KYBERNETIKA — VOLUME 12 (1976), NUMBER 3

Time Change of Objects and Problem of their Identification

ROMAN BEK

In present paper will be proposed a way of defining of time — terms within exact language. In such languages it is possible under some conditions, to solve the problem of "paradoxies of identity" for developing objects. One form of definition both of the timeterms and the relativised identity serves as the starting point for it.

I.

In this paper we will deal with some questions concerned with the time identification of physical objects undergoing time changes. The discourse will be concerned with relations between extralogical constants of a theoretical language and their denotates i.e. physical objects. Through sentences of a language we give statements on the properties of objects and on relations the objects enter. Assuming the permanent change of objects we are confronted with the problem of the truth of corresponding statements.

Let the object b have the property F at the time t_i. We describe this fact by the sentence

$$(I/1) \qquad F(b, t_i)$$

and this sentence is a true sentence of the language concerned.

Further let

$$(I/2) \qquad G(b, t_j)\,, \quad t_i \neq t_j$$

be an another sentence of the language. If we are to verify the truth of the other sentence we have to assign the same object to the constant b that was the denotate in the interpretation of the sentence (I/1). Then the question comes: Does there exist the same object at the time t_j that was spoken about in the sentence (I/1) and that was identifiable through its properties at the time t_i.

If we accept the Leibniz's idea of identity (i.e. the object existing at the time t_i denoted by b in the sentence (I/1) is identical with the object existing at the time t_j and denoted by b in (I/2)) then we have to admit that the object did not experience any changes in the closed time interval $\langle t_i, t_j \rangle$ i.e. it did not change any of its properties. If we are not inclined to accept a completely nonintuitive and empirically unverified assumption that physical objects do not change at all in their history, then we are moved to make the conclusion on the discrete character of time changes of objects, so that these objects do not change any of their properties in the course of certain time intervals and then they change some (at least) of their properties instantly.

(I/3) $\forall t(t_i \leqq t \leqq t_j \rightarrow (\forall F\, F(b, t) \wedge \forall t'(t_j < t' \rightarrow \exists G(\sim G(b, t) \wedge G(b, t')))))$

The sentence (I/3) drives us to a problem similar to that one above if the denotate of the constant b does not posses the priperty G in the time interval $\langle t_i, t_j \rangle$ and if the denotate of the constant b in the formula $G(b, t')$ does posses the property G at the time t_j then – from the Leibniz's point of view – we are delaing with different objects and by using the same symbol in all parts of the sentence (I/3) we offended against the uniteness of notation law.

This reasoning brings us to the following conclusion: When keeping the Leibniz's idea of identity we have to accept either the absolute invariability of objects throughout the world history or we have to take that a physical object becomes an another after its single property is changed; then in an exact language it must be denoted by an another constant.

The first metaphysical alternative is nonintuitive and it is easy to falsify it in an empirical way. The other alternative is drawing us to enormous complications in the area of language: every developing phase of an object should be denoted by a special extralogical constant accordingly. There should be as many extralogical constants as there are time changes in the history of an objects. So there would appear a set of distinct symbols for an object in any sentence speaking about a finite section in the history of the physical object. Suppose a sentence speaks about a variable metric quantity of a property of an object where the quantities assume rational numerical values, then for its complete formulation a countably infinite set of constants must be used.

If we use a variable for this purpose (the region of which is the class of the phases of the object changing in time) we have to use a term denoting a characteristic of the object unchanged in the section of the history concerned.

Suppose a sentence on the change of the magnitude of some property of an object in a given limited part of its history has the form

(I/4) $$\forall \beta(\beta \in H \rightarrow \boldsymbol{V}_\beta)$$

where $\boldsymbol{V}_\beta$ is a sentence in which free variable β occurs; H – the region of the variable β – is characterized through a property of the object that remained unchanged through the given part of its history. As far as this property is concerned the difficulties mentioned above remain if we want to deal with a series of subsequent parts of the history of the object, that are defined through changes of properties by which the regions of individual variables were defined.

The difficulties mentioned were dealt with by a good number of authors recently. In the following part of the paper I will seek possible solutions following the ideas of [2]. The solutions will be sought through a certain approach to the concept of "time" and through a relativisation of the concept of "identity".

II.

The difficulties mentioned in the part I are obviously connected with the variability of ontological entities spoken about in the languages or rigorous science. A language system unable – due to its structure – of the description of time changes is obviously inadequate for the purpose given. For this reason we will be dealing with a language system in whose sentences speaking about separate time events time terms – constants and variables – can occur. In the examples given above this sort of expressions was used yet. It was automatically supposed that the time structure spoken about was linear, metric and additive [6]. In the paper [6] also another time structures are studied. In the following reasonings I will seek a general solution; it will always be pointed out when a special time structure is being dealt with.

It is possible to give an interpretation of an exact language in a time structure that contains either an at most countable or an uncountable class of time points. First let us deal with a language the universe of which contains an at most countable time structure as a part. Its elements – time points – are denoted by time constants that make an at most countable set. By such a language we are to give an adequate description of time changes of physical objects that are at most countable too. If a quantified language is dealt with (in which relations between metric magnitudes of object properties are spoken about) then its mathematical formalism must not use operations leading to irrational values. This requirement is certainly not met by the language of classical physics based on theorems of classical mathematical analysis. On the other hand the requirement is obviously met by the language based on finite numerical mathematics.

Suppose a language J_t^1 describing just time changes of an object b is given. If we are to describe n different time changes of the object (e.g. changes of magnitude of a property A) then in the general case the alphabet of J_t^1 must contain at least n different time constants $t_1, t_2, t_3, \ldots, t_n$ along with relevant predicate constants.

Suppose further a language J_t^2 describing just time changes of metric magnitudes of a property A of an object b is given, where there are relations among the magnitudes. These relations are exponential functions. The alphabet of the language J_t^2 must contain the real time variable in the general case then.

Languages of described types can be built up in two ways as a matter of fact.

