KYBERNETIKA — VOLUME 10 (1974), NUMBER 6

A Real-Time Identification of Continuous Linear Systems

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The article presents the method of estimating the coefficients of linear differential equations based on measuring the input and output signals of identified systems. In comparison with other methods this method offers relatively very accurate results. It does not require a special type of input signal and is suitable for real-time identification under operating conditions.

1. INTRODUCTION

The basic requirement expected to be satisfied by the method suggested was achieving sufficient accuracy of estimation and finding a possibility of using it with an arbitrary, in advance undefined input signal.

As a digital computer is able to process only data sampled at discrete time instants, the system was identified by means of the hybrid technique which guarantees greater accuracy. The analog part was used for transforming both the input and output signals into such variables that can be sampled discretely without affecting the accuracy of the solution.

The problem of the continuous system identification was thus transformed into an equivalent problem which was then solved by means of the digital technique.

The unknown parameters were calculated by means of Bayes optimal estimation or its modification, which in some situations offers a more simple solution while retaining the basic characteristics of a good estimate.

2. EQUIVALENT PROBLEM

Suppose the identified system is described by a linear differential equation with constant coefficients

$$(1)\qquad \eta^{(k)} + \beta_k \eta^{(k-1)} + \ldots + \beta_1 \eta = \alpha_k \xi^{(k-1)} + \ldots + \alpha_1 \xi + \chi^{(k)} + \beta_k \chi^{(k-1)} + \ldots \\ \ldots + \beta_1 \chi ,$$

where $\xi(t)$ and $\eta(t)$ are input and output respectively and where $\chi(t)$ is additive output noise.

If we now use the symbols $\mathscr{L}_1$ and $\mathscr{L}_2$ to denote the suitable linear differential operators, equation (1) can be written as

$$(2) \qquad \mathscr{L}_1(\eta) = \mathscr{L}_2(\xi) + \mathscr{L}_1(\chi) .$$

Let $\mathscr{L}_0$ now be another stationary homogeneous linear differential operator and let the variables $x(t)$, $y(t)$ and $\mu(t)$ be defined by the relations

$$(3) \qquad \mathscr{L}_0(x) = \xi ,$$

$$(4) \qquad \mathscr{L}_0(y) = \eta ,$$

$$(5) \qquad \mathscr{L}_0(\mu) = \chi .$$

By inserting these equations into (2) we receive

$$\mathscr{L}_1(\mathscr{L}_0(y)) = \mathscr{L}_2(\mathscr{L}_0(x)) + \mathscr{L}_1(\mathscr{L}_0(\mu)) .$$

With regard to the commutativity and linearity of the operators, it is possible to write

$$(6) \qquad \mathscr{L}_0(\mathscr{L}_1(y)) = \mathscr{L}_0(\mathscr{L}_2(x) + \mathscr{L}_1(\mu))$$

or

$$(7) \qquad \mathscr{L}_0(\mathscr{L}_1(y) - \mathscr{L}_2(x) - \mathscr{L}_1(\mu)) = 0 .$$

The general solution y of this equation can then be expressed in the form

$$y = y_h + y_p ,$$

where y_h is a general solution of the homogeneous equation $\mathscr{L}_0(\mathscr{L}_1(y)) = 0$ and y_p is then an arbitrary particular solution of equation (6) or (7). If as this particular solution y_p a solution satisfying equation

$$(8) \qquad \mathscr{L}_1(y) = \mathscr{L}_2(x) + \mathscr{L}_1(\mu)$$

is chosen, then in the case when $y_h = 0$ the solution of equation (6) or (7) is given by the solution of equation (8). The condition $y_h = 0$ can be ensure by suitable choice of initial conditions or by making use of the equation $\lim_{t \to \infty} y_h = 0$ satisfying a stable system.

The identification problem of system (1) or (2) is then identical with the identification problem of system (8) where relations (3) and (4) hold for x and y. By a suitable choice of operator $\mathscr{L}_0$ we can then ensure that the variables $x, \dot{x}, \ldots, x^{(k)}, y, \dot{y}, \ldots$ $\ldots, y^{(k)}$ can be measured as outputs of linear filters which are connected to the input and/or output of the identified system.

