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Types of Preference

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The paper presents a semantical analysis of various types of preference on the base of probabilistic and value evaluations.

Motto:

... pour juger de ce que l'on doit faire pour obtenir un bien ou pour éviter un mal, il ne faut pas seulement considérer le bien et le mal en soi, mais la probabilité qu'il arrive ou n'arrive pas; et regarder géometriquement la proportion que toutes ces choses ont ensemble ...

Logique de Port-Royal

1. THE ROLE OF PREFERENCE AND PREFERENCE ORDERING

Questions concerning the formal status of preference relations have been raised in several scientific domains. It is a known fact e.g. that applications of the means and models of the statistical decision theory or the game theory are based on the supposition that there exist possible subjects of decision or game which are able to evaluate the considered alternatives from a point of view of relevant preference ordering. A preferential scheme also forms the base for any rational decision (in the Bayesian sense). This is also the reason why we find various systems of axioms or postulates in books or papers explaning the mathematical models of decision which must be satisfied by a rationally constructed preference relationship.* The construc-

* A suitable example of such a minimal system of axioms is mentioned by R. D. Luce and H. Raiffa [7]. The concept of preference based on the conbination of probability measure and desirability measure was introduced by R. C. Jeffrey [5]. A preferential ordering based on the distance measure was constructed by J. G. Kemeny and J. L. Snell [6]. It would be possible, of course, to mention other examples.

tion of preferential schemes is important in other domains: A good example can be found in modern prognostics. Prognostics today requires more than just pointing out the probability distribution of possible future states; it also requires the present and especially the future desirability of these states. This means the connection of these possible or expected future states with a certain system of values, criteria and preferences. Sometimes it is even required that prognostics should put emphasis on this sphere of values, decision criteria and preferences. Another domain in which the problems of preferences or preference schemes must be taken into consideration is data processing, data retrieval for various purposes or informatics.

We assume that a unique preference scheme does not and cannot exist. In works and papers concerning the statistical decision theory we usually find a conception describing preference relation as a binary relation between objects of any nature that can be *compared* on the basis of a certain evaluation. It is suitable to distinguish preference (which could be considered as a justification of decision) and the proper decision or choice of one alternative from the set of possible alternatives. Therefore it is sometimes stressed that preference indicates only a possible choice.* This assertion cannot be interpreted to mean that any decision or choice is a realized proference. It should be taken into consideration that decision can be accidental, irrational, motivated by other (and for observer unknown) preferences etc.

It is not possible to consider the relation between preference and preference ordering on the one hand and the decision or any other choice on the other hand as equivalence relation. The same reservation concerns the relation between preference ordering and a certain form of evaluation which can render a justification of the given preference. Nevertheless such an approach, i.e. the reconstruction of preference ordering on the basis of evaluation, is quite usual, even though it leads to some difficulties as we shall attempt to demonstrate in further explanations. The nature of these difficulties is found in the presupposition of substitution possibility – salva veritate – of the preference relation and of the evaluation which is able to render a justification of the given preference. This presupposition can be expressed in the simplest form as follows:

$$xPy[\equiv]\, w(x) > w(y)\,, \tag{1}$$

where P is the preference relation, x and y are preferable entities, w is the evaluation function assigning some (numerical) values to these entities and $[\equiv]$ is a relation with the properties of equivalence relation.

This approach is based on a simple application of the function w without taking into account the probability measure assignable to the entities x and y. This means e.g. that we do not take into consideration the likelihood of x and y or the chance that these entities might occur or any other probability measures. It is known that

* E. g. H. S. Houthakker [3].

exactly this was demanded by the famous Cartesian logic (Logique de Port-Royal) which was conceived as „l'art de penser". If p denotes such a probability measure which is supposed to fulfil the usual axioms of the probability theory, (1) can be transformed into the following notation:

$$xPy\,[\equiv]\,w(x)\,p(x) > w(y)\,p(y)\,. \tag{2}$$

In the case that x and y exhaust all possible alternatives and represent the complementary possibilities and that $p(x)$ has the value r, then (2) has the following shape:

$$xPy\,[\equiv]\,w(x)\,r > w(y)\,(1-r)\,. \tag{3}$$

It is therefore expedient to distinguish three different spheres among them the relations with properties of equivalence relation are interpolated:

(a) The sphere of proper decisions or choices between various alternatives. It is useful to suppose that this sphere is observable or that it has the nature of a behavior-type which can be objectively registered or reliably reconstructed on the basis of ascertained effects of the chosen alternatives.

(b) The sphere of preferences which can be reconstructed as a relational system between various alternative entities. This system can indicate, as was stressed earlier, the possible decision or possible choice if it is, of course, desirable that the decision or the choice must be justified by preferences.

(c) The sphere of evaluations which is conceived as simple or probability weighted evaluations or evaluations relativized to certain aims, criteria or postulates. It is desirable that these evaluations be expressed in numerical form or at least in a form anabling the complete ordering of the entities which are subjected to an evaluation.

The interpolation of equivalence relation between those, in principle different spheres, is connected with efforts to utilize the supposed interdependencies whose simplest notations are (1), (2) and (3) for reconstruction and justification of different steps in the above mentioned spheres. The most usual procedure of this justification can be expressed as follows: (c) $\Rightarrow$ (b) $\Rightarrow$ (a). It is, of course, possible to object that this justification is "built on the head" and that other reconstructions are also justified, e.g. in the form (a) $\Rightarrow$ (b) $\Rightarrow$ (c). In the following text we shall try to show that such a reconstruction has its limits (especially the presupposition of the interpolation of the equivalence relations which is able to demonstrate a certain ordering in one sphere by means of a corresponding ordering in the other sphere). In some cases these limits will be manifested by the interpretation possibility of the corresponding matrixes in only one direction and not inversely as it would be desirable in the case of equivalence relations and symmetrical dependencies.*

* This means that the operation which is interpolated between various spheres has the character of a connective but not an adjunctive operation in the Reichenbach sense [8]. A logical operation is, according Reichenbach, connective if the corresponding matrix can be interpreted only in a certain direction, and adjunctive if the sequence of interpretation is not decisive.