In the first case we introduce time constants as primitive individual constants. While interpreting a language built up in this way we have to accept the universe nonhomogeneous in that it contains objects the time changes of which we describe and time instants at the same time. In the other case we introduce time constants in the course of construction of the language through definitions where some predicate constants denoting periodically variable properties of an object chosen as a chronometer appear in the definiendum.

(i) In the first case the existence and character of the time structure is postulated. According to [6] in this case "time point" is a contents deprived case of events ... time is not individualized by events, events can be distinguished by means of time. Times are prior to events logically because events can be differentiated a identified only with reference to time".

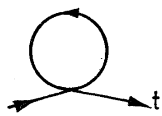

Fig. 1.

In the connection given they sometimes speak of an "absolute approach to time". An advantage of this approach is that while choosing postulates through which we speak of the character of time structure we are not bound by respects to the character of events described by the language as closely as it is in the other case. In [6] the author lays stress on the fact that by this approach to time we may assume time structures inadmissible for the other approach (the nonlinear time structure of the form given in Fig. 1 for example). Among the disadvantages of the "absolute approach to time" the above discussed nonhomogeneousness of the language universe of the language should be mentioned.

(ii) In the other case time constants of the language can be defined in two ways:

(a) In the first case descriptions of individual events are used for the definition. In the class of events a relation "to take place simultaneously" is defined. Time points are then identified with abstraction classes formed through this relation (which obviously is one of equivalence type).

In paper [1] the following form of the definition of an "event time" appears. "Let $S = \{x, y, z, \ldots\}$ be a class of events; let R be the relation "to take place simultaneously". Then the definition of the "time point at which the event x comes" is:

$$\text{Def}\,|x| = \hat{y}(y \in S \wedge R(x, y)).$$

(b) In the other case descriptions of isolated processes are used for the definition of time constants. In a class of processes $S' = \{p, q, r, \ldots\}$ a relation R' of being time equal is defined. Time intervals are defined as abstraction classes of S' with respect to the relation R':

$$\text{Def}\,|p| = \hat{q}(q \in S' \wedge R'(p, q))\,.$$

In [1] the author prefers the first type of the definition of time constants. He points out difficulties coming when the other way is followed: "events" cannot be defined through "processes"; "time points" cannot be defined through "time intervals" while the contrary procedure is possible.

In the connection with the approach (ii) they often speak about a "relativistic approach to time" [6]. It should be admitted that this approach inevitably implies complications of the syntactic structure of the language (definitions are introduced and relevant definition axioms are accepted). The interpretation of a language built in this way appears to be simpler from the following point of view: the class of time points (time intervals resp.) need not be established – contrary to the case of a language built in the first way. Ontology of the language is more complicated on the other hand. Time points (intervals resp.) are denoted by the first order predicate constants; classes of events (processes resp.), relations between time points (intervals resp.) are denoted by second order predicate constants. Using the approach (i) on the other hand time constants can be considered as a special kind of individual constants and relations between the time points can be denoted as first order predicate constants.

Let us go back to language of the type J_t by which time changes of objects are described. On acceptance of the "absolute approach to time" (i) time terms as primitive terms of the language are introduced and postulates are chosen so that they convey with the character of the region of physical reality to be described. The language of classical physics originating from the idea of time continuum would postulate in this approach the existence of an uncountable class of time points ordered by a connective relation "to take place earlier than". Individual time points would be assigned real numbers by metrization. Time structure (defined as an ordered pair of the class of time points and the order relation "to take place earlier than") would appear to be (through the postulates) linear, metric and additive.

Such a language should provide for a description of continuous time changes of properties of physical objects. If a language like this is built on the base of predicate logic with identity (using the Leibniz's approach to identity) the difficulties mentioned above would come. If we resign on the assumption of continuity of time changes of object properties, then of course the language with the postulated time structure appears to be too strong but there is no need to turn our back to it on this ground.

Suppose now that the "relativistic approach to time" was chosen for the construction of a type J_t language. It is easy to show that the definition of the "event time" puts some requirements on the choice of time structure of the language.

Events will be understood now as states of an object characterized by a certain (passing) property of the object.

Suppose there is an object c_1 with the property $T^{k,l}$. The event then is the state of the object described by the sentence: $T^{k,l}(c_1)$.

Suppose further that $T^{k,l}$ is a subclass of the class T decomposable into mutually disjoint subclasses $T^1, T^2, \ldots, T^k, \ldots$ and each of the subclasses is again decomposable into disjoint subclasses $T^{1,1}, T^{1,2}, \ldots, T^{2,1}, T^{2,2}, \ldots, T^{k,1}, T^{k,2}, \ldots, T^{k,l}, \ldots$. Let all subclasses of both kinds be ordered by the relation R^T (Fig. 2). These sub-

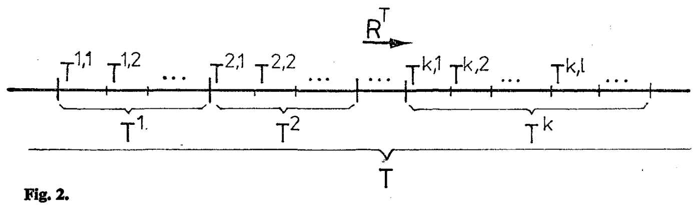

Fig. 2.

classes will be understood (due to the extensionality principle) as properties of the objects $c_1, c_2, c_3, \ldots$ belonging to the class C. The class C will be interpreted as the property "to be a given particular chronometer"; individual classes $T^{1,1}, T^{1,2}, \ldots$ wil be understood as distinct characteristics of the chronometer by which time is measured. If for example the chronometer is a certain quantity of a radioactive material, then the objects $c_1, c_2, \ldots$ are understood to be quantities of the material at particular stages of decomposition and classes $T^{1,1}, T^{1,2}, \ldots$ are understood to be the properties "one half of the original quantity is decomposed", "one half of the remaining half of the original quantity is decomposed", etc.

Under these assumptions languages of the type J_t with "the relativistic approach to time" can be built now.