Let us write equation (8) in the form

$$(9) \qquad y^{(k)} + \beta_k y^{(k-1)} + \ldots + \beta_1 y = \\ = \alpha_k x^{(k-1)} + \ldots + \alpha_1 x + \mu^{(k)} + \beta_k \mu^{(k-1)} + \ldots + \beta_1 \mu$$

and let us use a matrix form for equations (3) and (4)

$$(10) \qquad \dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{d} \,.\, \xi \,,$$

$$(11) \qquad \dot{\boldsymbol{y}} = A\boldsymbol{y} + \boldsymbol{d} \,.\, \eta$$

where

$$A = \begin{bmatrix} 0 & 1 \\ \gamma_1 & \gamma \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(k-1)} \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(k-1)} \end{bmatrix}.$$

The vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively can then easily be measured as the outputs of the linear continuous system described by equations (10) and (11) respectively with inputs ξ and η respectively. As χ cannot be measured, its derivatives cannot be measured either. In equation (9) we then put

$$\mu^{(k)} + \beta_k \mu^{(k-1)} + \ldots + \beta_1 \mu = \zeta \,.$$

If we further put

$$\boldsymbol{a}^{\mathrm{T}} = [\alpha_1, \ldots, \alpha_k] \,, \quad \boldsymbol{b}^{\mathrm{T}} = [\beta_1, \ldots, \beta_k]$$

where symbol $\boldsymbol{a}^{\mathrm{T}}$ denotes a matrix transposed to matrix $\boldsymbol{a}$, equation (9) can also be written in the form

$$(12) \qquad y^{(k)} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{b} = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{a} + \zeta \,.$$

Our task is then to find the optimal estimate $[\boldsymbol{a}^*, \boldsymbol{b}^*]$ of the vector $[\boldsymbol{a}, \boldsymbol{b}]$ on assumption that in equation (12) the variable $y^{(k)}(t)$ and the vectors $\boldsymbol{y}(t)$ and $\boldsymbol{x}(t)$ can be measured at arbitrary time instants t_j, $j = 1, 2, \ldots, m$. $\zeta(t)$ is then the noise.

3. OPTIMAL ESTIMATION

Let us assume now that the system of equations (12) for different time instants $t_j, j = 1, 2, \ldots, m$ can be written in the form

$$(13) \qquad \boldsymbol{w} = \boldsymbol{Z} \,.\, \boldsymbol{c} + \zeta$$

where $\boldsymbol{w}$ is an m-dimensional vector, $\boldsymbol{Z}$ is an $m \times n$-dimensional matrix, $\boldsymbol{c}$ is an n-dimensional vector of unknown parameters and ζ is an m-dimensional random vector. The vectors $\boldsymbol{c}$ and ζ are assumed to be mutually independent with a priori given probability densities $p(\boldsymbol{c})$ and $p(\zeta)$.

If we now call the positive definite function $\varrho(c^* - c)$ a risk, it is natural to choose the estimate c^* of the vector c in such a way as to minimize the risk. Under relatively general conditions [3], [4], [11] such optimal estimate c^* can be expressed

$$c^* = \int c \,.\, p(c \mid w, Z)\, \mathrm{d}c \tag{14}$$

where the a posteriori probability density $p(c \mid w, Z)$ is given by Bayes equation

$$p(c \mid w, Z) = \frac{p(w \mid Z, c)\, p(c \mid Z)}{\int p(w \mid Z, c)\, p(c \mid Z)\, \mathrm{d}c}$$

where the probability densities $p(w \mid Z, c)$ and $p(c \mid Z)$ can be calculated from the a priori probability densities $p(c)$ and $p(\zeta)$. Assuming that c is independent of the measured data Z

$$p(w \mid Z, c) = p_\zeta(w - Z \,.\, c)$$

and

$$p(c \mid Z) = p(c)\,.$$

The calculation of the optimal estimate c^* of the vector c according to equation (14) would on this level of generality be rather complicated. We shall therefore further assume that the necessary probability densities have a normal distribution. Let then

$$p(w \mid Z, c) = k_1 \exp\left[-\tfrac{1}{2}(w - Zc)^{\mathrm{T}} N^{-1}(w - Zc)\right], \tag{15}$$

$$p(c) = k_2 \exp\left[-\tfrac{1}{2}(c - c_0)^{\mathrm{T}} P_0^{-1}(c - c_0)\right]. \tag{16}$$

