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On Inverse of Linear Discrete-Time-Varying System

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The extension of the inverse problem to a linear nonstationary discrete-time system is given. It has importance for the synthesis of discrete-time or sampled-data time-varying systems.

INTRODUCTION

Various authors have recently found different mathematical forms of the inverse system in linear continuous-time stationary ([7] and [6]) as well as in nonstationary case ([8] and [3]). Kučera presented the useful results for linear discrete time stationary systems in [5].

The generalization of the inverse problem to linear discrete-time-varying systems is carried out here. Given the state equations of the original system the state space description of the inverse system and its coherence with the system relative order is derived. Finally the eigenvalues of the inverse system matrix are investigated.

FORMULATION OF THE INVERSE PROBLEM

Let us consider a single-input, single-output, linear discrete-time system G described by the state equations

$$\mathbf{x}(n + 1) = \mathbf{A}(n)\,\mathbf{x}(n) + \mathbf{b}(n)\,u(n)\,, \tag{1a}$$

$$y(n) = \mathbf{c}(n)\,\mathbf{x}(n) + d(n)\,u(n) \tag{1b}$$

where a system input and output are denoted by $u(n)$ and $y(n)$ respectively, $\mathbf{x}(n)$ is an $(s \times 1)$ state vector, $\mathbf{A}(n)$, $\mathbf{b}(n)$, $\mathbf{c}(n)$ and $d(n)$ are parameters of the proper dimensions.

The validity of the equations (1) for $n \in [n_0, n_f]$ is assumed if time variable n ranges over the integers.

Provided that the considered system is initially relaxed and we are interested in the output only, the state equations (1) with $\mathbf{x}(n_0) = \mathbf{0}$ may be viewed simply as the linear transformation of the input $u(n)$ into the output $y(n)$.

The inverse of G is now such a system denoted by G^{-1} which if subjected to the output $y(n)$ of G yields the output always identical with the input $u(n)$ of G. Obviously

$$(G^{-1})^{-1} = G\,. \tag{2}$$

STATE EQUATIONS OF THE INVERSE SYSTEM

Considering the equations (1) we define the operator $L_A^k\,\mathbf{c}(n)$ at first by the relations

$$L_A^k\,\mathbf{c}(n) = [L_A^{k-1}\,\mathbf{c}(n+1)]\,\mathbf{A}(n)\,; \quad k = 1, 2, \ldots, \tag{3}$$

with

$$L_A^0\,\mathbf{c}(n) = \mathbf{c}(n)$$

and the scalar $l_k^{-1}(n)$ as

$$l_k^{-1}(n) = [L_A^{k-1}\,\mathbf{c}(n+1)]\,\mathbf{b}(n)\,; \quad k = 1, 2, \ldots, \tag{4}$$

with

$$l_0^{-1}(n) = d(n)\,.$$

Let G be a system described by (1) with the property

$$l_{\varrho-j}^{-1}(n) = 0\,; \quad \varrho \geqq j \geqq 1 \tag{5}$$

and

$$l_\varrho^{-1}(n) \neq 0$$

for every $n \in [n_0, n_f]$.

Then the number ϱ $(0 \leqq \varrho \leqq s)$ is said to be the relative order of the system G (on the interval $[n_0, n_f]$). This relative order ϱ corresponds to the difference between the orders of the left and right side of the system s-order difference equation ([1] and [2]).

Theorem 1. *Given a linear, discrete-time system G described for $n \in [n_0, n_f]$ by the equations (1) with the property (5), the state equations of the inverse system G^{-1} have the form*

$$\mathbf{x}(n+1) = [\mathbf{A}(n) - l_\varrho(n)\,\mathbf{b}(n)\,L_A^\varrho\,\mathbf{c}(n)]\,\mathbf{x}(n) + l_\varrho(n)\,\mathbf{b}(n)\,y(n+\varrho)\,, \tag{6a}$$

$$u(n) = -l_\varrho(n)\,[L_A^\varrho\,\mathbf{c}(n)]\,\mathbf{x}(n) + l_\varrho(n)\,y(n+\varrho)\,. \tag{6b}$$

Using some results of Appendix A proof of this theorem is given in Appendix B.