Language J_T^u in that "event time" is defined

In the score of primitive constants of the language there are first order predicate constants:

$$\begin{array}{l} T, \\ T^1, T^2, T^3, \ldots \\ T^{1,1}, T^{1,2}, T^{1,3}, \ldots \\ T^{2,1}, T^{2,2}, T^{2,3}, \ldots \\ T^{3,1}, T^{3,2}, T^{3,3}, \ldots \\ \vdots \end{array}$$

where each of the classes denoted this way is a proper subclass of the class denoted T; further there are individual constants $c_1, c_2, c_3, \ldots$ and one-place predicate first order constant C so that it holds $c_1, c_2, \ldots \in C$; further a two-place second order predicate constant R^T denoting the order relation for proper subclasses of the class T, further there is a functor f^T together with an identity) in the Leibniz's sense notation. Further let the language contain the arithmetic of naturals.

Let in the language the following sentence holds:

$$\text{(II/1)} \quad \forall T^{i,m} \forall T^{j,n} \underset{c',c''\in C}{\forall c' \forall c''}((T^{i,m}(c') \wedge T^{j,n}(c'')) \rightarrow \forall F(F \nsubseteq T \rightarrow (F(c') \leftrightarrow F(c''))))$$

(The sentence (II/1) states in the interpretation intended that objects of the class C are identified on the base of all of their properties with the exception for the properties of the type $T^{k,l}$.)

Let us introduce definitions of the form

$$\text{(D II/1)} \qquad \text{Def } t_{i,m}(c) \leftrightarrow \exists T^{i,m} T^{i,m}(c)$$

By these definitions names of time points identified with subclasses of the type $T^{k,l}$ are introduced into the language. The definitions conform with the definition of the "event time" from [1] under assumption that the "chronometer" concept is specified through a given property C exclusively.

Let us define further on

$$\text{(D II/2)} \qquad \text{Def } t_{i,m} <^T t_{j,n} \leftrightarrow \exists T^{i,m} \exists T^{j,n} \underset{c',c''\in C}{\exists c' \exists c''}((T^{i,m}(c') \leftrightarrow t_{i,m}(c')) \wedge$$
$$\wedge (T^{j,n}(c'') \leftrightarrow t_{j,n}(c'')) \wedge R^T(T^{i,m}, T^{j,n}))$$

By the definition notation of an order relation on the class of time points is introduced.

Let in the language the following sentence holds:

$$\text{(II/2)} \qquad f^T(t_{i,m}) < f^T(t_{j,n}) \leftrightarrow t_{i,m} <^T t_{j,n}$$

(f^T should be understood as a notation for a function mapping the class of time points in the class of natural numbers now and the sentence (II/2) provides for the ordinal type of the scale of numerical magnitudes of individual time points.

Let us introduce definitions of the form

$$\text{(D II/3)} \qquad \text{Def } \Delta_{im,jn} = S \leftrightarrow \exists t_{i,m} \exists t_{j,n}(S(t_{i,m}) \wedge S(t_{j,n}) \wedge t_{i,m} <^T t_{j,n} \wedge$$
$$\wedge \sim\exists t_{k,o} \exists t_{l,p}(S(t_{k,o}) \wedge S(t_{l,p}) \wedge t_{k,o} <^T t_{i,m} \wedge t_{j,n} <^T t_{l,p}))$$

By these definitions notations of individual time intervals as closed intervals of time points are introduced.

Language J_t^p in that "process time" is defined.

The language contains the same primitive constants as the language J_T^u with the exception for the functor f^T in place of which the functor g^T is introduced. The language contains the arithmetic of naturals too.

In the language the concept of a "process" is defined. The process will be considered as an ordered pair $\langle A^i, A^j \rangle$ of classes A^i, A^j that are proper subclasses of the class A that are ordered by the relation R^A here. These subclasses are understood to be properties of objects $b_1, b_2, \ldots$ belonging to the class B:

$$(\text{D II}/4) \qquad \text{Def} \langle A^i, A^j \rangle^B_{\text{proc}} \leftrightarrow \underset{b', b'' \in B}{\exists b' \, \exists b''} (A^i(b') \wedge A^j(b'') \wedge \wedge R^A(A^i, A^j) \wedge \forall F(F \not\subseteq A \rightarrow (F(b') \leftrightarrow F(b''))))$$

(A pair of classes $\langle A^i, A^j \rangle$ is said to be a "process from the point of view of the class B" iff these classes are ordered by the relation R^A and are properties of objects from the class B identified by all of their properties with the exception for those belonging to the class A. Thus a process is understood to be a change of a property of some object while this change is oriented in some direction.)

For the subclasses of the class T and for objects of the class C it holds in particular

$$(\text{II}/3) \qquad \langle T^{i,m}, T^{j,n} \rangle^C_{\text{proc}} \leftrightarrow \underset{c', c'' \in C}{\exists c' \, \exists c''} (T^{i,m}(c') \wedge T^{j,n}(c'') \wedge R^T(T^{i,m}, T^{j,n}) \wedge \wedge \forall F(F \not\subseteq T \rightarrow (F(c') \leftrightarrow F(c''))))$$

Let us now introduce definitions of the form

$$(\text{D II}/5) \qquad \text{Def}\, \Delta t_{im,jn} = \langle T^{i,m}, T^{j,n} \rangle \leftrightarrow \underset{c', c'' \in C}{\exists c' \, \exists c''} (\langle T^{i,m}, T^{j,n} \rangle^C_{\text{proc}} \wedge T^{i,m}(c') \wedge T^{j,n}(c''))$$

By these definitions names of time intervals – identified with processes involving objects of the class C (chronometer) passed through due to oriented changes or properties of the type $T^{k,l}$ – are introduced. These definitions correspond with the definition of "process time" form [1] under the assumption that the concept of "chronometer" is specified by a particular property C.