The optimal estimate c^* according to equation (14) is then given by equation

$$c^* = (P_0^{-1} + Z^{\mathrm{T}}N^{-1}Z)^{-1}\,(P_0c_0 + Z^{\mathrm{T}}N^{-1}w) \tag{17}$$

or the equivalent relation

$$c^* = P_0Z^{\mathrm{T}}(ZP_0Z^{\mathrm{T}} + N)^{-1} \,.\, w + (I - P_0Z^{\mathrm{T}}(ZP_0Z^{\mathrm{T}} + N)^{-1}Z)\, c_0 \tag{18}$$

The optimal estimates (17) and (18) respectively are thus linear functions of the vector w. In the case of a distribution other than normal the optimal estimate (14) would become a nonlinear function of the measured vector w. The estimates (17) and (18) respectively would, however, remain the best estimates of the whole set of linear estimates [11].

It can easily be proved that the optimal estimate has the following properties

$$\mathscr{E}c^* = c$$

$$\mathscr{E}(c^* - c)\,(c^* - c)^{\mathrm{T}} = (P_0^{-1} + Z^{\mathrm{T}}N^{-1}Z)^{-1} =_{\mathrm{def}} P \tag{19}$$

where the symbol $\mathscr{E}\boldsymbol{x}$ denotes the expected value of the matrix $\boldsymbol{x}$. The optimal estimate $\boldsymbol{c}^*$ is thus an unbiased estimate of the unknown vector $\boldsymbol{c}$ with the covariance matrix $\boldsymbol{P}$, for which in all practical cases

$$\lim_{m \to \infty} \boldsymbol{P} = \boldsymbol{0} .$$

Thus the estimate $\boldsymbol{c}^*$ is also a consistent estimate.

4. RECURSIVE SOLUTION

Let us now divide the measurements into two series in which the noises will be mutually independent. Let then

$$\boldsymbol{w} = \begin{bmatrix} \boldsymbol{w}_1 \\ \boldsymbol{w}_2 \end{bmatrix}, \quad \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Z}_1 \\ \boldsymbol{Z}_2 \end{bmatrix}, \quad \zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N}_2 \end{bmatrix}.$$

By inserting these equations into (17) we obtain

$$(20) \quad \boldsymbol{c}^* = (\boldsymbol{P}_0^{-1} + \boldsymbol{Z}_1^T\boldsymbol{N}_1^{-1}\boldsymbol{Z}_1 + \boldsymbol{Z}_2^T\boldsymbol{N}_2^{-1}\boldsymbol{Z}_2)^{-1}(\boldsymbol{Z}_1^T\boldsymbol{N}_1^{-1}\boldsymbol{w}_1 + \boldsymbol{Z}_2\boldsymbol{N}_2^{-1}\boldsymbol{w}_2 + \boldsymbol{P}_0^{-1}\boldsymbol{c}_0)$$

With regard to (19) we define

$$(21) \qquad \boldsymbol{P}_0^{-1} + \boldsymbol{Z}_1^T\boldsymbol{N}_1^{-1}\boldsymbol{Z}_1 =_{\mathrm{def}} \boldsymbol{P}_1^{-1}$$

where $\boldsymbol{P}_1$ is apparently a covariance matrix of the estimate of the vector $\boldsymbol{c}$ calculated from the first series of measurements. Let the symbol $\boldsymbol{c}_1$ denote this estimate $\boldsymbol{c}^*$. Analogically, let the symbol $\boldsymbol{c}_2$ denote the optimal estimate after two series of measurements and the symbol $\boldsymbol{P}_2$ its covariance matrix. Equation (20) will then be of the form

$$\boldsymbol{c}_2 = (\boldsymbol{P}_1^{-1} + \boldsymbol{Z}_2^T\boldsymbol{N}_2^{-1}\boldsymbol{Z}_2)^{-1} . (\boldsymbol{Z}_1^T\boldsymbol{N}_1^{-1}\boldsymbol{w}_1 + \boldsymbol{P}_0^{-1}\boldsymbol{c}_0 + \boldsymbol{Z}_2^T\boldsymbol{N}_2^{-1}\boldsymbol{w}_2) .$$

