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On Formulation of State Equations for Linear Nonstationary Discrete-Time System

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Two different methods are given for the compact determination of a state space description of a linear nonstationary discrete-time system from the scalar difference equation.

Formulations of the state equations for a linear stationary system are well known and their discrete-time version can be found, e.g., in [1] and [2]. Nevertheless these methods are not directly applicable if system parameters vary with time.*

The methods of a state space description for linear continuous nonstationary systems as in [3] or [4] also cannot be used in similar discrete case simply by writing discrete instead of continuous functions in the results.

It is the purpose of this work to present two compact, non-recursive forms of a state space description from the scalar difference equation of a linear nonstationary discrete-time system. The first method corresponds to the continuous system way described in [3] and the second method is derived from the similar method for continuous nonstationary systems described in [4].

I. FORMULATION OF THE PROBLEM

A single-input, single-output, linear deterministic discrete time system is described by the equation

(1a) $$\sum_{i=0}^{s} a_i(k)\, y(k+i) = \sum_{i=0}^{s} b_i(k)\, u(k+i)$$

or

(1b) $$\sum_{i=0}^{s} a_i(k)\, E^i\, y(k) = \sum_{i=0}^{s} b_i(k)\, E^i\, u(k)$$

* Note that Freeman's statement on applicability of the formulations given in [1, pp. 22—27] is not quite clear. We cannot replace constant system parameters a_i, b_i by time-varying parameters $a_i(k)$, $b_i(k)$ in the resulting relations in both methods.

where an input and output are denoted by $u(k)$ and $y(k)$ respectively and the operator E is defined by

(2) $$E^j f(k) = f(k + j)$$

(j is an integer) with the property

(3) $$E^j[f(k)\, g(k)] = E^j f(k)\, E^j g(k)\,.$$

The system action period is assumed here to be $T = 1$ for simplicity, i.e., discrete values of time equal k.

Initial conditions of (1) are

(4) $$E^i\, y(k_0)$$

where $i = 0, 1, \ldots, s - 1$.

A system (1) can be described in a state space form by two equations

(5) $$\mathbf{x}(k + 1) = \mathbf{A}(k)\, \mathbf{x}(k) + \boldsymbol{\beta}(k)\, u(k)\,,$$

(6) $$y(k) = \mathbf{c}(k)\, \mathbf{x}(k) + d(k)\, u(k)$$

where

(7) $$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_s(k) \end{bmatrix}$$

is an $(s \times 1)$ vector of state variables $x_i(k)$, $\mathbf{A}(k)$ is an $(s \times s)$ matrix, $\boldsymbol{\beta}(k)$ is an $(s \times 1)$ vector, $\mathbf{c}(k)$ is an $(1 \times s)$ vector and $d(k)$ is a scalar.

The output initial conditions (4) are transformed into the state initial conditions by

(8) $$\mathbf{x}(k_0) = \mathbf{P}(k_0)\, \mathbf{y}(k_0) - \mathbf{R}(k_0)\, \mathbf{u}(k_0)$$

where

(9) $$\mathbf{y}(k) = \begin{bmatrix} y(k) \\ y(k + 1) \\ \vdots \\ y(k + s - 1) \end{bmatrix},$$

(10) $$\mathbf{u}(k) = \begin{bmatrix} u(k) \\ u(k + 1) \\ \vdots \\ u(k + s - 1) \end{bmatrix}$$

are $(s \times 1)$ vectors and $\mathbf{P}(k)$ and $\mathbf{R}(k)$ are $(s \times s)$ matrices.

The methods for determination of the vector-matrix equations (5), (6) and (8) from the scalar equations (1) and (4) are shown and proved in the following text.