Further a concept of "time interval summation" is introduced. Behind it a formation of a class containing just all subclasses of the type $T^{k,l}$ contained in the interval of subclasses of this type through which time interval was defined, is understood:

$$\begin{aligned}(\text{D II}/6) \qquad \text{Def} \textstyle\sum \Delta t_{im,jn} = U \leftrightarrow\ & \exists T^{i,m} \exists T^{j,n} (\Delta t_{im,jn} = \langle T^{i,m}, T^{j,n} \rangle \wedge \\ & \wedge \exists T^{i,m+1} \exists T^{i,m+2} \ldots \exists T^{i,m+o_1} \\ & \exists T^{i+1,1} \exists T^{i+1,2} \ldots \exists T^{i+1,o_2} \\ & \ldots \exists T^{i+n,1} \exists T^{i+n,2} \ldots \exists T^{i+n,o_{n+1}} \\ & \exists T^{j,1} \exists T^{j,2} \ldots \exists T^{j,o_{n+2}} \\ & (\sim \exists T^{i,m+o_1+1} R^T(T^{i,m_{o_1+1}}, T^{i+1,1}) \wedge \\ & \wedge \sim \exists T^{i+1,o_2+1} R^T(T^{i+1,o_2+1}, T^{i+2,1}) \wedge \\ & \ldots \wedge \sim \exists T^{i+n,o_{n+1}+1} R^T(T^{i+n,o_{n+1}+1}, T^{j,1}) \wedge \\ & \wedge \sim \exists T^{j,o_{n+2}+1} R^T(T^{j,o_{n+2}+1}, T^{j,n})) \wedge \\ & \wedge U = \{T^{i,m}, T^{i,m+1}, \ldots, T^{j,1}, \ldots, T^{j,n}\}) .\end{aligned}$$

In scheme we get Fig. 3.

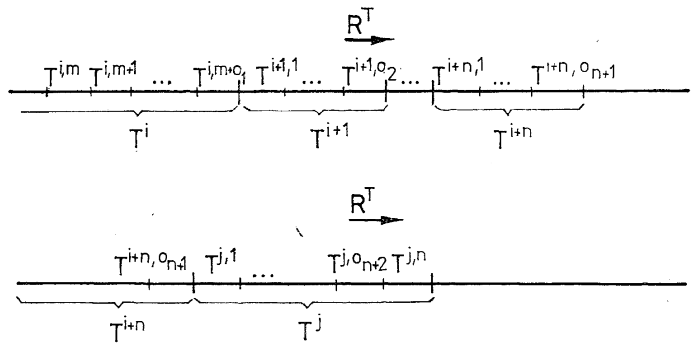

Fig. 3.

Further let us introduce definitions of the form:

(D II/7) $$\text{Def } u(\Delta t_{im,jn}) = NC \sum \Delta t_{im,jn}$$

by which the number of elements (cardinality) of the class, the summation of the given interval, is defined.

Further the definition

(D II/8) $$\text{Def } \Delta t_{im,jn} <_T \Delta t_{ko,lp} \leftrightarrow \exists v_1 \exists v_2 (v_1 = u(\Delta t_{im,jn}) \wedge v_2 = u(\Delta t_{ko,lp}) \wedge \wedge v_1 < v_2)$$

is introduced.

Let in the language a sentence enabling us to measure magnitudes of time intervals

(II/4) $$g^T(\Delta t_{im,jn}) = u(\Delta t_{im,jn})$$

be given. (By this sentence the number equal to the number of elements of its summation is assigned by the function named g^T to each time interval. In the interpretation intended this number gives the duration length of the interval. It is obvious that the magnitude of the time unit of duration is given by the norm of the decomposition of the class T into classes of the type $T^{k,l}$.

Additivity of the introduced time structure is assured by the next sentence (II/5) if required. Let us define the "union of two time intervals" first:

(D II/9) $$\text{Def } \Delta t_{im,jn} +_T \Delta t_{ko,lp} = \Delta t_{im,lp} \leftrightarrow \exists U_1 \exists U_2 \exists U_3 (U_1 = \sum \Delta t_{im,jn} \wedge \wedge U_2 = \sum \Delta t_{ko,lp} \wedge U_3 = \sum \Delta t_{im,lp} \wedge \wedge U_3 = U_1 \cup U_2)$$

 The postulated sentence on additivity is

$$g^T(\Delta t_{im,jn} +_T \Delta t_{ko,lp}) = g^T(\Delta t_{im,jn}) + g^T(\Delta t_{ko,lp}) \tag{II/5}$$

In the languages under construction names of time points can be defined by definitions of the form

$$\text{Def } t_{im} = v \leftrightarrow \exists \Delta t_{ko,lp}(v \in \sum \Delta t_{ko,lp}) \tag{D II/10}$$

now. (Thus time point is defined as an element of the summation class of a given interval.)

It obviously follows from the sentences and definitions presented that the term "time interval" can be defined in the ranks of the language J_t^u so that it meets all requirements formulated in the language J_t^p. Similarly on the other hand the term "time point" can be defined within the language J_t^p so that it meets all requirements formulated by the sentences of the language J_t^u. Hence there is no need in prefering one approach to "relativistic time" to the other one.

Sentences in both kinds of languages obviously hold if finiteness of the number of subclasses of the type $T^{k,l}$ of the class T is assumed. The conclusions reached can be generalized for the countable cardinality of subclasses of the class T in the following way.

(1) We will prove that the conclusions hold when the class T is decomposed into two subclasses T^1, T^2 where each of the subclasses is decomposed in another two $T^{1,1}$, $T^{1,2}$, $T^{2,1}$, $T^{2,2}$ again.

(2) Further it will be proved: Suppose the conclusions hold for decompositions of the class T into n subclasses of the type T^k where each of them is decomposed into l_o subclasses of the type T^{k,l_o} $(1 \leqq o \leqq n)$ again, then the conclusions hold for any decomposition of T into $(n+1)$ subclasses of the type T^k and also for decompositions of each of the subclasses into l_{o+1} subclasses of the type $T^{k,l_{o+1}}$.

It should be agreed upon that the identification of a time point with the whole of a subclass of the type $T^{i,m}$ (where further decomposability is not excluded) is not very intuitive from the traditional point of view. We are tempted to consider time points as sections in time intervals from this traditional standpoint. Under assumption of the "relativistic approach to time" there are considerable troubles implied by the "event" definition of "time points" understood in this way. In the definition from (1) mentioned above it is supposed in view of the interpretation intended that we are able to verify the condition "the event x took place simultaneously with the event y". Of course this sentence is unverifiable in the strict sense of word due to the finiteness of our observing and measuring capacity. If verifiability in certain (historically given) tolerance limits in stating simultaneousness is admitted, then we arrive exactly at the same concept of "time point" as used in the languages J_t^u, J_t^p suggested.