However, according to (17) and (21)

$$\boldsymbol{Z}_1^T\boldsymbol{N}_1^{-1}\boldsymbol{w}_1 + \boldsymbol{P}_0^{-1}\boldsymbol{c}_0 = \boldsymbol{P}_1^{-1}\boldsymbol{c}_1$$

and thus

$$\boldsymbol{c}_2 = \boldsymbol{P}_2(\boldsymbol{P}_1^{-1}\boldsymbol{c}_1 + \boldsymbol{Z}_2^T\boldsymbol{N}_2^{-1}\boldsymbol{w}_2)$$

where

$$\boldsymbol{P}_2 = (\boldsymbol{P}_1^{-1} + \boldsymbol{Z}_2^T\boldsymbol{N}_2^{-1}\boldsymbol{Z}_2)^{-1} .$$

If we now make several series of such measurements, the estimate of the unknown parameter $\boldsymbol{c}$ and its covariance matrix can be calculated from the following equations

$$\boldsymbol{c}_r = \boldsymbol{K}_r\boldsymbol{w}_r + (\boldsymbol{1} - \boldsymbol{K}_r\boldsymbol{Z}_r)\,\boldsymbol{c}_{r-1} , \qquad r = 1, 2, \ldots, q$$

and

$$P_r = (I - K_r Z_r) P_{r-1}, \qquad r = 1, 2, \ldots, q$$

where

$$K_r = (P_{r-1}^{-1} + Z_r^T N_r^{-1} Z_r)^{-1} Z_r N_r^{-1}, \quad r = 1, 2, \ldots, q$$

or

$$K_r = P_{r-1} Z_r^T (N_r + Z_r P_{r-1} Z_r^T)^{-1}, \qquad r = 1, 2, \ldots, q.$$

The expected value c_0 and the covariance matrix P_0 of the a priori probability density of the vector c are supposed to be given. The noise covariance matrix N_r, $r = 1, 2, \ldots, q$ is also considered known. It can be shown that the estimate does not require a precise knowledge of the a priori characteristics. If we do not know them, we can choose for example $c_0 = 0$, $P_0^{-1} = \varepsilon \,.\, I$, where $\varepsilon \to 0$, and $N_r = I$ [11].

5. OPTIMAL ESTIMATION MODIFICATION

In the above given optimal estimate of the unknown vector c it is supposed that the structure of the identified system is given by equation (13) where the vectors c and ζ are independent. This equation applies to the case when the identified system is described by a special type of differential equation or when the use of filter allows us to neglect the dependence of the vector ζ on the vector c. In a more general case it is, however, necessary, to consider even the more general structure of the identified system which can be written in the form

$$w + Yb = Xa + \zeta(b). \tag{22}$$

It could easily be shown that the classical application of Bayes estimate to this case does not lead to a solution suitable for practical use. We shall, therefore, present a substitute solution which, although it does not offer an estimate minimalizing the expected risk, has one great advantage – simple calculation.

Let us assume then that in equation (22) b is the given constant. Then only the vector a is left to be determined. Let us add the vector $X_+ a_+$, $a_+ = 0$ i.e. the zero vector to the right side of equation (22) and let us, at the beginning, regard a_+ as unknown.