Note. The inverse system corresponds to the physically unrealizable (anticipatory) system with the input $y(n)$ in the case of original system having the relative order $\varrho > 0$. To describe such a system by the state equations we must use the input $y(n + \varrho)$ translated by ϱ time units as it can be seen from the equations (6).

CONTROL CANONICAL FORM

The widely used type of state equations (1) has the form

$$\mathbf{x}(n+1) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a_0(n) & a_1(n) & a_2(n) & \dots & a_{s-1}(n) \end{bmatrix} \mathbf{x}(n) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(n), \tag{7}$$

$$y(n) = [c_0(n)\, c_1(n)\, c_2(n) \dots c_{s-1}(n)]\, \mathbf{x}(n) + d(n)\, u(n).$$

This so called control canonical form (CCF) ([4] and [5]) of the state space description can be obtained from general state equations by the transformation of the state coordinates. Scalar difference equation or weighting function of the system can always be directly brought to the CCF of state equations too.

The system relative order ϱ is immediately evident from the CCF of state equations.

Theorem 2. *Let G be a linear, discrete-time system of the relative order $\varrho > 0$ described by the equations (7) in CCF. Then*

$$d(n) = 0, \tag{8}$$

$$c_{s-i}(n) = 0, \quad i < \varrho$$

and

$$c_{s-\varrho}(n) \neq 0. \tag{9}$$

This theorem is proved in Appendix C.

THE MATRIX OF INVERSE SYSTEM

Theorem 3. *Let G be a linear, discrete-time system described by the equations (1) the order and relative order of which are s and ϱ respectively; $0 < \varrho \leqq s$. Then the inverse system matrix*

$$\mathbf{A}^*(n) = \mathbf{A}(n) - l_\varrho(n)\, \mathbf{b}(n)\, L_A^\varrho\, \mathbf{c}(n) \tag{10}$$

has always one zero eigenvalue $\lambda_1 = 0$.

Proof. Using the relations (3) and (4) $\mathbf{A}^*(n)$ may be written as

$$(11) \qquad \mathbf{A}^*(n) = [\mathbf{I} - \mathbf{b}(n) \{[L_A^{\varrho-1} \mathbf{c}(n+1)] \mathbf{b}(n)\}^{-1} L_A^{\varrho-1} \mathbf{c}(n+1)] \mathbf{A}(n)$$

where $\mathbf{I}$ denotes the unit matrix.

Choosing the eigenvector of $\mathbf{A}^*(n)$ as

$$(12) \qquad \mathbf{v}_1(n) = \mathbf{A}^{-1}(n) \mathbf{b}(n)$$

we get

$$(13) \quad \mathbf{A}^*(n) \mathbf{v}_1(n) = \mathbf{b}(n) - \mathbf{b}(n) \{[L_A^{\varrho-1} \mathbf{c}(n+1)] \mathbf{b}(n)\}^{-1} [L_A^{\varrho-1} \mathbf{c}(n+1)] \mathbf{b}(n) = \mathbf{0}$$

and really $\lambda_1 = 0$.

Possible other generalized eigenvectors $\mathbf{v}_i(n)$, $i > 1$, with the property

$$(14) \qquad \mathbf{A}^*(n) \mathbf{v}_i(n) = \mathbf{v}_{i-1}(n)$$

do not exist provided that the equations (1) have a general form.

The other eigenvalues of $\mathbf{A}^*(n)$ depend on the concrete type of the equations (1); they can be easily determined if the system is described in the CCF.

Theorem 4. *Let G be a linear, discrete-time system* ($0 < \varrho \leqq s$) *described in* CCF *by the equations* (7).