The difficulties mentioned appear to be still more obvious iff the given concept of an "event" is taken for the start point. Definitions of the "time point" of the form (D II/2) would need the continuous character of changes of the property T of the chronometer C. If on the other hand the objects c_1, c_2, c_3, ... (considered as development stages of the chronometer) are identical by the sentence (II/1) in all their properties with the exception for those of the class T (phases of the property change T) then uncountably many such objects must be considered. Statements of the form $T^{k,l}(c)$ are unverifiable in an empirical way and the chance of "exact recognition of what time is right now by chronometer" is lost. Hence it seems that the author of [1] is not quite right when he prefers the "event approach". Difficulties appear not only at the stage where the "time point" should be defined but also much sooner when explanation of what it actually is an "event" and the question of how to describe it in an exact language are asked.

In the course of construction of the languages J_t^u, J_t^p we have mentioned a possible definition of the "time point" by which these make an at most countable class. Classical physics on the other hand takes on the hypothesis of time-continuum just at this start point. Suggested languages give us a chance of a step by step growth of density of the class of time points by further decomposition of subclasses of the type $T^{k,l}$. (The subclass $T^{i,m}$ can be – for example – further decomposed into another two subclasses $T^{i,m,1}$, $T^{i,m,2}$ and these again into subclasses $T^{i,m,1,1}$, $T^{i,m,1,2}$, $T^{i,m,2,1}$, ...) When building up the language this will be reflected in growing numbers of primitive constants $T^{k,l}$, $T^{k,l,i}$ etc.

If potentially unlimited decomposability of any of the subclasses of the class T is assumed then a way to the time-continuum may be looked upon as a limiting process of decompositions of subclasses of the class T. The assumption of such unlimited decomposability is unverifiable in an empirical way of course. It must be postulated in the metalanguage by which the construction of the type J_t languages is described. The aim is to get sentences specific for the continuous character of the class of time points in the language constructed for the "relativistic approach to time". The language built would be enriched by new postulates then. A sentence on connectivity of the relation R^T in the class of all thinkable subclasses of the class T could be postulated for example. This of course would bring some formal changes in some other sentences of the language.

In traditional discourses a linear, metric and additive character of the time structure was mostly assumed. There is no need in keeping this assumption when constructing languages in suggested ways. The specific character of the time structure where time points and time intervals are defined in suggested ways can be assured by means of further postulates through which the meaning of the term R^T denoting the order relation for subclasses of the class T and that of the term f^T (g^T resp.) through which the metric character of the time structure is postulated.

By means of specific postulates e.g. a nonlinear character of the class of time points, additive or nonadditive character of the time metric can be given.

The circular character of the class of time points in the case of languages discussed with an at most countable class of subclasses of T could be formulated by an additional postulate of the form:

"There is i and m so that it holds:

$\{[\text{if } k \neq i, l \neq m \text{ then: } \forall T^{k,l}\, \exists T^{k,l+1}\, R^T(T^{k,l}, T^{k,l+1})$, if $T^{k,l}$ is not the last subclass of the class T in order R^T) and simultaneously $\forall T^{k,l}\, \exists T^{k+1,1}\, R^T(T^{k,l}, T^{k+1,1})$ if $T^{k,l}$ is the last subclass of the class T^k in order R^T)] and simultaneously $R^T(T^{i,m}, T^{1,1})\}$.

In scheme, see Fig. 4.

In all cases of the languages there holds a sentence a modification of which gives N. Rescher in [6] in a form of a "chronometric axiom":

$$\underset{c \in C}{\forall c}\, \forall t_{im}\, \forall t_{jn} (\exists T^{i,m}\, \exists T^{j,n} ((t_{im}(c) \leftrightarrow T^{i,m}(c)) \wedge (t_{jn}(c) \leftrightarrow T^{j,n}(c)) \wedge T^{i,m} = T^{j,n}) \to$$
$$\to t_{im} = t_{jn}) .^*$$

Let us note that due to the definitions given in J_t^u, J_t^p the "relativists" need not refuse the time structure of the form shown in Fig. 1 as Rescher states in [6]. Let the class T ordered in this way be of the form given in Fig. 5. The form is specified by giving

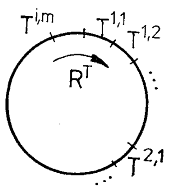

Fig. 4.

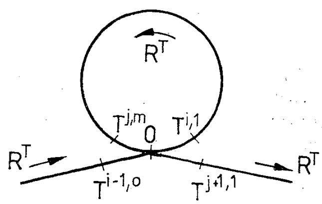

Fig. 5.

particular postulates in the languages. The point of intersection O need not mean that in the corresponding time point an object must have and need not have a given property at the same time. In our approach where time points are identified not with the points but with the classes it is certainly possible to describe e.g. the following changes of properties of the object b so that the contradiction law is respected:

$$F(b, t_{i-1,o}) \wedge F(b, t_{i,1}) \wedge \ldots \wedge F(b, t_{j,m}) \wedge \sim F(b, t_{j+1,1}) .$$

Hence we need not prefer the "absolute approach to time" as stated in [6].

* In [6] the axiom is given in the form: $R_{t'}[C_{t'}(p)] \supset t = t'$, where $R_{t'}(p)$ is interpreted as "p holds at the time t". The sentence mentioned above is in good consent with this sentence as far as meaning is concerned.

In connection with the discussion of the order relation R^T and the metric functions f^T, g^T it is necessary to add that when switching from one norm of decompositions of the class T to an another one at a certain level of decompositions a change in the character of the relation and the functions can be fell upon. For example it may be assumed that when subclasses of the type $T^{k,l}$ ordered by the relation R^T in the class T are decomposed further then the subclasses of the type $T^{k,l,o}$ (obtained as products of the decomposition) may appear to be ordered by an another relations $R^{T'}$ and relevant time points (or intervals resp.) may refer to another metric functions $f^{T'}$, $g^{T'}$. These changes of order relations and those of metric functions can be formulated in the languages by means of further postulates speaking about the fact that the topology and may be the metric of the ontologic structure is being changed within certain limits. One can feel that languages modified in this way could meet the demand of modern physics.

