)(b \in G \wedge b \neq a \rightarrow$$
$$\rightarrow \boldsymbol{name}(a) \neq \boldsymbol{name}(b))))$$

Let $A = \langle n, P, G\rangle$ be a dynamic class. We define its *name* as n, its *definition set* as P, its *characteristics* as G and its *domain* resp. *existence* as the projections of its definition set. We say that an element x is present in a dynamic class at time t if $\langle x, t\rangle$ is in its definition set.

$$\boldsymbol{Name}(A) = n$$
$$\boldsymbol{Def}(A) = P$$
$$\boldsymbol{Char}(A) = G$$
$$\boldsymbol{Dom}(A) = \{x \mid \exists t(\langle x, t\rangle \in \boldsymbol{Def}(A))\}$$
$$\boldsymbol{Ex}(A) = \{t \mid \exists x(\langle x, t\rangle \in \boldsymbol{Def}(A))\}$$
$$\boldsymbol{Pres}(x, t, A) \equiv \langle x, t\rangle \in \boldsymbol{Def}(A)$$

Corollary. Let a be an attribute of the characteristics of a dynamic class. A. Then $\boldsymbol{pres}(x, t, a) \equiv \boldsymbol{Pres}(x, t, A)$, $\boldsymbol{ex}(a) = \boldsymbol{Ex}(A)$ and $\boldsymbol{dom}(a) = \boldsymbol{Dom}(A)$.

Similarly as for dynamic attributes, also for dynamic classes the word *dynamic* will be omitted in the present paper.

5. DYNAMIC SYSTEMS

A *dynamic quasisystem* is a set of dynamic classes:

$$\boldsymbol{QSYS}(\Sigma) \equiv (A)(A \in \Sigma \rightarrow \boldsymbol{Class}(A))$$

We define its *definition set* resp. its *domain*, *existence* and *characteristics* as the set union of the definition sets resp. domains, existence and characteristics of all classes which are in it. We say an element x is present in a dynamic quasisystem at time t if $\langle x, t\rangle$ is in its definition set.

$$\mathbf{DEF}(\Sigma) = \bigcup_{A\in\Sigma} \mathbf{Def}(A)$$

$$\mathbf{DOM}(\Sigma) = \bigcup_{A\in\Sigma} \mathbf{Dom}(A)$$

$$\mathbf{EX}(\Sigma) = \bigcup_{A\in\Sigma} \mathbf{EX}(A)$$

$$\mathbf{CHAR}(\Sigma) = \bigcup_{A\in\Sigma} \mathbf{Char}(A)$$

$$\mathbf{PRES}(x, t, \Sigma) \equiv \langle x, t\rangle \in \mathbf{DEF}(\Sigma)$$

Let us mention that an empty set is also a dynamic quasisystem; in this case we take $\bigcup_{A\in\Sigma} F(A)$ as to be the empty set.

Corollary. Let Σ be a dynamic quasisystem and $A \in \Sigma$. Then $\mathbf{Pres}(x, t, A) \rightarrow \rightarrow \mathbf{PRES}(x, t, \Sigma)$.

A *dynamic system* is a non-empty dynamic quasisystem Σ satisfying the following conditions:

5.1. $(x)(t_1)(t_2)(t_2 \in \mathbf{EX}(\Sigma) \wedge t_1 < t_2 \wedge \mathbf{PRES}(x, t_1, \Sigma) \wedge \sim\mathbf{PRES}(x, t_2, \Sigma) \rightarrow \rightarrow (t)(t > t_2 \rightarrow \sim\mathbf{PRES}(x, t, \Sigma)))$;

5.2. $(A)(x)(t_1)(t_2)(A \in \Sigma \wedge t_2 \in \mathbf{EX}(\Sigma) \wedge \mathbf{Pres}(x, t_1, A) \wedge \sim\mathbf{Pres}(x, t_2, A) \rightarrow \rightarrow \sim\mathbf{PRES}(x, t_2, \Sigma))$;

5.3. $(a)(x)(t)(a \in \mathbf{CHAR}(\Sigma) \wedge \mathbf{ptr}(a) \wedge \mathbf{pres}(x, t, a) \rightarrow \mathbf{PRES}(a(x, t), t, \Sigma) \vee \vee a(x, t) = \mathbf{none})$.