2. THE COMPARABILITY PRINCIPLE AS A PRESUPPOSITION OF RECONSTRUCTION OF PREFERENCE SYSTEM

The sentences (1), (2) and (3) or any other analogical sentences indirectly express the postulate which is usually characterized as the comparability principle.* If we use as a base the sentence (1) then the comparability principle means that one of the following possibilities is realizable:

$$w(x) > w(y)\,,$$

$$w(x) < w(y)\,,$$

$$w(x) = w(y)\,.$$

It should be stressed that these possibilities express the comparability principle *indirectly*, i.e. express the comparability principle in the sphere (b) by means of (c). Moreover it is supposed that the corresponding evaluation is conceived in numerical form, i.e. is mapped in the set of real numbers. But this presupposition involves some difficulties connected with the discrimination limits, with the limited application of the density principle etc.

Because of this it is more useful to express the comparability principle only in sphere (b). For the present we shall put aside the problem of the nature of arguments in the relation xPy. (In the following explanation we shall try to show that for the logical interpretation of the preference relation the interpretation of these arguments as denotations of statements or proposition-like-entities or other analogical entities is more suitable.**) From the intuitive point of view we can take as comparable those entities from the class of all considered entities $\mathscr{X}$ that have at least one common property. It is moreover desirable that this property be assignable in various degrees which can be discriminated. If the predicate C denotes such a property, then the subclass $(\hat{x})\, Cx$ $((\hat{x})\, Cx \subset \mathscr{X})$ is comparable. C can be, e.g., interpreted so that all elements of the class $(\hat{x})\, Cx$ are connected with various expenceses, gaines or losses, render certain advantages, are products of activity connected with certain effects ete.

If C denotes a property which can be designed to all the elements of a given class in various degrees then it is possible (with respect of any arbitrary pair of elements) to decide if this property can be assigned in a higher or lower or equal degree. If the preference relation is relativized to C we can construct a very elementary form of a preference system by means of the pair $\langle P_C, I_C \rangle$ where P_C is the preference relation with respect to C and I_C the indifference relation (or value coincidence) with respect to C.

* See e. g. S. Halldén [1].

** One of the pioneers of preference logic G. H. von Wright speaks of proposition-like-entities [12] [13].

The properties of P_C and I_C are expressed by the following requirements:

$$(\mathbf{R}\ 1)\quad (\forall x)(\forall y)(\forall z)\,[(xP_Cy\,.\,yP_Cz) \rightarrow xP_Cz]\,,$$

$$(\mathbf{R}\ 2)\quad (\forall x)(\forall y)\,\{xI_Cy \rightarrow [\sim(xP_Cy)\ \vee\ \sim(yP_Cx)]\}\,,$$

$$(\mathbf{R}\ 3)\quad (\forall x)(\forall y)\,(xP_Cy\ \vee\ xI_Cy\ \vee\ yP_Cx)\,,$$

$$(\mathbf{R}\ 4)\quad (\forall x)\,(xI_Cx)\,,$$

$$(\mathbf{R}\ 5)\quad (\forall x)(\forall y)\,(xI_Cy \rightarrow yI_Cx)\,,$$

$$(\mathbf{R}\ 6)\quad (\forall x)(\forall y)(\forall z)\,[(xI_Cy\,.\,yI_Cz) \rightarrow xI_Cz]\,.$$

The requirement (**R** 3) can be interpreted as an expression of the comparability principle. It is obvious that P_C is transitive (**R** 1), I-reflexive (**R** 2) and that I_C is reflexive (**R** 4), symmetrical (**R** 5) and transitive (**R** 6). On the basis of the mentioned requirements (or axioms) we can formulate various theorems, e.g.

$$(\forall x)(\forall y)(\forall z)\,[(xP_Cy\,.\,yI_Cz) \rightarrow xP_Cz]\,,$$

$$(\forall x)(\forall y)\,(xP_Cy \rightarrow \sim yP_Cx)\,,\quad \text{etc.}$$

The pair $\langle P_C, I_C\rangle$ realizes a quasi-serial order in the class $(\hat{x})\,Cx \subseteq \mathscr{X}$ and the preference relation has all the properties of comparative predicates.*

The procedure used for the justification of the comparability principle was based on the presupposition that a preference system is realizable in the class $\mathscr{X}$ or in a subclass of this class provided that a common property C (discernible in degree) can be fixed so that

$$(\hat{x})\,Cx \subseteq \mathscr{X}$$

and

$$(\hat{x})\,Cx \neq 0\,.$$

The comparability principle can be connected with a somewhat different meaning. If a common property C is unknown but the entities forming the given class are comparable, then the comparability can indicate that there is one or more common properties assignable to all entities of the given class. A similar consideration formed the base for the known Marx's procedure leading to the introduction of the concept "value" (in the economic sense). It is also possible to introduce a binary relation K defined as follows:

$$xKy \equiv xPy\ \vee\ yPy\ \vee\ xIy\,.$$

It can be proved that this relation is symmetrical, reflexive and transitive.

* The analysis of comparative predicates and the reconstruction of a quasi-serial order is explained in [11].