Then

a) *the inverse system matrix* $\mathbf{A}^*(n)$ *has always* ϱ *zero eigenvalues* $\lambda_i = 0$; $i = 1, 2, \ldots, \varrho$;

b) *the remaining* $s - \varrho$ *eigenvalues* $\lambda_j(n)$, $j = \varrho + 1, \ldots, s$, *of* $\mathbf{A}^*(n)$ *satisfy the equation*

$$(15) \qquad c_{s-\varrho}(n+\varrho) \lambda^{s-\varrho}(n) + c_{s-\varrho-1}(n+\varrho) \lambda^{s-\varrho-1}(n) + \ldots$$
$$\ldots + c_1(n+\varrho) \lambda(n) + c_0(n+\varrho) = 0\,.$$

Proof. If a system G is described by the equations (7) we can easily derive that

$$(16) \qquad L_A^{\varrho-1} \mathbf{c}(n+1) = [\underbrace{0 \ldots 0}_{\varrho-1} \;\; c_0(n+\varrho) \;\; c_1(n+\varrho) \ldots c_{s-\varrho}(n+\varrho)]$$

and

$$(17) \qquad l_\varrho^{-1}(n) = c_{s-\varrho}(n+\varrho)\,.$$

Substituting the expressions (16) and (17) into the relation (11) we obtain

(18)

$$A^*(n) = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline 0 & \underbrace{0 \ldots 0}_{\varrho}; \quad -\dfrac{c_0(n+\varrho)}{c_{s-\varrho}(n+\varrho)}; \quad -\dfrac{c_1(n+\varrho)}{c_{s-\varrho}(n+\varrho)}; \quad \ldots; \quad -\dfrac{c_{s-\varrho-1}(n+\varrho)}{c_{s-\varrho}(n+\varrho)} \end{array}\right].$$

The characteristic equation of the matrix $A^*(n)$ has the form

(19) $\det[A^*(n) - \lambda(n)\,I] =$

$$= \left|\begin{array}{cccccc} -\lambda(n) & 1 & 0 & \ldots & 0 & 0 \\ 0 & -\lambda(n) & 1 & \ldots & 0 & 0 \\ 0 & 0 & -\lambda(n) & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -\lambda(n) & 1 \\ \hline \underbrace{0 \ldots 0}_{\varrho}; & -\dfrac{c_0(n+\varrho)}{c_{s-\varrho}(n+\varrho)}; & \ldots; & & -\dfrac{c_{s-\varrho-2}(n+\varrho)}{c_{s-\varrho}(n+\varrho)}; & -\dfrac{c_{s-\varrho-1}(n+\varrho)}{c_{s-\varrho}(n+\varrho)} - \lambda(n) \end{array}\right| = 0.$$

If the determinant in (19) is expanded in the terms of its last row the equation (19) can be rewritten after the small rearrangement as

$$(20) \qquad c_{s-\varrho}(n+\varrho)\,\lambda^s(n) + c_{s-\varrho-1}(n+\varrho)\,\lambda^{s-1}(n) + \ldots + c_0(n+\varrho)\,\lambda^{\varrho}(n) = 0\,.$$

We can see that the root $\lambda(n) = 0$ of the multiplicity ϱ satisfies the equation (20) as it is given by the statement a). The other eigenvalues $\lambda_j(n)$; $j = \varrho + 1, \ldots, s$, are really the solutions of the equation (15) and the statement b) is proved.

Lemma. *Let G be a linear, discrete-time system described in the* CCF *by the equations* (7) *the order and the relative order of which are s and $\varrho = 0$, respectively.*

Then the inverse system matrix $A^*(n)$ has s eigenvalues $\lambda_j(n)$; $j = 1, \ldots, s$, which satisfy the equation

$$(21) \qquad \lambda^s(n) + \left[\frac{c_{s-1}(n)}{d(n)} - a_{s-1}(n)\right]\lambda^{s-1}(n) + \ldots + \left[\frac{c_1(n)}{d(n)} - a_1(n)\right]\lambda(n) + \\ + \frac{c_0(n)}{d(n)} - a_0(n) = 0\,.$$