Condition 5.1 corresponds to 2.3, condition 5.2 corresponds to 2.2 and 5.3 corresponds to a certain closeness of the dynamic system relating to its pointers; a similar property characterizes the static systems among the static quasisystems, as it has been presented in par. 4 of [2].

In the present paper the word *dynamic* will be omitted also in connection with the terms *quasisystem* and *system* in the following text. Let Σ be a system and x any element of its domain. Then the following theorems hold.

Theorem 1. If $A \in \Sigma$, $t_2 \in \mathbf{EX}(\Sigma)$ and $t_1 < t_2$ such that $\mathbf{Pres}(x, t_1, A)$ and $\sim\mathbf{Pres}(x, t_2, A)$, then for any $t > t_2$ there is $\sim\mathbf{Pres}(x, t, A)$.

Proof. According to 5.2 there is $\sim\mathbf{PRES}(x, t_2, \Sigma)$. According to the corollary presented in this paragraph there is $\mathbf{PRES}(x, t_1, \Sigma)$. Because of 5.1 there is $\sim\mathbf{PRES}(x, t, \Sigma)$ for any $t > t_2$. The mentioned corollary then implies $\sim\mathbf{Pres}(x, t, A)$.

Theorem 2. If $a \in \boldsymbol{CHAR}(\Sigma)$, $t_2 \in \boldsymbol{EX}(\Sigma)$ and $t_1 < t_2$ such that $\boldsymbol{pres}(x, t_1, a)$ and $\sim \boldsymbol{pres}(x, t_2, a)$, then for any $t > t_2$ there is $\sim \boldsymbol{pres}(x, t, a)$.

Proof. The definition of $\boldsymbol{CHAR}$ implies that there is a class A of Σ such that $a \in \boldsymbol{Char}(A)$; it must satisfy $\boldsymbol{Pres}(x, t_1, A)$ and $\sim \boldsymbol{Pres}(x, t_2, A)$ because of the corollary presented in par. 4. The preceding theorem states that in this case $\sim \boldsymbol{Pres}(x, t, A)$ for any $t > t_2$; according to the metioned corollary, it is $\sim \boldsymbol{pres}(x, t, a)$, too.

For $\boldsymbol{Ex}(A) \subseteq \boldsymbol{EX}(\Sigma)$ in case of $A \in \Sigma$, we can modify theorem 1 so that instead of $t_2 \in \boldsymbol{EX}(\Sigma)$ we put $t_2 \in \boldsymbol{Ex}(A)$. Because of similar reasons we can modify theorem 2 so that we put $t_2 \in \boldsymbol{ex}(a)$ instead of $t_2 \in \boldsymbol{EX}(\Sigma)$. Thus any class of a dynamic system and any attribute of its characteristics must satisfy a very similar condition to 5.1. It is not true, that if a quasisystem satisfies 5.2 and 5.3 and its classes resp. attributes of its characteristics satisfy the mentioned conditions, the quasisystem is a system. We can illustrate it at the following example: the quasisystem has two classes with disjoint nonepty domains; the existence of the first class is the union of the intervals (0, 1) and (2,3), the existence of the second class is the union of the intervals (1, 2) and (3, 4). Every element of the domain of a class is present in it during its whole existence; we can assume arbitrary standard attributes and some pointers in the characteristics of any class with the range being a subset of the closure (see [2], par. 2) of the domain of the same class.

6. EVENTS AND DYNAMICS

Let a be an attribute, t_0 an element of its existence. We say that a set U is a *history* of t_0 in a if U is a nonempty subset of the existence of a, contains only numbers less than t_0 and no element of the existence of a is greater than the supremum of U (in $\mathscr{R}$) and less than t_0. History in a class resp. in a quasisystem is defined similarly.