From the theoretical point of view we could suppose further interpretations of the comparability principle. We shall start with the following question: If we are able (in the class $\mathscr{X}$ with a common property) to prefer x before y, can we always expect to be able to find such a z that z could be interpolated between x and y in the given preference system? In other words, is it justified to assert that

$$(\forall x)\,(\forall y)\,(\exists z)\,(xPy\,.\,xPz\,.\,zPy)\,?$$

It seems that this is not possible in all circumstances: First, it should be stressed that the class of entities could be finite so that it is impossible to find a z with the mentioned properties. Furthermore it holds that any proference discrimination is connected with certain limits of discriminability. This means that the preference distance between x and y can be so insignificant that no other entity could be interpolated between x and y in such a manner that this entity could be discriminated from x and y in the preference order. What is merely guaranted is the following: If there is a z such that

$$z \in (\hat{x})\,Cx\,,$$

it holds that

(4) $$xP_Cy \rightarrow (xP_Cz \vee zP_Cy)\,.$$

It is obvious that this expression which could be called a *comparability principle in the narrow sense* does not exclude

$$xI_Cz \vee yI_Cz\,.$$

In this connection a possibility is offered to modify the system of requirements (**R** 1)–(**R** 6) so that these requirements would involve the comparability principle in the narrow sense. But the comparability principle in the narrow sense is valid only if xP_Cy and if there exist a z which is an element of $(\hat{x})\,Cx$. Because of this it is not possible to accept

(5) $$(\forall x)\,(\forall y)\,(\exists z)\,[xP_Cy \rightarrow (xP_Cz \vee zP_Cy)]\,.$$

Also because of this it is not possible to substitute (**R** 5) by the expression (5) and this modification is not passable.

3. PREFERENCE OF THINGS AND PREFERENCE OF STATES OF AFFAIRS

Upto now we have considered the preference concept as a two-placed predicate whose arguments are not specified entities. This conception can be characterized as preference of things and obviously has a very limited application. Moreover this conception does not correspond to the intuitive interpretation of preference which is evident from the following examples: If we ask whether a peaceful coexistence is better than war, then the preference concerns *states of affairs*. Moreover it is possible to realize the preference relation between a state of affairs and a negation of this state asking, e.g. if it is more advantageous in the present period that it is raining or that it is not raining. It seems to be sure that the majority uses of the preference relationship in common sense language has the character of preference of states of affairs more that of preference of things. Assuming that we prefer a family house to a tenament house we understand thereby that we prefer living in a family house to living a tenament house.

The conception of the preference relationship as preference of states of affairs, of course, changes the logical status of preference. As arguments of preference relation are taken entities which can be characterized as states of affairs or proposition-like-entities [13]. This conception of preference relation is supported by most authors studying the logic of preference. N. Rescher speaks e.g. on propositional preference ordering [10, p. 292]. The expression pPq is then understood to mean that the states of affairs denoted by p have preference before the states of affairs denoted by q. It is obvious that the meaning of the sign P is somewhat different from the meaning of P used as a comparative predicate. It should be stressed that the expression pPq does not state the preference relation between p and q taken as statements or sentences, i.e. the preference between the linguistic forms, but it states the preference between states of affairs or possible or alternative facts denoted by p and q. Since any state of affairs can be taken as realized or not realized, we must suppose positive and negative instances of states of affairs. (For the negative instances we shall use the sign $\sim$.) Then the expression

$$pP \sim p$$

means that the state of affairs denoted by p is preferable before the negation of the state denoted by $\sim p$.

If we consider as arguments of the preference relation states of affairs or otherwise characterized proposition-like-entities, we have to admit as arguments denotations of molecular expressions, e.g.

$$(p \,.\, r)\, Pq\,,$$

$$(p \,.\, \sim q)\, P(\sim p \,.\, q)\,,$$

$$pP(q \vee r)\,,$$

etc.

The fact that arguments of a preference relation are formed not only by atomic expressions but also by negations and molecular expressions leads to the new claims on the concept of preference. Difficulties raised in this connection concern especially the possibilities the sphere of evaluations (i.e. the sphere (c)). If it holds e.g. that

$$pPq\,,$$

 then it is obvious that this implies

$$\sim(qPp).$$

If we put $w(p) = \mathbf{A}$ and $w(q) = \mathbf{B}$, then

$$pPq\ [\equiv]\ \mathbf{A} > \mathbf{B},$$

$$pIq\ [\equiv]\ \mathbf{A} = \mathbf{B}.$$

In various systems of preference logic is mostly accepted as valid,* i.e. as axioms or theorems of the preference system, the following sentences:

(6) $$pPq \rightarrow \sim qP \sim p,$$

(7) $$pPq \rightarrow (p\,.\sim q)\,P(\sim p\,.\,q),$$

(8) $$\sim qP \sim p \rightarrow (p\,.\sim q)\,P(\sim p\,.\,q).$$

These sentences are valid e.g. in the system of preference logic elaborated by S. Halldén [1] whose preference system **A** is introduced by the following axioms:

(**A** 1) $pPq \rightarrow \sim(qPp)$,

(**A** 2) $pPq\,.\,qPr \rightarrow pPr$,

(**A** 3) pIp,

(**A** 4) $pIq \rightarrow qIp$,

(**A** 5) $pIq\,.\,qIr \rightarrow pIr$,

(**A** 6) $pPq \rightarrow (p\,.\sim q)\,P(\sim p\,.\,q)$,

(**A** 7) $pIq \rightarrow (p\,.\sim q)\,I(\sim p\,.\,q)$.