We obtain

$$w + Yb = [X, X_+] \begin{bmatrix} a \\ a_+ \end{bmatrix} + \zeta(b).$$

As the vectors a and a_+ are independent of $\zeta(b)$, it is possible to use relation (17) for their estimation. With the denotation

$$Z_+ = [X, X_+]; \quad c_+ = \begin{bmatrix} a \\ a_+ \end{bmatrix}$$

we obtain

$$(23) \qquad c_+^* = (P_{+0}^{-1} + Z_+^T N^{-1} Z_+)^{-1} (P_{+0}^{-1} c_{+0} + Z_+^T N^{-1}(w + Yb))$$

where c_{+0} and P_{+0} are the expected value and the covariance matrix of the a priori probability density of the vector c_+ respectively.

If, however, we consider the vector a_+ given and if, on the contrary, we do not know the vector b, under suitable conditions the estimate of the vector a and b can be calculated form equation (23) and written

$$(24) \qquad \begin{bmatrix} a^* \\ b^* \end{bmatrix} = \begin{bmatrix} P_{01}^{-1} + X^T N^{-1} X\,; & -X^T N^{-1} Y \\ X_+^T N^{-1} X\,; & -X_+^T N^{-1} Y \end{bmatrix}^{-1} . \left(\begin{bmatrix} X^T \\ X_+^T \end{bmatrix} N^{-1} w + \begin{bmatrix} P_{01}^{-1} a_0 \\ 0 \end{bmatrix} \right)$$

where a_0 and P_{01} are the expected value and the covariance matrix of the a priori probability density of the vector a respectively. Practically, it is possible to choose $P_{01}^{-1} = 0$. Then we obtain

$$(25) \qquad c^* = (Z_+^T N^{-1} Z)^{-1} Z_+^T N^{-1} w$$

where

$$c^* = \begin{bmatrix} a^* \\ b^* \end{bmatrix}, \quad Z = \begin{bmatrix} X \\ -Y \end{bmatrix}.$$

Putting $N = \sigma^2 I$ yields

$$(26) \qquad c^* = (Z_+^T Z)^{-1} Z_+^T w\,.$$

It could easily be proved that estimate (24) and thus even estimates (25) and (26) are consistent and asymptotically ubiased.

6. NOTES ON PRACTICAL APPLICATION

When using the above mentioned method of identification it is necessary to connect continuous linear filters satisfying relations (10) and (11) to the input and output of the identified system. This can be done by means of an analog computer.

In practical cases it is necessary to make sufficient use of the range of the analog computer in order that measurements might be carried out with sufficient accuracy.

It can be demonstrated, however, that completely controllable filters can be connected to the input and output of the identified system

$$(27) \qquad \dot{u} = Eu + f\xi$$

and

$$(28) \qquad \dot{v} = Gv + h\eta$$

which are mutually bound by the relations

(29) $$QG = EQ$$

and

(30) $$Qh = f$$

where Q is the arbitrary regular matrix. The matrices E, f, G and h must then be chosen so that the filters would operate with satisfactory accuracy and at the same time be sufficiently simple [11]. As the vectors u and v can be regarded as the trans-

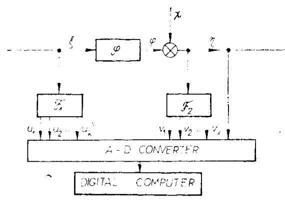

Fig. 1.

formed vectors x and y, it is necessary to transform the relations for the calculation of the optimal estimate as well. It is sufficient to use this transformation at the beginning and at the end of the calculation only.

In identifying the system described by the equation

(31) $$\eta^{(k)} + \beta_k \eta^{(k-1)} + \ldots + \beta_1 \eta = \alpha_k \xi^{(k-1)} + \ldots + \alpha_1 \xi + v$$

on assumption that the noise v is negligible after passing through the output filter, it is possible to proposeed according to Fig. 1 where the symbol $\mathscr{S}$ denotes the identified system and $\mathscr{F}_1$ and $\mathscr{F}_2$ are the input and output filters respectively described by relations (27) to (30).