III.

We will deal with the problem of the identification of objects undergoing time changes in the next part of our discourse. The introductory discussion of problems connected with the willingness of keeping the Leidniz's ideas on identity when time changes of objects are described will be pursued further here.

The following example is given in [6]. Let a, b be identical objects. Then it holds by Leibniz:

(III/1) $$\forall F(F(a) \leftrightarrow F(b)) .$$

Suppose further that the object a posseses the property F at the time t, while the object b does not possess it:
$F(a)$, $\sim F(b)$, hence $\sim(a = b)$ which contradicts the sentence (III/1).

Utilizing the way of formulation used in this discourse we can also write

$$F(a, t) \wedge \sim F(b, t)$$

A weakening of identity with respect to time is suggested by Rescher in [6]: The following weakened concept of identity can be accepted according to it:

$$a = b \leftrightarrow \forall F \, \forall t(F(a, t) \leftrightarrow F(b, t)) .$$

If the object b does not posses the property F at an another time t' i.e.

$$F(a, t) \wedge \sim F(b, t')$$

the contradiction is avoided now.

Another trouble will be faced on the other hand when the following possibility is admitted: at all time points preceding t_0 and at all time points following t_1 all properties of the objects a and b are equal; in the time interval $\langle t_0, t_1 \rangle$ the objects differ in the property F (see Fig. 6). Let us suggest a still more weakened concept of identity now.

$$a = b \leftrightarrow \exists t\, \forall F(F(a, t) \leftrightarrow F(b, t))\,.$$

All traditional laws of identity can not be deduced then yet. The identity treated in this way is no more transitive as illustrated by the scheme given in Fig. 7. It holds: $a = b$ at t_0, $b = c$ at t_1, but it does not hold $a = c$ at any time.

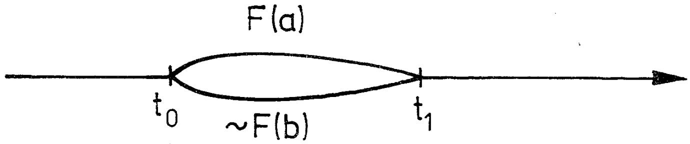

Fig. 6.

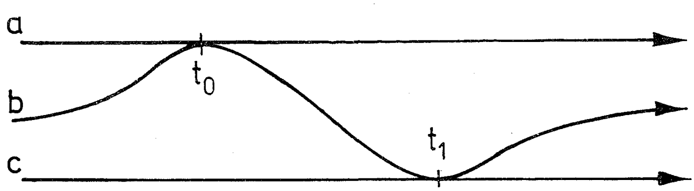

Fig. 7.

Let us supply some notes to the reasonings taken from [6]. First of all it is obvious that the weakened treatment of identity is not identical with the Leibniz's one. So both terms defined must be denoted by distinct symbols. Let us define the notion of identity "relativized with respect to the time interval $\Delta t_{i,j}$" in the following way:

(D III/1) $$\text{Def}\; x \underset{\Delta t_{ij}}{=} y \leftrightarrow \underset{t \in \Delta t_{ij}}{\forall t}\; \forall F(F(x, t) \leftrightarrow F(y, t))\,.$$

It is obvious that the "relativized identity" defined in this way gets along with the modified identity laws. From the definition axiom it can be deduced:

$$\forall x\, \forall \Delta t_{ij}(x \underset{\Delta t_{ij}}{=} x)\,,$$

$$\forall x\, \forall y\, \forall \Delta t_{ij}(x \underset{\Delta t_{ij}}{=} y \rightarrow y \underset{\Delta t_{ij}}{=} x)\,,$$

$$\forall x\, \forall y\, \forall z\, \forall \Delta t_{ij}((x \underset{\Delta t_{ij}}{=} y \wedge y \underset{\Delta t_{ij}}{=} z) \rightarrow x \underset{\Delta t_{ij}}{=} z)\,,$$

$$\forall x\, \forall y\, \forall z\, \forall \Delta t_{ij}((x \underset{\Delta t_{ij}}{=} z \wedge y \underset{\Delta t_{ij}}{=} z) \rightarrow x \underset{\Delta t_{ij}}{=} y)\,,$$

what appear to be the time relativized versions of identity laws.

Let us place the following question now: is an object undergoing time changes identical with itself? The search for an answer to this question was characteristic for quite a number of various philosophical schools in the past.

By tradition we identify an object by means of all its properties, what is well noticeable from the form of the definition of an individual:

$$\text{Def } a = x \leftrightarrow F(x)$$

where F is a molecular predicate reflecting the union of all properties of the object whose name is being defined.

The form of the corresponding definition axiom will certainly be: $\forall x\, \forall F(a = x \leftrightarrow F(x))$, where in the left part of the equivalence the traditional concept of identity is used.

In a language with a time variable the axiom could be formulated as follows: $\forall x\, \forall F\, \forall t(a = x \leftrightarrow F(x, t))$. Suppose now that the object a will be changed in some of its properties in the course of time. Let us denote by a_1 and a_2 two different phases of the changing object a. Let us suppose that the changing object is identical with itself: $a_1 = a_2$; but it holds at the same time: $\exists F\, \exists t_1\, \exists t_2(F(a_1, t_1) \wedge \sim F(a_2, t_2))$ and hence by means of the definition axiom of identity contradiction can be drawn:

$$(a_1 = a_2) \wedge \sim(a_1 = a_2)\,.$$

The paper [5] dealing with this sort of problems offers a suggestion of a weakened notion of identity that is based on substantial properties of identical objects here. In a simplified form*):

$$\text{Def } x = y \leftrightarrow \forall F(P(F) \rightarrow (F(x) \leftrightarrow F(y)))$$

where the one place second order predicate constant P denotes the property of "being a substantial property".