It is obvious that (6), (7) and (8) are axioms or theorems in this system of preference logic.

If we write down now the values of the evaluation function $w(s_i)$ where s_i is the corresponding expression in the following scheme:

	$w(s_i)$
p	**A**
q	**B**
$\sim p$	**C**
$\sim q$	**D**

* Only Chisholm and Sosa [4] reject sentences (6), (7) and (8) as inacceptable, but the reason of this rejection is incoherent with the evaluation sphere, but is justified by their hedonistic conception of preferability.

we should always obtain as valable that

$$\text{if} \quad \mathbf{A} > \mathbf{B}, \quad \text{then} \quad \mathbf{D} > \mathbf{C}.$$

But it is quite well realizable e.g. the following evaluation:

	$w(s_i)$
p	10
q	3
$\sim p$	−2
$\sim q$	−4

which evidently does not satisfy the above mentioned requirement. It also means that the suppesed equivalence relation interpolated between the sphere (b) and (c) which should guarantee the mutual correspondance of decisions in both spheres is violated.

If we take into consideration as arguments of preference relation molecular expressions, the correspondance of sphere (b) and sphere (c) is more sophisticated. We can confront the following evaluation schemes:

(I)

	$w(s_i)$
p	$\mathbf{A}$
q	$\mathbf{B}$
$\sim p$	$\mathbf{C}$
$\sim q$	$\mathbf{D}$

(II)

	$w(s_i)$
$p \,.\, q$	$\mathbf{a}$
$p \,.\, \sim q$	$\mathbf{b}$
$\sim p \,.\, q$	$\mathbf{c}$
$\sim p \,.\, \sim q$	$\mathbf{d}$

If we accept sentences (7) and (8) as axioms or theorems of a preference system it would be desirable that the following be valid:

$$\text{if} \quad \mathbf{A} > \mathbf{B}, \quad \text{then} \quad \mathbf{b} > \mathbf{c}$$

and

$$\text{if} \quad \mathbf{D} > \mathbf{B}, \quad \text{then} \quad \mathbf{b} > \mathbf{c}.$$

It is, of course, obvious that there are no a priori guarantees for this.

Another problem raised herewith is the question whether the values of the evaluation function in scheme (I) can be determinated independently from those in scheme (II) and vice versa. Which of the mentioned schemes can be taken as the initial scheme?

An answer to these questons was presented by N. Rescher [9] [10]. It can be shown that this solution involves the calculation of values in scheme (I) on the basis of scheme (II), in other words, table (II) must be considered as initial. In the opposite case the problem is not solvable. Therefore constraints are given for the acceptance of sentences (6), (7) and (8) or of other analogical expressions. (This concerns e.g. the expression

$$pIq \rightarrow (p \,.\, {\sim}q)\, I({\sim}p \,.\, q)$$

etc.) Rescher's approach leads to the conclusion that only one concept of preference is not sufficient. Therefore Rescher presents two different concepts of preference.

The justification of Rescher's conception is based on semantics: If p, q, r, ..., etc. are atomic propositions, then we must suppose that for any atomic or molecular proposition there exists a set of states of affairs or of possible worlds (in the Leibnizian sense) or a set of denotations of admissible state descriptions (in the sense of Carnap). Having a triplet of propositions p, q, r and taking into account only those propositions and their negations, then the proposition p would correspond to possible state descriptions which can be expressed as the disjunction

$$p \,.\, q \,.\, r \vee p \,.\, q \,.\, {\sim}r \vee p \,.\, {\sim}q \,.\, r \vee p \,.\, {\sim}q \,.\, {\sim}r \,.$$

On this semantical base Rescher introduces two different concepts of preference. The preference denoted by $P^{\#}$ is based on the average (arithmetical mean) of the values of those possible worlds involved by the used propositions. The expression $pP^{\#}q$ is true if the average value of the possible worlds involved in p is higher than the average value of the possible worlds involved in q. (It should be stressed that this approach does not solve the evaluation of a contradiction.) For the sake of simplicity we shall suppose only four possible worlds with evaluations:

possible worlds:		their evaluations:	
	$p \,.\, q$		a
	$p \,.\, {\sim}q$		b
	${\sim}p \,.\, q$		c
	${\sim}p \,.\, {\sim}q$		d

The evaluation of p based on this approach and denoted by $w^{\#}(p)$ is formed by the average of evaluations of two possible worlds denoted by $p \,.\, q$ and $p \,.\, {\sim}q$, i.e.

$$w^{\#}(p) = \frac{a + b}{2}$$

and similarly

$$w^{\#}(q) = \frac{a + c}{2},$$

$$w^{\#}({\sim}p) = \frac{c + d}{2},$$

$$w^{\#}({\sim}q) = \frac{b + d}{2}.$$

Rescher's conception supposes that all possible worlds are equally probable. But it is useful – as was the case with the probabilization of Carnapian state descriptions – to suppose that various possible worlds are not equally probable. Therefore we shall introduce a weighted evaluation which corresponds to the above mentioned requirement of Cartesian logic. This is shown in the following scheme (where $r_1 + r_2 + r_3 + r_4 = 1$):

possible worlds or states of affairs	probability of the state $p(s_i)$	genuine evaluation $w(s_i)$	weighted evaluation $p(s_i)\, w(s_i)$
$p \,.\, q$	r_1	e_1	$r_1 \,.\, e_1 = a$
$p \,.\, {\sim}q$	r_2	e_2	$r_2 \,.\, e_2 = b$
${\sim}p \,.\, q$	r_3	e_3	$r_3 \,.\, e_3 = c$
${\sim}p \,.\, {\sim}q$	r_4	e_4	$r_4 \,.\, e_4 = d$