$$\boldsymbol{hist}(U, t_0, a) \equiv t_0 \in \boldsymbol{ex}(a) \wedge U \subseteq \boldsymbol{ex}(a) \wedge U \neq \emptyset \wedge (t)(t \in U \rightarrow t < t_0) \wedge \wedge \sim \exists t (t \in \boldsymbol{ex}(a) \wedge \sup(U) < t < t_0)$$

$$\boldsymbol{Hist}(U, t_0, A) \equiv t_0 \in \boldsymbol{Ex}(A) \wedge U \subseteq \boldsymbol{Ex}(A) \wedge U \neq \emptyset \wedge (t)(t \in U \rightarrow t < t_0) \wedge \wedge \sim \exists t (t \in \boldsymbol{Ex}(A) \wedge \sup(U) < t < t_0)$$

$$\boldsymbol{HIST}(U, t_0, \Sigma) \equiv t_0 \in \boldsymbol{EX}(\Sigma) \wedge U \subseteq \boldsymbol{EX}(\Sigma) \wedge U \neq \emptyset \wedge (t)(t \in U \rightarrow t < t_0) \wedge \wedge \sim \exists t (t \in \boldsymbol{EX}(\Sigma) \wedge \sup(U) < t < t_0)$$

If $a \in \boldsymbol{Char}(A)$ then $\boldsymbol{hist}(U, t_0, a) \equiv \boldsymbol{Hist}(U, t_0, A)$, because $\boldsymbol{ex}(a) = \boldsymbol{Ex}(A)$ (see Corollary of par. 4). It is not true that $A \in \Sigma$ would imply the equivalence or any implication between $\boldsymbol{Hist}(U, t_0, A)$ and $\boldsymbol{HIST}(U, t_0, \Sigma)$ as we can illustrate on the example of a system Σ with two classes A and B so that $\boldsymbol{Def}(A) = \{\langle x, 1\rangle, \langle y, 3\rangle\}$

and $\boldsymbol{Def}(B) = \{\langle z, 2\rangle\}$, where $x \neq y \neq z$: $\boldsymbol{Hist}(\{1\}, 3, A)$ but not $\boldsymbol{HIST}(\{1\}, 3, \Sigma)$ and $\boldsymbol{HIST}(\{2\}, 3, \Sigma)$ but not $\boldsymbol{Hist}(\{2\}, 3, A)$.

Let a be an attribute. We say x to *change* a at time t_0 if $\boldsymbol{pres}(x, t_0, a)$ and if there is a history U of t_0 in a such that $a(x, t)$ is defined and different from $a(x, t_0)$ for any $t \in U$.

$$\boldsymbol{change}(x, t_0, a) \equiv \boldsymbol{pres}(x, t_0, a) \wedge \exists U(\boldsymbol{hist}(U, t_0, a) \wedge (t)(t \in U \rightarrow \rightarrow \boldsymbol{pres}(x, t, a) \wedge a(x, t) \neq a(x, t_0)))$$

Theorem 3. Let Σ be a system and $a \in \boldsymbol{CHAR}(\Sigma)$; then $\boldsymbol{change}(x, t_0, a)$ is equivalent with $\boldsymbol{pres}(x, t_0, a) \wedge \exists V(\boldsymbol{HIST}(V, t_0, \Sigma) \wedge (t)(t \in V \rightarrow \boldsymbol{pres}(x, t, a) \wedge a(x, t) \neq \neq a(x, t_0)))$.