Proof. Provided that a system G of zero relative order is given in CCF by the equations (7) the inverse system matrix $A^*(n)$ has the form

$$(22) \qquad A^*(n) = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline a_0(n) - \dfrac{c_0(n)}{d(n)}; & a_1(n) - \dfrac{c_1(n)}{d(n)}; \quad \ldots; \quad a_{s-1}(n) - \dfrac{c_{s-1}(n)}{d(n)} \end{array}\right].$$

Writing the characteristic equation of $\mathbf{A}^*(n)$ in the polynomial form we get

(23)
$$\left[a_{s-1}(n) - \frac{c_{s-1}(n)}{d(n)} - \lambda\right](-\lambda)^{s-1} - \left[a_{s-2}(n) - \frac{c_{s-2}(n)}{d(n)}\right](-\lambda)^{s-2} + \ldots +$$
$$+ (-1)^{s+2}\left[a_1(n) - \frac{c_1(n)}{d(n)}\right](-\lambda) + (-1)^{s+1}\left[a_0(n) - \frac{c_0(n)}{d(n)}\right] = 0\,.$$

The equation (23) can be expressed after the small rearrangement in the form (21) and the validity of the lemma is proved.

EXAMPLE

The system G is described for $n \in [0, \infty)$ in CCF by the equations

$$\mathbf{x}(n+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -ne^{-n} & e^{-(n+2)} \end{bmatrix} \mathbf{x}(n) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(n)\,,$$
$$y(n) = [e^{-n} \; 2 \; 0]\, \mathbf{x}(n)\,.$$

We shall determine the inverse system G^{-1} and the eigenvalues of the inverse system matrix.

1. It follows directly from the theorem 2 that $\varrho = 2$.

Using the theorem 4 the eigenvalues of inverse system matrix can be obtained at first. We have

$$\lambda_1 = \lambda_2 = 0$$

and the equation (15) as

$$2\,\lambda(n) + e^{-(n+2)} = 0\,.$$

Hence

$$\lambda_3(n) = -0{\cdot}5e^{-(n+2)}\,.$$

2. According to (4)

$$l_0^{-1}(n) = l_1^{-1}(n) = 0$$

and

$$l_2^{-1}(n) = 2\,.$$

Then using the relations (3) and (6) the state equations of the inverse system have the form

$$\mathbf{x}(n+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -0{\cdot}5e^{-(n+2)} \end{bmatrix} \mathbf{x}(n) + \begin{bmatrix} 0 \\ 0 \\ 0{\cdot}5 \end{bmatrix} y(n+2)\,,$$
$$u(n) = [1 \;\; ne^{-n} \;\; -1{\cdot}5e^{-(n+2)}]\, \mathbf{x}(n) + 0{\cdot}5\, y(n+2)\,.$$

APPENDIX A

THE RELATION BETWEEN STATE EQUATIONS AND INPUT-OUTPUT DIFFERENCE EQUATION

Let us consider a system described by the state equations (1) and seek the corresponding input-output scalar difference equation.

Gradually applying the translation operator

$$E^i\, y(n) = y(n+i)\,; \quad i = 0, 1, \ldots, s\,, \tag{A·1}$$

to the equation (1b) and using the designation (3) we obtain the following set of equations:

$$\begin{aligned} y(n+i) = {} & [L_A^i\, \mathbf{c}(n)]\, \mathbf{x}(n) + [L_A^{i-1}\, \mathbf{c}(n+1)]\, \mathbf{b}(n)\, u(n) + \ldots \\ & + [L_A^0\, \mathbf{c}(n+i)]\, \mathbf{b}(n+i-1)\, u(n+i-1) + d(n+i)\, u(n+i)\,; \\ & \qquad i = 0, 1, \ldots, s-1\,, \end{aligned} \tag{A·2}$$

and

$$\begin{aligned} y(n+s) = {} & [L_A^s\, \mathbf{c}(n)]\, \mathbf{x}(n) + [L_A^{s-1}\, \mathbf{c}(n+1)]\, \mathbf{b}(n)\, u(n) + \ldots \\ & \ldots + [L_A^0\, \mathbf{c}(n+s)]\, \mathbf{b}(n+s-1)\, u(n+s-1) + d(n+s)\, u(n+s)\,. \end{aligned} \tag{A·3}$$