To make the defined weakened identity follow the laws of identity (the property of "being a substantial property" can change in the course of time itself: what is substantial now need not be substantial in future) let us relativize the identity with respect to a certain set of properties that remain to be substantial properties of the object concerned in all the time interval given. Thus a double relativization of identity is reached: one with respect to a given time interval, the other with respect to a given set of substantial properties:

(D III/2) $$\text{Def } x \underset{\Delta t_{ij}}{=^P} y \leftrightarrow \forall F \underset{t \in \Delta t_{ij}}{\forall t} (P(F, t) \rightarrow (F(x, t) \leftrightarrow F(y, t)))\,.$$

Double relativized versions of identity axioms can be drawn from the definition

* In [5] multiplace predicates are dealt with as well.

165 axiom (D III/2):

$$\forall x\, \forall P\, \forall \Delta t_{ij}(x \underset{\Delta t_{ij}}{=^{P}} x)\,,$$

$$\forall x\, \forall y\, \forall P\, \forall \Delta t_{ij}(x \underset{\Delta t_{ij}}{=^{P}} y \rightarrow y \underset{\Delta t_{ij}}{=^{P}} x)\,,$$

$$\forall x\, \forall y\, \forall z\, \forall P\, \forall \Delta t_{ij}((x \underset{\Delta t_{ij}}{=^{P}} y \wedge y \underset{\Delta t_{ij}}{=^{P}} z) \rightarrow x \underset{\Delta t_{ij}}{=^{P}} z)$$

$$\forall x\, \forall y\, \forall z\, \forall P\, \forall \Delta t_{ij}((x \underset{\Delta t_{ij}}{=^{P}} z \wedge y \underset{\Delta t_{ij}}{=^{P}} z) \rightarrow x \underset{\Delta t_{ij}}{=^{P}} y)\,.$$

In accordance with the concept of double relativization stated an object changing in time can be considered as "identical with itself" in a given time interval if its substantial properties remain unchanged throughout this interval:

(III/2) $$\forall F \underset{t, t_1, t_2 \in \Delta t_{ij}}{\forall t\, \forall t_1\, \forall t_2}((P(F, t) \rightarrow ((F(a_1, t_1) \leftrightarrow F(a_2, t_2)) \rightarrow a_1 \underset{\Delta t_{ij}}{=^{P}} a_2))$$

for a given time interval and a given set of substantial properties.

Under these circumstances the existence of nonsubstantial properties changing in this time interval can be described by the sentence:

$$\exists G \underset{t', t'' \in \Delta t_{ij}}{\exists t'\, \exists t''}(\sim P(G, t') \wedge \sim P(G, t'') \wedge G(a_1, t') \wedge \sim G(a_2, t''))$$

which – together with the sentence (III/2) concerned with the "identity of an object with itself" – does not bring us to contradictions any more.

The relativization of identity suggested is quite intuitive. Let us keep in mind that we consider as (pragmatically) substantial just those of the properties of an object undergoing changes that experience no chages in the given period and through which the object can enter relatively stable relations (important to us).

If the suggested notion of identity is adopted the "motion paradoxes of identity" can be avoided and still the identity laws (in their relativized form) can be kept.

At the same time a weak point of the approach suggested should be stated: it brings considerable complications of the syntactic structure of the language in question.

For the sake of individual time intervals and for individual given sets of substantial properties the language is conservatively being enriched by definitions of further new predicate constants-notations of distinctly relativized identities.

The "motion paradoxes of identity" will appear neither in a language in which names of time points are considered to be primitive constants (the "absolute approach to time") nor in the languages J_t^u, J_t^p in which time terms (names of time points and time intervals) are introduced by definitions (the "relativistic approach to time") provided the identity is double relativized. Hence all these languages can be built up (from the point of view of avoiding the "identity paradoxes") on the classical logical base.

Let us consider properties that do not belong to the set P. Suppose that two phases a_1, a_2 of a changing object are identical in the sence of (D III/2) from the point of view of the property set P and the time interval $\Delta t_{i,j}$.

Let us consider a property $A \notin P$, an object b undergoing changes in the limits of the interval $\Delta t_{i,j}$. When the "relativistic approach to time" is adopted then the description of time changes of the object b obviously appears not to be a paradox only if the following condition is met:

$$\sim \underset{t_{k,l} \in \Delta t_{ij}}{\exists t_{k,l}} (A(b_1, t_{k,l}) \wedge \sim A(b_1, t_{k,l}))$$

The case when the condition is not met can be visualized in the manner given in Fig. 8 when the definition of the term "time point" is given either in the language J_t^u or in J_t^p.

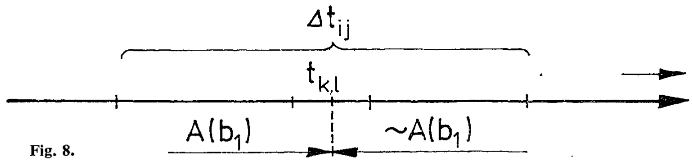

Fig. 8.

If the relativistic approach to time is adopted, then no property can be changed within a time point. Certainly then the objects the time changes of which are being dealt with must not change continuously. What was said now refers to changes of metric magnitudes of properties too if these metric magnitudes of properties are understood to be classes – what is quite common nowadays.

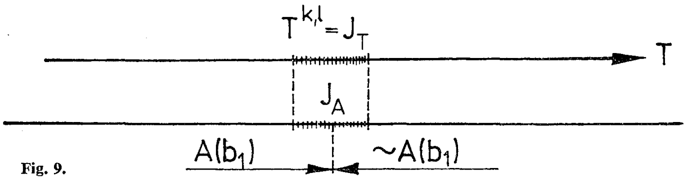

Fig. 9.