If it holds that

$$w^{\#}(p) = A\,,$$
$$w^{\#}(q) = B\,,$$
$$w^{\#}({\sim}p) = C\,,$$
$$w^{\#}({\sim}q) = D\,,$$

it follows that

$$A = a + b\,,$$
$$B = a + c\,,$$
$$C = c + d\,,$$
$$D = b + d\,.$$

It can be easily shown that this approach does not impose any restriction on the evaluations of the possible worlds or states of affairs if this evaluation is taken as an initial. This means that A, B, C, D are calculable provided that a, b, c, d are given. These constraints can be expressed by the following relations:

$$A + C = B + D \quad (\text{i.e. } w^{\#}(p) + w^{\#}({\sim}p) = w^{\#}(q) + w^{\#}({\sim}q))$$
$$A + c = B + b\,,$$
$$D + c = C + b\,.$$

If we establish evaluations schemes for the possible worlds or states of affairs on the one hand and the evaluation scheme of atomic propositions on the other hand, we can read the correspondance of these schemes in one direction only. This fact represents a certain analogy of connective operations as opposed to adjunctive operations which allow for interpretations in both directions.*

Similar consequences are obtained by the analysis of the second preference concept which Rescher denotes by P^*. If we take as a base the mentioned scheme of weighted evaluation we can introduce evaluation corresponding to P^* so that we take into consideration not only the weighted evaluation of the possible worlds involved in the used propositions, but also the weighted evaluation of those worlds excluded in the used propositions, according to the following rule:

$$w^*(s_i) = w^{\#}(s_i) - w^{\#}(\sim s_i) .$$

Since s_i and $\sim s_i$ are complementary and involve the whole universe it also holds that

$$w^*(s_i) + w^*(\sim s_i) = 0 .$$

The constraints corresponding to P^* are then expressed by the following relations:

$$\begin{aligned} w^*(p) &= \mathbf{A} = \mathbf{a} + \mathbf{b} - \mathbf{c} - \mathbf{d} , \\ w^*(q) &= \mathbf{B} = \mathbf{a} - \mathbf{b} + \mathbf{c} - \mathbf{d} , \\ w^*(\sim p) &= \mathbf{C} = -\mathbf{a} - \mathbf{b} + \mathbf{c} + \mathbf{d} , \\ w^*(\sim q) &= \mathbf{D} = -\mathbf{a} + \mathbf{b} - \mathbf{c} + \mathbf{d} . \end{aligned}$$

It follows then that

$$\begin{aligned} \mathbf{A} + \mathbf{C} = 0 , \quad &\text{i.e.} \quad w^*(p) + w^*(\sim p) = 0 , \\ \mathbf{B} + \mathbf{D} = 0 , \quad &\text{i.e.} \quad w^*(q) + w^*(\sim q) = 0 . \end{aligned}$$

It can easily be shown that both preference concepts, i.e. $P^{\#}$ and P^* satisfy, when the above mentioned connective interpretation of corresponding schemes is respected, sentences (6), (7) and (8).

4. PREFERENCE "CETERIS PARIBUS"

Till now we have distinguished two concepts of preference. The first one denoted by $P^{\#}$ could be characterized as the *concept of involving preference*, the second one denoted by P^* as the *concept of differential preference.* Of course not all possible and used conceptions of preference have been exhausted. Let us confront the following preference statements:

(α) It is better to have a sparrow in the hand than a pigeon on the roof.
(β) I prefer tea to coffee at all meals.

* The distinction of adjunctive and connective operations was introduced by H. Reichenbach [8].

The preference statement (α) is an example of a *relativised preference*, statement (β) is an example of *preference cateris paribus.* Relativized preference represents a preference ordering of two components which are under different circumstances. Preference ceteris paribus is a preference ordering of two components under all concidered circumstances. From the concept of preference ceteris paribus we can distinguish another type of preference which can be described as *preference under explicity determined circumstances*, e.g.

(γ) I prefer tea to coffee with an English breakfest of ham, eggs, cheese and toast.

In common sense language the differences between (β) and (γ) need not be clearly and exactly distinguishable which is likewise evident on the basis of the above mentioned examples (β) and (γ). Because of this and for the sake of higher exactness we shall introduce a schematic example: Suppose that we have to ascertain the preference order between p and q provided that other components r and t must be taken into consideration. Then the possible states of affairs or the possible worlds can be demonstrated by the following scheme which also involves the weighted evaluation:

	p	q	r	t	$p(s_i)\ w(s_i)$
s_1	1	1	1	1	a_1
s_2	1	1	1	0	a_2
s_3	1	1	0	1	a_3
s_4	1	1	0	0	a_4
s_5	1	0	1	1	a_5
s_6	1	0	1	0	a_6
s_7	1	0	0	1	a_7
s_8	1	0	0	0	a_8
s_9	0	1	1	1	a_9
s_{10}	0	1	1	0	a_{10}
s_{11}	0	1	0	1	a_{11}
s_{12}	0	1	0	0	a_{12}
s_{13}	0	0	1	1	a_{13}
s_{14}	0	0	1	0	a_{14}
s_{15}	0	0	0	1	a_{15}
s_{16}	0	0	0	0	a_{16}

If we denote the preference ceteris paribus by P_{cp} and preference under the explicitely determined circumstances by P_{ec}, we obtain using the evaluation function $w^{\#}$:

$$pP_{cp}q\ [\equiv]\ w^{\#}(s_5 \vee s_6 \vee s_7 \vee s_8) > w^{\#}(s_9 \vee s_{10} \vee s_{11} \vee s_{12})$$
$$[\equiv]\ (a_5 + a_6 + a_7 + a_8) > (a_9 + a_{10} + a_{11} + a_{12})\ .$$