For the calculation of the optimal estimate it is necessary to form the square matrix F defined by the relation

(32) $$F = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_k \end{bmatrix} = \begin{bmatrix} f^{\mathrm{T}} \\ f_1 E^{\mathrm{T}} \\ \vdots \\ f_{k-1} E^{\mathrm{T}} \end{bmatrix}$$

where E and f are the matrices from equation (27) and the column matrix d for which

(33) $$d^{\mathrm{T}} = [0, 0, \ldots, 0, 1] .$$

On the basis of the row vector $\boldsymbol{r}_1$ given by the relation

$$\boldsymbol{r}_1^{\mathrm{T}} = \boldsymbol{F}^{-1}\boldsymbol{d} \tag{34}$$

the square matrix $\boldsymbol{R}$ is formed

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \\ \vdots \\ \boldsymbol{r}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_1\boldsymbol{E} \\ \vdots \\ \boldsymbol{r}_{k-1}\boldsymbol{E} \end{bmatrix} \tag{35}$$

and the $\boldsymbol{S}$ matrix calculated

$$\boldsymbol{S} = \boldsymbol{R}\boldsymbol{Q} \tag{36}$$

where the matrix $\boldsymbol{Q}$ is taken from equations (29) or (30). Next we form the matrix $\boldsymbol{T}$

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S} \end{bmatrix}.$$

If now symbol $\hat{\boldsymbol{Z}}_r$

$$\hat{\boldsymbol{Z}}_r = \begin{bmatrix} u_1(t_{r,1}), & \ldots, & u_k(t_{r,1}), & -v_1(t_{r,1}), & \ldots, & -v_k(t_{r,1}) \\ u_1(t_{r,2}), & \ldots, & u_k(t_{r,2}) & -v_1(t_{r,2}), & \ldots, & -v_k(t_{r,2}) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ u_1(t_{r,m}), & \ldots, & u_k(t_{r,m}), & -v_1(t_{r,m}), & \ldots, & -v_k(t_{r,m}) \end{bmatrix}$$

is used to denote the matrix obtained from the measured values of the output variables of the filters $\mathscr{F}_1$ and $\mathscr{F}_2$ in the r-th measurement series, and the symbol $\boldsymbol{w}_r$ to denote the matrix for which

$$\boldsymbol{w}_r^{\mathrm{T}} = [w(t_{r,1}), w(t_{r,2}), \ldots, w(t_{r,m})],$$

where

$$w(t_{r,i}) = \boldsymbol{s}_k\boldsymbol{G}\boldsymbol{v}(t_{r,i}) + \eta(t_{r,i}), \quad i = 1, 2, \ldots, m \tag{37}$$

where $\boldsymbol{s}_k$ is the k-th row of the matrix $\boldsymbol{S}$, and if the symbols $\boldsymbol{c}_0$ and $\boldsymbol{P}_0$, which are assumed to be given, are used to denote the expected value and the covariance matrix of the a priori probability density of the vector $\boldsymbol{c}$ respectively, then the characteristic of the a posteriori probability density after the q-th measurement series can be obtained from the relations

$$\hat{\boldsymbol{c}}_0 = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{c}_0, \tag{38}$$

$$\hat{\boldsymbol{P}}_0 = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{P}_0\boldsymbol{T} \tag{39}$$

$$\hat{\boldsymbol{K}}_r = \hat{\boldsymbol{P}}_{r-1}\hat{\boldsymbol{Z}}_r^{\mathrm{T}}(\boldsymbol{N}_r + \hat{\boldsymbol{Z}}_r\hat{\boldsymbol{P}}_{r-1}\hat{\boldsymbol{Z}}_r^{\mathrm{T}})^{-1}; \quad r = 1, 2, \ldots, q \tag{40}$$

or the equivalent equation

$$\hat{K}_r = (\hat{P}_{r-1}^{-1} + \hat{Z}_r^T N_r^{-1} \hat{Z}_r)^{-1} \hat{Z}_r^T N_r^{-1} ; \quad r = 1, 2, \dots, q$$

where

(41) $$N_r = \mathscr{E} \zeta_r \zeta_r^T , \quad r = 1,2 , \dots, q$$

is the noise covariance matrix. Next we compute the matrices

(42) $$\hat{c}_r = \hat{K}_r w_r + (I - \hat{K}_r \hat{Z}_r) \hat{c}_{r-1} , \quad r = 1, 2, \dots, q$$

and

(43) $$\hat{P}_r = (I - \hat{K}_r \hat{Z}_r) \hat{P}_{r-1} , \quad r = 1, 2, \dots, q$$

which can be used as a priori information for the following estimate calculation or as a basis for computing the a posteriori characteristics of the vector c