The condition given seems to be quite a strong requirement at the first sight. Let us keep in mind that every commonly used observatory or measuring apparatus by means of which we find whether a property of an object studied was changed or not has its precision limits characterized by its confidence interval. The following requirement can be drawn from this fact: It we are to avoid the "motion paradoxes" of the type $F(a, t) \wedge \sim F(a, t)$ when describing time changes that are formulated in any of the languages J_t^u, J_t^p from the standpoint of any property of an object, then the confidency interval that comes when any of these properties is measured must be "larger in its width" than is the "width" of any time point defined in this language by means of the respective class $T^{k,l}$.

The requirement is quite intuitive (see Fig. 9).

The formulation of the requirement given is far from being precise of course. What is the meaning of the "width of an interval" in comparison to the "time point width"?

The form of the requirement can be made more precise under the assumption that metric additive properties and a metric additive time structure is being dealt width.

Suppose that in the language J_t^u or J_t^p the class $T^{k,l}$ by means of which the time point $t_{k,l}$ is defined is just equal to the confideney interval of the apparatus through which the changes of the property T of the chronometer C is observed. Let us consider the class $T^{k,l}$ as an interval of subclasses ordered by the relation R^T in $T^{k,l}$ where the number of the subclasses in $T^{k,l}$ is increasing beyond any limits. Suppose T_1 is the first, T_2 is the last of all the subclasses. Let t_1, t_2 be the time terms assigned to the classes T_1, T_2 by means of the definitions analoguous to that one assigning the term $t_{k,l}$ to the class $T^{k,l}$.

Let further f^A be a metrizing function mapping the magnitude of the property A in the class of numbers.

Let J_A be a numerically given confidency interval of the apparatus by means of which the magnitude of the property A of the object B is being measured in the given instant.

Then for every property A, every object b and every interval $T^{k,l}$ it must hold

$$|f^T(t_1) - f^T(t_2)| = J_T \rightarrow |f^A(A(b, t_1)) - f^A(A(b, t_2))| < J_A .$$

The requirement stated is formulated in a metalanguage speaking about the language J_t^u (J_t^p resp.); unlimited decomposability of the basic subclass $T^{k,l}$ is assumed.

If the requirement is satisfied the motion paradoxes are avoided when nonsubstantial properties are dealt with. Let us realize that the requirement implies the following: along with any narrowing of the confidency interval the confidency interval of the "time" property T changes observation of the chronometer C must be made narrower too. If the last condition is not satisfied then there is a danger of paradoxes when informations on time changes are formulated in a given language; if we want to avoid the paradoxes in a language then it may appear necessary to rebuild. the language e.g. by switching to a nonclassical logical base (e.g. through the adoption of a third value for propositions "unstated" of the language) from the classical one. The papers [6; 7; 3; 4] should be mentioned at this place.

In languages based on the "absolute approach to time" the requirement is satisfied in a trivial manner if the existence of time continuum is postulated there. In the other case (i.e. in the case of "discrete time") we may be forced to rebuild the language in which new empirical informations are to be formulated if they were found by means of a more precise apparatus. It can be done so that we either postulate a "condensation" of the class of time points or we rebuild the logical base of the language (e.g. by switching from the classical logical base to the nonclassical one).

IV.

Our reasonings could be summarized as follows:

1. When trying to describe time changes of objects in an exact language in which the Leibniz's treatment of identity is adopted we may come to well known paradoxes leading to contradictions in the language.

2. Time changes of objects are described by means of time terms in an exact language. Time terms can be introduced as primitive constants in the language built. Special time postulates speak about the character of time structure in a model of the language then. Certain complication of the language universe that is nonhomogeneous from a certain standpoint appears to be a disadvantage of the construction.

Time terms can be introduced in the language built by means of definitions too. In such a case language must have the term T, series of terms T^1, T^2,,..., $T^{1,1}$, $T^{1,2}$, ..., $T^{2,1}$, $T^{2,2}$, ..., the term R^T, the terms c_1, c_2, ... and the term C among its primitive constants. By means of them subsequent changes of a chosen property of a chronometer C through which time points and intervals are defined, are described in the intended interpretation of the language.

Time terms can be introduced in two ways. The first one refers to the "event approach to time" the other refers to the "process approach to time". It was shown that both ways in the introduction of time terms are idempotent for a language construction if an at most countable set of the type $T^{k,l}$ of subclasses of the class T is assumed. By means of specific postulates that refer to primitive terms R^T and f^T (g^T resp.) a special topological and metric character of time structure can be assured in languages constructed. By the approach described some weak points of the "relativistic approach to time" spoken about by some authors earlier can be avoided.

3. A concept of double relativized identity (i.e. that of identity relativized with respect to a given set of substantial properties and with respect to a given time interval) was suggested. By this concept basic laws of identity in their relativized form remain observed. This way "motion paradoxes of identity" of the type "an object is and is not identical with itself at a given time" are avoided.

By a certain assumption concerned with the time structure of the language "motion paradoxes" drawing contradictions in exact languages and facing us with necessary changes in the logical base of the language can be avoided. The assumption was formulated for both the "relativistic" and the "absolute" approach to time. Time changes of physical objects can be described through empirical statements without paradoxes and without any need in change of classical logical base of the language if the assumption is satisfied.

(Received October 22, 1975.)

REFERENCES

[1] З. Аугустынек: Два определения времени. Вопросы философии *6* (1970), 35, 65—76.
[2] R. Bek: K popisu fyzikálních procesů pomocí klasifikačních, topologických a metrických termínů. Teorie a metoda *4* (1972), 3, 87—109.
[3] R. Bek: Sémantický aspekt popisu reality pomocí přesných množin. Acta Polytechnica *1* (1975), 4, 23—43.
[4] St. Körner: Zkušeňost a teorie. Svoboda, Praha 1970.
[5] L. Tondl: Problémy identifikace a podobnosti. Filosofický časopis *2* (1968), 16, 222—233.
[6] Rescher - Urquard: Temporal Logic. Springer, N. York 1971.
[7] А. Зиновев: Логическая физика. Наука, Москва 1972.

Doc. PhDr Roman Bek, CSc., Katedra aplikované matematiky, strojní fakulta ČVUT (Department of Applied Mathematics, Faculty of Mechanical Engineering, The Czech Technical University), Suchbátarova 4, 160 00 Praha 6. Czechoslovakia.