It should be noted that we could also use the differential evaluation w^*, but this use leads to the equivalent solution. Therefore we need not distinguish the involving preference ceteris paribus and the differential preference ceteris paribus. Using w^* we obtain

$$pP_{cp}q\ [\equiv]\ (-a_1 - a_2 - a_3 - a_4 + a_5 + a_6 + a_7 + a_8 - a_9 - a_{10} - a_{11} - a_{12} - a_{13} - a_{14} - a_{15} - a_{16}) > (-a_1 - a_2 - a_3 - a_4 - a_5 - a_6 - a_7 - a_8 + a_9 + a_{10} + a_{11} + a_{12} - a_{13} - a_{14} - a_{15} - a_{16})$$

which is equivalent with the above mentioned expression.

The concept of preference under the explicitely determined circumstances supposes that further components are explicitely determined. If it is determined that preference relation between p and q has to be fixed under the circumstances $r\ .\ \sim t$, then

$$pP_{ec}q\ (ec \equiv r\ .\ \sim t)\ [\equiv]\ w^{\#}(p\ .\ \sim q\ .\ r\ .\ \sim t) > w^{\#}(\sim p\ .\ q\ .\ r\ .\ \sim t)$$
$$[\equiv]\ a_6 > a_{10}$$

It should also be added that here the use of w^* is connected with the equivalent results so that the distinction between involving and differential concept of preference under the explicitely determined circumstances is not needed.

We shall now notice one important property of preference under the explicitely determined circumstances. While expressions

$$pP^{\#}q\ .\ qP^{\#}p\ ,$$
$$pP^{*}q\ .\ qP^{*}p$$

and

$$pP_{cp}q\ .\ qP_{cp}p$$

are to be considered as inconsistent, there is no justification to consider as inconsistent the following expression

$$pP_{ec_1}q\ .\ qP_{ec_2}p\ ,$$

if e.g. in the above mentioned expression

$$ec_1 \equiv r\ .\ \sim t$$

and

$$ec_2 \equiv \sim r\ .\ t$$

and if it holds that

$$a_6 > a_{10} \quad \text{and at the same time} \quad a_{11} > a_7\ .$$

From the logical point of view it is important to notice that the preference ceteris paribus, and likewise the preference under the explicitly determined circumstances, the

involving preference and the differential preference fulfil the sentences (6), (7), (8).
This, however, does not hold for the relativized preference. What about the relativized preference denoted here by P_{rel} it may fulfil only (**A** 1) so that

$$pP_{rel}q \rightarrow \sim(qP_{rel}p),$$

but it is hardly possible that the relativized preference is transitive and moreover can fulfil the sentences (6), (7), (8). It is also natural that the relativized preference is irreflexive so that it holds:

$$\sim(pP_{rel}p).$$

It is obvious that the relativized preference satisfies only very minimal claims and therefore it would be wrong to take the relativized preference as a starting point for the construction of preference systems.

5. THE CONCEPT OF "PREFERABLE STATES OF AFFAIRS" AS QUALITATIVE CONCEPT

Upto now we have distinguished five concepts of preference. The majority of them (with the exception of relativized preference) had preperties of comparative concepts, i.e. are irreflexive, assymmetrical and transitive. Since comparative concepts are in the logical methodology construed as a superstructure in a domain of qualitative concepts which enable the assignment of a certain quality discernible in degree, a concept of preferability may be supposed which would correspond to qualitative concepts. In logical systems of preference it is quite usual to introduce qualitative concepts based on the concept of preference or on the concept of indifference, respectively. Thus a triplet of qualitative concepts can be introduced:

Bp (the state denoted by p is preferable, i.e. good),
Mp (the state denoted by p is not preferable, i.e. bad),
Lp (the state denoted by p is indifferent)

and defined as follows:

$$Bp \equiv pP \sim p,$$
$$Mp \equiv \sim pPp,$$
$$Lp \equiv pI \sim p.$$

This procedure is used by S. Halldén [1] and von Wright in his first study [12]. Because of very similar reasons von Wright consders the following expression as a tautology:

$$(pP \sim p) . (\sim qPq) \rightarrow (pPq) \quad (9)$$

which can be interpreted as follows: If p is preferable and q unpreferable, then we must prefer p to q.

The difficulties cennected with (9) are, of course, quite evident, if we put for $w(p)$, $w(q)$, $w(\sim p)$ and $w(\sim q)$ as in the above mentioned schemes $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$. Then we should accept as valid

$$[(\boldsymbol{A} > \boldsymbol{C}) . (\boldsymbol{D} > \boldsymbol{B})] \rightarrow (\boldsymbol{A} > \boldsymbol{B}) . \tag{10}$$

If $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are real numbers we have undoubtedly no a priori guarrantee to consider (10) as tautology. If e.g.