(44) $$c_r = (T^{-1})^T \hat{c}_r , \quad r = 1, 2, \dots, q$$

and

(45) $$P_r = (T^{-1})^T \hat{P}_r T^{-1} , \quad r = 1, 2, \dots, q .$$

In the case when the influence of the noise on the output vector $v(t)$ cannot be neglected, it is possible to proceed according to Fig. 2.

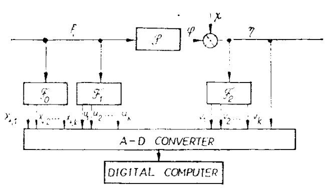

Fig. 2.

Suppose that the identified system is again described by the differential equation (31) and that relations (27) to (30) hold for the filters $\mathscr{F}_1$ and $\mathscr{F}_2$. Moreover, there is the filter $\mathscr{F}_0$ here, for which

$$\dot{x}_+ = D_1 x_+ + D_2 u + g\xi$$

where the basic requirement is that the vector x_+ be linearly independent of the vector u. The most advantageous course would then be making these vectors orthogonal, which can be done by a suitable choice of the matrices D_1, D_2 and g.

If we then use the symbols

$$\begin{aligned} X_+^{\mathrm{T}} &= [x_+(t_1),\ x_+(t_2),\ \ldots,\ x_+(t_m)]\,, \\ U^{\mathrm{T}} &= [u(t_1),\ u(t_2),\ \ldots,\ u(t_m)]\,, \\ V^{\mathrm{T}} &= [v(t_1),\ v(t_2),\ \ldots,\ v(t_m)]\,, \\ w^{\mathrm{T}} &= [w(t_1),\ w(t_2),\ \ldots,\ w(t_m)]\,, \end{aligned}$$

to denote the matrices of the measured values where relation (37) holds for $w(t_i)$, the estimate $[a^*, b^*]$ of the vector $[a, b]$ is given by the relations

$$\begin{aligned} \hat{a}_0 &= R^{\mathrm{T}} a_0\,, \\ \hat{P}_{01} &= R^{\mathrm{T}} P_{01} R\,, \end{aligned}$$

$$\begin{bmatrix} \hat{a}^* \\ \hat{b}^* \end{bmatrix} = \begin{bmatrix} \hat{P}_{01}^{-1} + U^{\mathrm{T}} N^{-1} U; & -UN^{-1}V \\ X_+^{\mathrm{T}} N^{-1} U; & -X_+^{\mathrm{T}} N^{-1} V \end{bmatrix}^{-1} . \left(\begin{bmatrix} U^{\mathrm{T}} \\ X_+^{\mathrm{T}} \end{bmatrix} N^{-1} w + \begin{bmatrix} \hat{P}_{01}^{-1} \\ 0 \end{bmatrix} \hat{a}_0 \right),$$

$$\begin{bmatrix} a^* \\ b^* \end{bmatrix} = \begin{bmatrix} R^{\mathrm{T}}, & 0 \\ 0, & S^{\mathrm{T}} \end{bmatrix}^{-1} . \begin{bmatrix} \hat{a}^* \\ \hat{b}^* \end{bmatrix}$$

where a_0 and P_{01} denote the excepted value and the covariance matrix of the a priori probability density of the vector a respectively and where the matrices R and S are given by relations (32) to (36).