II. FIRST METHOD

The results of this method are

(11)
$$\mathbf{A}(k) = \begin{bmatrix} -\frac{\alpha_{s-1}(k)}{\alpha_s(k)} & 1 & 0 & 0 & \dots & 0 \\ -\frac{\alpha_{s-2}(k)}{\alpha_s(k)} & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \\ -\frac{\alpha_1(k)}{\alpha_s(k)} & 0 & 0 & 0 & \dots & 1 \\ -\frac{\alpha_0(k)}{\alpha_s(k)} & 0 & 0 & 0 & \dots & 0 \end{bmatrix},$$

(12)
$$\boldsymbol{\beta}(k) = \begin{bmatrix} \beta_{s-1}(k) - \alpha_{s-1}(k)\frac{\beta_s(k)}{\alpha_s(k)} \\ \beta_{s-2}(k) - \alpha_{s-2}(k)\frac{\beta_s(k)}{\alpha_s(k)} \\ \vdots \\ \beta_1(k) - \alpha_1(k)\frac{\beta_s(k)}{\alpha_s(k)} \\ \beta_0(k) - \alpha_0(k)\frac{\beta_s(k)}{\alpha_s(k)} \end{bmatrix},$$

(13)
$$\mathbf{c}(k) = \left[\frac{1}{\alpha_s(k)}\ 0\ \dots\ 0\right]$$

and

(14)
$$d(k) = \frac{\beta_s(k)}{\alpha_s(k)}$$

where

(15)
$$\alpha_i(k) = E^{-i}\, a_i(k)\,,$$

(16)
$$\beta_i(k) = E^{-i}\, b_i(k)\,;$$

$$i = 0, 1, \dots, s - 1\,.$$

For the initial conditions (8) we have

$$\mathbf{P}(k) = \begin{bmatrix} \alpha_s(k) & 0 & 0 & \dots 0 & \\ \alpha_{s-1}(k) & \alpha_s(k+1) & 0 & \dots 0 & \\ \vdots & \vdots & \vdots & \vdots & \\ \alpha_2(k) & \alpha_3(k+1) & \alpha_4(k+2) & \dots 0 & \\ \alpha_1(k) & \alpha_2(k+1) & \alpha_3(k+2) & \dots & \alpha_s(k+s-1) \end{bmatrix} \tag{17}$$

and

$$\mathbf{R}(k) = \begin{bmatrix} \beta_s(k) & 0 & 0 & \dots 0 & \\ \beta_{s-1}(k) & \beta_s(k+1) & 0 & \dots 0 & \\ \vdots & \vdots & \vdots & \vdots & \\ \beta_2(k) & \beta_3(k+1) & \beta_4(k+2) & \dots 0 & \\ \beta_1(k) & \beta_2(k+1) & \beta_3(k+2) & \dots & \beta_s(k+s-1) \end{bmatrix}. \tag{18}$$

Proof. Assume that the following set of equations is valid

$$x_1(k) = \alpha_s(k)\, y(k) - \beta_s(k)\, u(k)\,, \tag{19a}$$

$$\begin{aligned} x_2(k) &= \alpha_{s-1}(k)\, y(k) - \beta_{s-1}(k)\, u(k) + x_1(k+1)\,, \\ &\vdots \\ x_{s-1}(k) &= \alpha_2(k)\, y(k) - \beta_2(k)\, u(k) + x_{s-2}(k+1)\,, \\ x_s(k) &= \alpha_1(k)\, y(k) - \beta_1(k)\, u(k) + x_{s-1}(k+1)\,, \end{aligned} \tag{19b}$$

$$0 = \alpha_0(k)\,(yk) - \beta_0(k)\, u(k) + x_s(k+1)\,. \tag{19c}$$

If $x_1(k)$ from the equation (19a) is substituted into the first equation (19b), and the resulting $x_2(k)$ is substituted into the next equation (19b), and so on, all of the $x_i(k)$ are eliminated and the equations (19a), (19b) can be written as