Proof. Because of a similarity of logical expressions defining ***change*** and equivalent with ***change*** according to the theorem it is only to demonstrate that U of the definition can be replaced by V of the theorem and vice versa, namely that $\boldsymbol{HIST}(U, t_0, \Sigma)$ and $\boldsymbol{hist}(V, t_0, a)$. Let $A \in \Sigma$ such that $a \in \boldsymbol{Char}(A)$; $\boldsymbol{ex}(a) = \boldsymbol{Ex}(A) \subseteq \boldsymbol{EX}(\Sigma)$. $U \subseteq \boldsymbol{ex}(a)$ and thus $U \subseteq \boldsymbol{EX}(\Sigma)$. $U \neq \emptyset$ and for any $t \in U$ there is $t < t_0$. Let t_1 be an element of U. If $t \in \boldsymbol{Ex}(A) = \boldsymbol{ex}(a)$, then it is not possible that $\sup(U) < t < t_0$. Let us suppose that there is $t \in \boldsymbol{EX}(\Sigma) - \boldsymbol{Ex}(A)$ such that $\sup(U) < t < t_0$; then $t_1 < t < t_0$, $\boldsymbol{Pres}(x, t_0, A)$, $\boldsymbol{Pres}(x, t_1, A)$ and $\sim\boldsymbol{Pres}(x, t, A)$, and thus $\boldsymbol{PRES}(x, t_0, \Sigma)$ and $\boldsymbol{PRES}(x, t_1, \Sigma)$. Because of 5.2, $\sim\boldsymbol{PRES}(x, t, \Sigma)$, but 5.1 then implies $\sim\boldsymbol{PRES}(x, t_0, \Sigma)$ which is a contradiction. Thus there is no $t \in \boldsymbol{EX}(\Sigma) - \boldsymbol{Ex}(A)$ such that $\sup(U) < < t < t_0$ and therefore U is a history of t_0 in Σ. From the condition $(t)(t \in V \rightarrow \rightarrow \boldsymbol{pres}(x, t, a))$ of the theorem, $V \subseteq \boldsymbol{ex}(a)$ follows. $V \neq \emptyset$ and for any t of V there is $t < t_0$ because of $\boldsymbol{HIST}(V, t_0, \Sigma)$. $t \in \boldsymbol{ex}(a)$ implies $t \in \boldsymbol{EX}(\Sigma)$ and thus there is no t of V for which the relation $\sup(V) < t < t_0$ would be satisfied. Therefore V is the history of t_0 in a.

Let Σ be a quasisystem. We say x to *enter* Σ at time t_0 if x is present at t_0 in Σ and there is a history U of t_0 in Σ such that x is not present at t in Σ for any $t \in U$. We say x to *leave* Σ at time t_0 if x is not present in Σ at t_0 and there is a history U of t_0 in Σ such that x is present at t in Σ for any $t \in U$.

$$\boldsymbol{ENTER}(x, t_0, \Sigma) \equiv \boldsymbol{PRES}(x, t_0, \Sigma) \wedge \exists U(\boldsymbol{HIST}(U, t_0, \Sigma) \wedge (t)(t \in U \rightarrow \rightarrow \sim\boldsymbol{PRES}(x, t, \Sigma)))$$

$$\boldsymbol{LEAVE}(x, t_0, \Sigma) \equiv \sim\boldsymbol{PRES}(x, t_0, \Sigma) \wedge \exists U(\boldsymbol{HIST}(U, t_0, \Sigma) \wedge (t)(t \in U \rightarrow \rightarrow \boldsymbol{PRES}(x, t, \Sigma)))$$

We say that at time t an *event* of a system Σ is present if t is the minimum of $\boldsymbol{EX}(\Sigma)$ (in case it exists) or if there is an element of $\boldsymbol{DOM}(\Sigma)$ such that it leaves or enters Σ or changes some attribute of $\boldsymbol{CHAR}(\Sigma)$ at t. We define *dynamics* of a system as a set of all times when events are present in it.

$$\textit{\textbf{EVENT}}(t, \Sigma) \equiv t = \min(\textit{\textbf{EX}}(\Sigma)) \vee \exists x(\textit{\textbf{ENTER}}(x, t, \Sigma) \vee \textit{\textbf{LEAVE}}(x, t, \Sigma) \vee$$
$$\vee\ \exists a(a \in \textit{\textbf{CHAR}}(\Sigma) \wedge \textit{\textbf{change}}(x, t, a)))$$
$$\textit{\textbf{DYN}}(\Sigma) = \{t \mid \textit{\textbf{EVENT}}(t, \Sigma)\}$$

It would be possible to define entering, leaving, events and dynamics also for attributes and classes, but such notions have no importance in the considerations presented in this paper.

We say that a system is *continuous* if its dynamics is equal to its existence and the existence is an interval of real numbers. We call a system *discrete* if its dynamics is nowhere dense in $\mathscr{R}$. A system is *non-degenerated* if its dynamics has more than one element.