If we now put $P_{01}^{-1} = 0$ then by using the relations

$$T = \begin{bmatrix} R, & 0 \\ 0, & S \end{bmatrix}, \quad \hat{Z} = [U, V]\,; \quad \hat{Z}_+ = [U, X_+]\,; \quad c = \begin{bmatrix} a \\ b \end{bmatrix}$$

we obtain

$$c^* = (T^{-1})^{\mathrm{T}} (\hat{Z}_+^{\mathrm{T}} N^{-1} \hat{Z})^{-1}\, \hat{Z}^{\mathrm{T}} N^{-1} w$$

and in the case when $N = \sigma^2 . I$ we obtain

$$c^* = (T^{-1})^{\mathrm{T}} (\hat{Z}_+^{\mathrm{T}} \hat{Z})^{-1}\, \hat{Z}_+^{\mathrm{T}} w\,.$$

7. EXPERIMENTAL TESTING OF THE METHOD

The above mentioned method was tested in identifying a given linear dynamic system modelled on the analog computer MEDA T and described by the differential equation

$$c_1 \xi + \ldots + c_k \xi^{(k-1)} = c_{k+1} \varphi + \ldots + c_{2k} \varphi^{(k-1)} + \varphi^{(k)}\,.$$

The input $\xi(t)$ obtained from the random generator and the signal $\eta(t)$ given by the relation

$$\eta(t) = \varphi(t) + \chi(t)$$

were measured on the identified system. In the above given equation $\chi(t)$ is the additive corruptive noise simulated by means of another, independent random generator. The unknown coefficients were calculated according to relations (38) to (45) by the digital computer ODRA 1204.

Table I presents the actual and estimated coefficients of the fourth order system the output of which was not corrupted by any noise. The actual and estimated step responces are shown in Fig. 3. The whole calculation took about 50 seconds.

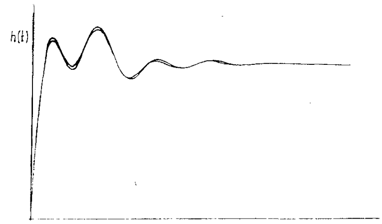

Fig. 3.

Table I.

c	0,500	1,000	1,000	1,000	0,500	1,000	2,000	1,000
c^*	0,511	1,020	1,017	0,966	0,513	1,027	2,055	1,006

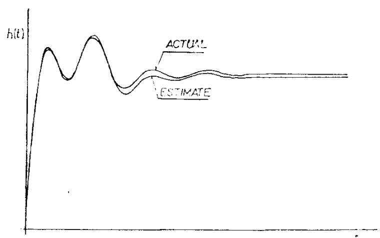

Fig. 4.

Table II.

c	0,500	1,000	1,000	1,000	0,500	1,000	2,000	1,000
c^*	0,469	1,310	0,868	1,094	0,484	1,320	2,070	1,128

Table II and Fig. 4 show the results of the identification of the same system the output of which, however, was corrupted by the noise χ of approximately the same level as that of the signal φ. In order to suppress the influence of the corruptive noise it was necessary to calculate the estimate for about two hours.

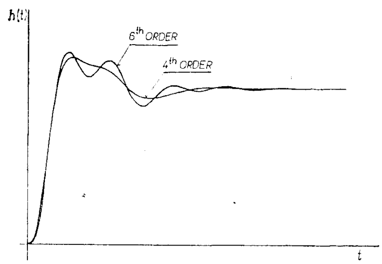

Fig. 5.

Table III.

c	0,500	1,000	1,000	1,000	0,000	0,000	0,500	1,500	3,500	4,000	4,000	2,000
c^*	0,444	0,436	0,379	-0,098	—	—	0,446	0,890	2,095	1,029	—	—

Table III and Fig. 5 present the results of the identification of the sixth order system, which was approximated by a fourth order differential equation. The corruptive noise was zero and the calculation took about 50 seconds.

8. CONCLUSION

Judging from the principle and the testing experiments it can be said that the presented method of identification of linear systems with constant parameters offers, in comparison with other methods, relatively very accurate results and can even be used to identify the higher order systems. It is of great advantage that this method does not require a special type of input signal and allows the system identification to be carried out directly under opetation without impairing its normal function. It can be used for real-time estimation, even in the case of high-speed systems identification. The principle of the method allows its use even in identifying multivariable linear dynamic systems.

(Received October 3, 1973.)

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