$$\boldsymbol{A} = 5 , \quad \boldsymbol{B} = 6 , \quad \boldsymbol{C} = 4 , \quad \boldsymbol{D} = 7 ,$$

it is obvious that (10) and thereby (9) are not valid. In order to make this expression acceptable it is not possible to take any concept of preference as initial, but only a certain type of preference and thereby the corresponding constraints for $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$. It can be shown that (9) is satisfied provided that P is interpreted as involving preference, i.e. as $P^{\#}$. Then it holds that

$$\boldsymbol{A} + \boldsymbol{C} = \boldsymbol{D} + \boldsymbol{B}$$

and (10) is acceptable. Likewise the concept of differential preference P^* satisfies (9) since it holds that

$$\boldsymbol{A} + \boldsymbol{C} = 0$$

and

$$\boldsymbol{B} + \boldsymbol{D} = 0 .$$

Contrariwise the concept of preference ceteris paribus does not satisfy (9) because if it held that

$$[(pP_{\mathrm{cp}} \sim p) . (\sim qP_{\mathrm{cp}}q)] \rightarrow (pP_{\mathrm{cp}}q) ,$$

it should also have held, using the above mentioned scheme with 16 possible states on the basis of components p, q, r, t, that

$$\{[(\boldsymbol{a}_1 + \boldsymbol{a}_2 + \boldsymbol{a}_3 + \boldsymbol{a}_4 + \boldsymbol{a}_5 + \boldsymbol{a}_6 + \boldsymbol{a}_7 + \boldsymbol{a}_8) > (\boldsymbol{a}_9 + \boldsymbol{a}_{10} + \boldsymbol{a}_{11} + \boldsymbol{a}_{12} + \boldsymbol{a}_{13} + \boldsymbol{a}_{14} + \boldsymbol{a}_{15} + \boldsymbol{a}_{16})] . [(\boldsymbol{a}_1 + \boldsymbol{a}_2 + \boldsymbol{a}_3 + \boldsymbol{a}_4 + \boldsymbol{a}_9 + \boldsymbol{a}_{10} + \boldsymbol{a}_{11} + \boldsymbol{a}_{12}) > (\boldsymbol{a}_5 + \boldsymbol{a}_6 + \boldsymbol{a}_7 + \boldsymbol{a}_8 + \boldsymbol{a}_{13} + \boldsymbol{a}_{14} + \boldsymbol{a}_{15} + \boldsymbol{a}_{16})]\} \rightarrow [(\boldsymbol{a}_5 + \boldsymbol{a}_6 + \boldsymbol{a}_7 + \boldsymbol{a}_8) > (\boldsymbol{a}_9 + \boldsymbol{a}_{10} + \boldsymbol{a}_{11} + \boldsymbol{a}_{13})] ,$$

for which, of course, no a priori guarrantee is given.

In contradistinction to the concept of preference ceteris paribus the concept of preference under explicitly determined circumstances satisfies (9). If we take in the used scheme

$$\mathrm{ec} \equiv r . \sim t ,$$

we obtain

$$[(a_6 > a_{10}) \cdot (a_6 > a_{10})] \rightarrow (a_6 > a_{10})$$

which is evidently a tautology.

We may conclude that the corresponding qualitative concepts, i.e. concepts of preferable, unpreferable and indifferent states of affairs can be introduced on the basis of preference concepts only if some of the constraints are accepted. From the above introduced concepts of preference involving preference, differential preference and preference under the explicitly determined circumstances can be used as such a base.

The sentence (9) corresponds, at first sight, to the trivial requirement that good states of affairs are preferable to bad states of affairs. But this problem is more complicated, since the used qualitative concepts are not primitive, but concepts defined on the basis of preference concepts, which take into account the mentioned constraints. From the intuitive point of view the most favourable concept is the concept of differential preference taken as the initial one: As far as this concept is concerned it holds that the sum of evaluations connected with complementary states is always equal to zero, i.e. that for any s_i

$$w^*(s_i) + w^*(\sim s_i) = 0 .$$

It should be added that this approach and this justification of preference correspond to such a mode of deliberation where any choice from the set of possible alternatives calculates both with the sum of gains and likewise with the sum of losses. It is unnecessary to think of the concept of gain or loss only in the economic or financial sense. Then a certain simplification of the mentioned definitions of B, M, and L is possible:

$$Bp \ [\equiv] \ w^*(p) > 0 ,$$
$$Mp \ [\equiv] \ w^*(p) < 0 ,$$
$$Lp \ [\equiv] \ w^*(p) = 0 .$$

6. PREFERENCE AS A PROPOSITIONAL ATTITUDE

The expression pPq, no matter what type of preference – the fore mentioned or other types – is taken into consideration, represents in fact a certain simplification: This expression is an impersonal and untemporal modification of the following expression: "x in the time t prefers p to q". Because of this it seems to be suitable to relativize preference expressions to a certain subject and to a certain time.

Since arguments of the relation P are states of affairs or denotations of prositions (in all the above mentioned types of preference) it is expedient to consider P as a *special form of a propositional attitude* or as a special epistemic-modal operator. As propositional attitudes we usually regard statements like

$$x \text{ believes that } p\,,$$

$$x \text{ knows that } p\,,$$

$$x \text{ doubts } p\,, \quad \text{etc.}$$

Logical theory of propositional attitudes or epistemic-modal operators was elaborated in details in many systems of asserton logic or epistemic logic.* One of the characteristic features of the propositional attitudes is the non-extensional contextual concatenation of the operator and of the proposition itself, which brought these problems closer to the problems of various systems of modal logic. In contradistinction to the usual epistemic-modal operators which fix a certain attitude of the speaker to one proposition or to an interpretation (usually denotation) of one proposition, the preference expression involves the attitude of the speaker to the donotation of two propositions or, in other words, to a certain preference ordering of the propositions.