The equations (A·2) can be written in the vector-matrix form as

$$\mathbf{y}(n) = \mathbf{Q}(n)\, \mathbf{x}(n) + \mathbf{R}(n)\, \mathbf{u}(n) \tag{A·4}$$

where the $(s \times 1)$ vectors

$$\mathbf{y}(n) = \begin{bmatrix} y(n) \\ y(n+1) \\ \vdots \\ y(n+s-1) \end{bmatrix}, \tag{A·5}$$

$$\mathbf{u}(n) = \begin{bmatrix} u(n) \\ u(n+1) \\ \vdots \\ u(n+s-1) \end{bmatrix} \tag{A·6}$$

and the $(s \times s)$ matrix

$$\mathbf{Q}(n) = \begin{bmatrix} L_A^0\, \mathbf{c}(n) \\ L_A^1\, \mathbf{c}(n) \\ \vdots \\ L_A^{s-1} \mathbf{c}(n) \end{bmatrix}. \tag{A·7}$$

The $(s \times s)$ matrix $\mathbf{R}(n)$ of the triangular structure

(A·8)

$$\mathbf{R}(n) = \begin{bmatrix} r_0(n) & 0 & 0 & \dots 0 & 0 \\ r_1(n) & r_0(n+1) & 0 & \dots 0 & 0 \\ r_2(n) & r_1(n+1) & r_0(n+2) & \dots 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{s-1}(n) & r_{s-2}(n+1) & r_{s-3}(n+2) & \dots r_1(n+s-2) & r_0(n+s-1) \end{bmatrix}$$

has the elements determined by the relations

(A·9) $$r_0(n) = l_0^{-1}(n) = d(n)\,,$$
$$r_k(n) = l_k^{-1}(n) = [L_A^{k-1}\,\mathbf{c}(n+1)]\,\mathbf{b}(n)\,; \quad k = 1, 2, \dots, s\,.$$

Solving the equation (A·4) for $\mathbf{x}(n)$ we get

(A·10) $$\mathbf{x}(n) = \mathbf{Q}^{-1}(n)\,[\mathbf{y}(n) - \mathbf{R}(n)\,\mathbf{u}(n)]$$

provided $\mathbf{Q}(n)$ is nonsingular, i.e. the system is completely observable on the interval $[n_0, n_f]$.

Substituting now the solution (A·10) into the equation (A·3) we can write

(A·11) $$y(n+s) = [L_A^s\,\mathbf{c}(n)]\,\mathbf{Q}^{-1}(n)\,\mathbf{y}(n) +$$
$$+ \{\mathbf{r}(n) - [L_A^s\,\mathbf{c}(n)]\,\mathbf{Q}^{-1}(n)\,\mathbf{R}(n)\}\,\mathbf{u}(n) + d(n+s)\,u(n+s)$$

where the elements of the $(1 \times s)$ vector

(A·12) $$\mathbf{r}(n) = [r_s(n) \;\; r_{s-1}(n+1) \dots r_1(n+s-1)]$$

stand in (A·9).