7. BASED DYNAMIC SYSTEMS

Any subset of a system is a quasisystem. This trivial consequence of the definitions presented above has no importance for proving theorems but it enables to apply definitions introduced for quasisystems in par. 5 for subsets of systems. We say that an attribute is *constant* (in time) if any element of its domain is present in it at any time of its existence and if its value does not depend on time.

$$\textit{\textbf{const}}(a) \equiv \textit{\textbf{def}}(a) = \textbf{dom}(a) \times \textbf{ex}(a) \wedge (x)(t_1)(t_2)(x \in \textbf{dom}(a) \wedge t_1 \in \textbf{ex}(a) \wedge$$
$$\wedge\ t_2 \in \textbf{ex}(a) \rightarrow a(x, t_1) = a(x, t_2))$$

Let Σ be a system, M its subset. We say that M is a *pseudobase* in Σ if any class of it contains only such elements in its domain which are present in the system at any time of its existence.

$$\textit{\textbf{PSB}}(M, \Sigma) \equiv M \subseteq \Sigma \wedge (x)(x \in \textbf{DOM}(M) \rightarrow (t)(t \in \textit{\textbf{EX}}(\Sigma) \rightarrow \textit{\textbf{PRES}}(x, t, \Sigma)))$$

Corollary. $\textit{\textbf{PSB}}(\emptyset, \Sigma)$; if $\textit{\textbf{PSB}}(M, \Sigma)$ and $\textit{\textbf{PSB}}(N, \Sigma)$ then $\textit{\textbf{PSB}}(N \cup M, \Sigma)$. Any system has its maximal pseudobase, containing any other pseudobase of the same system as its subset.

Let M be a pseudobase in Σ. We say that an attribute is *internal* in Σ relating to M if it is a pointer of the characteristics of some class from M and if its values are **none** or are present in the domains of classes of M. We say that an attribute is *external* in Σ relating to M if it is a pointer of the characteristics of some class from M and if its values are not present in the domain of any class of M. We say that a set of attributes is *internal* in Σ relating to M if its elements are only attributes internal in Σ relating to M; similarly a set is *external* in Σ relating to M if its elements are only attributes external in Σ relating to M.

$$\mathbf{int}(a, M, \Sigma) \equiv \mathbf{ptr}(a) \wedge a \in \mathbf{CHAR}(M) \wedge (x)(t)(a(x, t) \neq \text{none} \rightarrow$$
$$\rightarrow \mathbf{PRES}(a(x, t), t, M))$$

$$\mathbf{ext}(a, M, \Sigma) \equiv \mathbf{ptr}(a) \wedge a \in \mathbf{CHAR}(M) \wedge (x)(t)(\sim \mathbf{PRES}(a(x, t), t, M))$$

$$\mathbf{Int}(X, M, \Sigma) \equiv (a)(a \in X \rightarrow \mathbf{int}(a, M, \Sigma))$$

$$\mathbf{Ext}(X, M, \Sigma) \equiv (a)(a \in X \rightarrow \mathbf{ext}(a, M, \Sigma))$$

We say that a pseudobase of Σ is its *base* if all pointers of the characteristics of its classes can be divided into two groups so that one group contains only external attributes in Σ relating to the pseudobase and the other group contains only internal attributes in Σ relating to the pseudobase, which are constant.

$$\mathbf{BASE}(M, \Sigma) \equiv \mathbf{PSB}(M, \Sigma) \wedge \exists X\, \exists Y(\mathbf{Int}(X, M, \Sigma) \wedge \mathbf{Ext}(Y, M, \Sigma) \wedge$$
$$\wedge (a)(\mathbf{ptr}(a) \wedge a \in \mathbf{CHAR}(M) - Y \rightarrow a \in X \wedge \mathbf{const}(a)))$$

Evidently, every system has one base – an empty set. It need not have a maximal base, containing every of its other bases; we can simply illustrate it by a system composed of two classes A and B, where in $\mathbf{Char}(A)$ there are only some pointers which are not constant and the values of which are in $\mathbf{Dom}(B)$, and in $\mathbf{Char}(B)$ there are only pointers with symmetrical properties; moreover, let $\{A, B\}$ be a pseudobase. Then $\{A\}$ and $\{B\}$ are bases but $\{A, B\}$ is not a base. Because of such cases, we must introduce the following definition:

A *based dynamic system* is an ordered pair of a dynamic system and its base.