Likewise as in the other cases of propositional attitudes the temporal specification is not without importance for preference expressions. The change of temporal specification can lead to changes in the preference attitudes of the given speaker. Because of similar reasons the following concatenation of two different preference attitudes of the same speaker need not be considered as inconsistent: "x prefers at the time t p to q and x prefers at the time t' (different from t) q to p". It can not be excluded that the value system of the speaker x can be subjected to changes in the interval between t and t'. If we write down the speaker of the preference attitude and the temporal determination of this attitude as indexes of P (no matter which interpretation of the sign P is chosen), then the following expressions are to be considered as consistent:

$$pP_{x,t}q \,.\, qP_{x,t'}p \quad (\text{provided that } t \neq t')$$

or

$$pP_{x,t}q \,.\, qP_{y,t}p \quad (\text{provided that } x \neq y)\,.$$

Regarding the preference as a special form of the propositional attitude (therefore we speak about preference attitudes) we must take into account that as speakers or subjects of preference attitudes we can admit not real persons but also legal persons, certain groups of persons (e.g. voters of the given electoral domain etc.). Then a manifestation system of preference attitudes is needed (e.g. choice of one of possible alternatives, decision of the corresponding authorities representing the legal person,

* The problem of propositional attitudes was already mentioned in Whitehead's and Russell's Principia Mathematica and later in Carnap's Meaning and Necessity. In the last period new interesting conceptions of a logical theory of the propositional attitudes were presented by J. Hintikka [2] and N. Rescher [10].

electoral system etc.). As a subject of a preference attitude we can also imagine a constructive entity, e.g. an ideal rational being, a Bayesian robot considered by R. C. Jeffrey [5]. An assertion that such beings do not exist in reality does not exclude the possibility of reconstructing the preference attitudes of such beings.

Because of similar reasons some authors considered the distinction between the concept of "preference" and the concept of "preferability" as reasonable (e.g. von Wright in his last paper [13]). The concept of "preferability" or "preferable" can be thought of as "being reasonably better than" or as preference attitude of an ideal being or similarly. Therefore statements of preferability need not be realitivized to a certain person and time. Since preference attitudes of various persons need not coincide with preference attitude of an ideal rational being (denoted here as pBq ("p is preferable before q"), the espression

$$pP_{x,t}q \cdot qBp$$

can be considered as consistent.

Analysing various preference attitudes and consistency problems of statements which express these attitudes we can fix various claims to what is or must be at the subject's disposal. If we can, e.g. concede that this subject established his preference attitude accordind to a value system different from an ideal or rationaly justified value system of a Bayesian robot (in the spirit of the principle "de gustibus non est disputandum") we will evidently show less readiness to concede the ignorance of logical principles, e.g. the ignorance of contradiction. In the sense of these claims the preference expression

$$(p \cdot \sim p)\, P_x q$$

will be now considered as evidently inconsistent. This means that from the logico-semantic point of view we shall be more willing to tolerate the plurality of value systems and distinctions between various subjects and less willing to tolerate the violation of elementary logical, syntactical and semantical principles valid for the linguistic means used in preference expressions.

CONCLUSIONS

1. An analysis of the logical status of preference supports the conception that we must distinguish various concepts of preference. Our analysis introduced five concepts: involving preference, differential preference, preference ceteris paribus, preference under the explicitly specified circumstances and relativized preference. These concepts satisfy in different ways principles considered as tautologies of a preference calculus. (In our paper we have selected sentences (6), (7), (8), (9).)

2. As a semantical base of the involving preference we can consider procedures corresponding to the state description method or to Carnap's concept of logical range.

Differential preference corresponds to the procedures used in econometric decisions under the supposition of zero sum, i.e. if it holds for any s_i that

$$w^*(s_i) + w^*(\sim s_i) = 0 .$$

Preference ceteris paribus reflects all possible circumstances taken into consideration at a preference ordering. Since the number of these circumstances can be very large it is obvious that the justification of preference on this basis can be very sophisticated. Preference under explicitly specified circumstances is evidently the most exact starting point of a reliable preference ordering.

3. Preference expressions can be considered as a special form of propositional attitude and the concept of preference itself as a epistemic-modal operator.

4. Varous types and forms of preference expressions suppose various conditions of consistency. The absence of violation of elementary logical, syntactical and semantical rules must be considered as a minimal consistency condition.

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REFERENCES

[1] Sören Halldén: On the Logic of "Better". Library of Theoria. Uppsala 1957.
[2] Jaakko Hintikka: Knowledge and Belief. Ithaca, New York 1962.
[3] H. S. Houthakker: On the Logic of Preference and Choice. In: A. T. Tymieniecka, ed.: Contributions to Logic and Methodology in Honor of J. M. Bocheński. Amsterdam 1965.
[4] Roderick M. Chisholm, Ernest Sosa: Intrinsic Preferability and the Problem of Superogation. Synthese *16* (1966), 3/4.
[5] Richard C. Jeffrey: The Logic of Decision. Mc Graw-Hill. New York—London—Sydney 1965.
[6] John G. Kemeny, J. Laurie Snell: Mathematical Models in the Social Sciences. Blaisdell Publ. Comp. New York—Toronto—London 1962.
[7] R. D. Luce, H. Raiffa: Games and Decisions. Introduction and Critical Survey. New York—London—Sydney 1957.
[8] Hans Reichenbach: Elements of Symbolic Logic. New York 1947.
[9] Nicholas Rescher: Notes on Preference, Utility and Cost. Synthese *16* (1966), 3/4.
[10] Nicholas Rescher: Topics in Philosophical Logic. D. Reidel Publ. Comp., Dordrecht 1968.
[11] Ladislav Tondl: Prerequisities for Quantification in the Empricial Sciences. Theory and Decision *2* (1972), 3.
[12] G. H. von Wright: The Logic of Preference. Edinburgh 1963.
[13] G. H. von Wright: The Logic of Preference Reconsidered. Theory and Decision *3* (1972), 2.

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