If we denote

(A·13) $$\boldsymbol{\alpha}(n) = [\alpha_0(n) \;\; \alpha_1(n) \dots \alpha_{s-1}(n)] = -[L_A^s\,\mathbf{c}(n)]\,\mathbf{Q}^{-1}(n)\,,$$

(A·14) $$\boldsymbol{\beta}(n) = [\beta_0(n) \;\; \beta_1(n) \dots \beta_{s-1}(n)] = \mathbf{r}(n) - [L_A^s\,\mathbf{c}(n)]\,\mathbf{Q}^{-1}(n)\,\mathbf{R}(n)$$

and

(A·15) $$\beta_s(n) = r_0(n+s) = d(n+s)$$

the equation (A·11) corresponds to the input-output difference equation

(A·16) $$y(n+s) + \sum_{i=0}^{s-1} \alpha_i(n)\,y(n+i) = \sum_{j=0}^{s} \beta_j(n)\,u(n+j)\,.$$

The coefficients $\alpha_i(n)$ and $\beta_j(n)$ result from (A·13)–(A·15); obviously the relation

(A·17) $$\boldsymbol{\beta}(n) = \mathbf{r}(n) + \boldsymbol{\alpha}(n)\,\mathbf{R}(n)$$

is also valid.

APPENDIX B

PROOF OF THEOREM 1

Let us substitute the relations (A·8) and (A·12)–(A·14) into the equation (A·17) and compare the elements of row vectors on both sides. Hence the following set of equations is obtained if the relation (A·15) is written at first:

$$\text{(B·1)}\quad \begin{aligned} \beta_s(n) &= r_0(n+s)\,, \\ \beta_{s-1}(n) &= r_1(n+s-1) + \alpha_{s-1}(n)\, r_0(n+s-1)\,, \\ &\vdots \\ \beta_1(n) &= r_{s-1}(n+1) + \alpha_{s-1}(n)\, r_{s-2}(n+1) + \dots \\ &\quad \dots + \alpha_2(n)\, r_1(n+1) + \alpha_1(n)\, r_0(n+1)\,, \\ \beta_0(n) &= r_s(n) + \alpha_{s-1}(n)\, r_{s-1}(n) + \dots + \alpha_1(n)\, r_1(n) + \alpha_0(n)\, r_0(n)\,. \end{aligned}$$

If now

$$\text{(B·2)}\quad \begin{aligned} &\beta_{s-i}(n) = 0 \quad \text{for} \quad i < \varrho\,, \\ &\beta_{s-i}(n) \neq 0 \quad \text{for} \quad i \geqq \varrho\,; \quad i = 0, 1, \dots, s\,; \quad n \in [n_0, n_f]\,, \end{aligned}$$

then ϱ is the relative order of the system (1).

The relations (5) follow directly from the conditions (B·2) and the equations (B·1) if the designations (A·9) are applied.

Considering (4) and (5) in the equation (A·2; $i = \varrho$) we have

$$\text{(B·3)}\qquad y(n+\varrho) = [L_A^{\varrho}\, \mathbf{c}(n)]\, \mathbf{x}(n) + [L_A^{\varrho-1}\, \mathbf{c}(n+1)]\, \mathbf{b}(n)\, u(n)\,.$$

Solving the equation (B·3) for $u(n)$ we obtain the output equation (6b). Then the state equation (6a) results from substitution of this $u(n)$ into the equation (1a).

In the case $\varrho = 0$ the condition $d(n) \neq 0$ is valid and the state equations of inverse system might be directly obtained from the original equations (1).

APPENDIX C

PROOF OF THEOREM 2

General validity of the relation (8) has been proved above.

Assuming the state equations of the system to be in the control canonical form (7) we can determine

$$\text{(C·1)}\quad \begin{aligned} r_1(n) &= c_{s-1}(n+1)\,, \\ r_2(n) &= c_{s-2}(n+2) + a_{s-1}(n+1)\, c_{s-1}(n+2)\,, \\ r_3(n) &= c_{s-3}(n+3) + a_{s-1}(n+1)\, c_{s-2}(n+3) + \\ &\quad + [a_{s-2}(n+2) + a_{s-1}(n+2)\, a_{s-1}(n+1)]\, c_{s-1}(n+3)\,, \\ &\vdots \\ &\text{etc.} \end{aligned}$$

According to (A·9) and (4)–(5)

(C·2) $$r_i(n) = 0\,, \quad i < \varrho$$

and

$$r_\varrho(n) \neq 0\,.$$

The relations (9) follow immediately from the equations (C·1).

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