$$\mathbf{BSYS}(Z) \equiv \exists \Sigma\, \exists M(Z = \langle \Sigma, M \rangle \wedge \mathbf{SYS}(\Sigma) \wedge \mathbf{BASE}(M, \Sigma))$$

If $Z = \langle \Sigma, M \rangle$ is a based system, we call Σ its *essence* and M its *main base*; we write $\mathbf{ESSE}(Z) = \Sigma$ and $\mathbf{MBASE}(Z) = M$. The elements of $\mathbf{DOM}(\mathbf{MBASE}(Z))$ are called *activities* of Z and the elements of $\mathbf{DOM}(\mathbf{ESSE}(Z)) - \mathbf{DOM}(\mathbf{MBASE}(Z))$ are called *transactions* or Z. *Pseudoactivities* of Z are the elements of the domain of the maximal pseudobase of its essence. Every activity of a system Z is its pseudoactivity but not vice versa (for example if the main base is an empty set). We say that a based system is *of type A* if its main base is equal to its essence. We say that a based system is *of type T* if its main base is empty. We say that a based system is *of type AT* if it is neither of type A nor of type T.

8. DISCUSSION AND CONCLUSION

We could define notions introduced for dynamic systems also for based dynamic systems, according to a rule that if F is a function defined for quasisystems in par. 5, then F_B is defined for based dynamic systems as $F_B(Z) = F(\mathbf{ESSE}(Z))$. But those definitions have no importance for developing the theory of simulation.

In [1] the conception of dynamic system has been introduced so that it was defined as a mapping of a subset of $\mathscr{R}$ into a set of static systems. That conception has led to the detailed elaboration of the theory of static systems in [2], but in order to formulate that the mentioned mapping satisfies the conditions mentioned in par. 2 the definition of dynamic system had to be completed by an auxiliary notion of dynamic attribute; one axiom had to be expressed that at time t of the existence of the system the value of any static attribute for its argument x is the same as the value of a dynamic attribute for arguments x, t. Such a doubling was contrary to the estetics and elegance of mathematics and therefore the conception of dynamic system has been modified in [4]: dynamic system has been introduced as a set of dynamics classes, where dynamic class has been defined similarly as in the present paper; the only exception has been that a dynamic attribute has been defined at the cartesian product of its domain and of the existence of the system. In order to permit systems the maximal pseudobase of which is not equal to the whole system, the definition had to be completed by a predicate *presence* with two arguments: an element of the system domain and an element of the system existence.

Therefore dynamic system has been defined as an ordered pair of a set of dynamic classes and of its presence, satisfying certain axioms; the definition could be enlarged so that dynamic system is an ordered triplet, where the third member is its main base. Such definition are more elegant than the original one, but the values $a(x, t)$ of attributes have no importance in case x is not present at t; namely in any axiom of model we had to put the presence of the concerned element at the concerned moment into the premise.

Thus the definition presented in this paper has risen. We have separated the development of the theory of dynamic systems from the theory of based dynamic systems because of the simplicity of the structure of the entier theory of simulation: the conception of dynamic and simulation model does not depend on the main base and thus it is more convenient to introduce all necessary terms independently on any conception of base. The conception of based system enables however to classify dynamic systems relating to their transactions and activities and to exply some techniques of use of simulation languages (e.g. neglecting the activities if using SIMULA: even a system which has the maximal base can form the essence of a based system with the empty main base).

The notion of dynamics enables to distinguish between discrete systems and continuous ones, independently on the decision whether a discrete system exists during an entier interval of real numbers or only in moments of its events. We have not incorporated any traditional concept of continuous attributes as that of continuous functions, because it is not a principal problem of the difference between the continuous and discrete systems, between continuous and discrete simulation and between continuous system simulation languages and discrete event ones; we can state that excepting degenerated systems the continuous systems (according the presented definition) figure in continuous simulation and the universes of semantics of continuous system simulation languages, while discrete systems (according to the presented definition) figure similarly in discrete simulation and appropriate languages. The relation between continuous systems and their images in the discrete systems of digital computer is the subject of the theory of dynamic and simulation models, which has not been included into the present paper more.

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