$$\begin{aligned} x_1(k) &= \alpha_s(k)\, y(k) - \beta_s(k)\, u(k)\,, \\ x_2(k) &= \alpha_{s-1}(k)\, y(k) + \alpha_s(k+1)\, y(k+1) - \beta_{s-1}(k)\, u(k) - \\ &\quad - \beta_s(k+1)\, u(k+1)\,, \\ &\vdots \\ x_{s-1}(k) &= \alpha_2(k)\, y(k) + \alpha_3(k+1)\, y(k+1) + \dots + \alpha_s(k+s-2)\, y(k+s-2) - \\ &\quad - \beta_2(k)\, u(k) - \beta_3(k+1)\, u(k+1) - \dots - \beta_s(k+s-2)\, u(k+s-2)\,, \\ x_s(k) &= \alpha_1(k)\, y(k) + \alpha_2(k+1)\, y(k+1) + \dots + \alpha_s(k+s-1)\, y(k+s-1) - \\ &\quad - \beta_1(k)\, u(k) - \beta_2(k+1)\, u(k+1) - \dots - \beta_s(k+s-1)\, u(k+s-1)\,. \end{aligned} \tag{20}$$

Then the equation (19c) with respect to (3) is

$$\sum_{i=0}^{s} E^i \alpha_i(k)\, E^i\, y(k) = \sum_{i=0}^{s} E^i \beta_i(k)\, E^i\, u(k)\,. \tag{21}$$

Now if the relations (15) and (16) are valid then the equations (1) and (21) are identical.

Hence the state variables of a system (1) are defined by the equations (20).

The state equations result from the equations (19b) and (19c).

If

$$y(k) = \frac{1}{\alpha_s(k)} x_1(k) + \frac{\beta_s(k)}{\alpha_s(k)} u(k) \tag{22}$$

from the equation (19a) is substituted into equations (19b) we can write (19b) and (19c) in the form

$$\begin{aligned} x_1(k+1) &= -\frac{\alpha_{s-1}(k)}{\alpha_s(k)} x_1(k) + x_2(k) + \left[\beta_{s-1}(k) - \alpha_{s-1}(k)\frac{\beta_s(k)}{\alpha_s(k)}\right] u(k), \\ x_2(k+1) &= -\frac{\alpha_{s-2}(k)}{\alpha_s(k)} x_1(k) + x_3(k) + \left[\beta_{s-2}(k) - \alpha_{s-2}(k)\frac{\beta_s(k)}{\alpha_s(k)}\right] u(k), \\ &\vdots \\ x_{s-1}(k+1) &= -\frac{\alpha_1(k)}{\alpha_s(k)} x_1(k) + x_s(k) + \left[\beta_1(k) - \alpha_1(k)\frac{\beta_s(k)}{\alpha_s(k)}\right] u(k), \\ x_s(k+1) &= -\frac{\alpha_0(k)}{\alpha_s(k)} x_1(k) + \left[\beta_0(k) - \alpha_0(k)\frac{\beta_s(k)}{\alpha_s(k)}\right] u(k). \end{aligned} \tag{23}$$

From the equations (23) $\mathbf{A}(k)$ is given by (11) and $\boldsymbol{\beta}(k)$ is given by (12).

According to (22) $\mathbf{c}(k)$ stands in (13) and $d(k)$ stands in (14).

The matrices $\mathbf{P}(k)$ and $\mathbf{R}(k)$ are given by (17) and (18) respectively as it can be seen from the equations (20).

III. SECOND METHOD

If we let $a_s(k) = 1$ for the system (1) (without loss of generality) it is possible to prove that the coefficients of (5) and (6) can be determined as

$$\mathbf{A}(k) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0(k) & -a_1(k) & -a_2(k) & \dots & -a_{s-1}(k) \end{bmatrix}, \tag{24}$$

$$\mathbf{c}(k) = \mathbf{c} = [1 \; 0 \; \dots \; 0], \tag{25}$$

$$d(k) = b_s(k-s) \tag{26}$$

and

$$\boldsymbol{\beta}(k) = E \sum_{\nu=0}^{s} L_A^\nu \, \mathbf{b}_\nu(k) \tag{27}$$

where

$$\mathbf{b}_\nu(k) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_\nu(k) \end{bmatrix} \tag{28}$$

is an $(s \times 1)$ vector and the operator L_A^ν is defined as

$$L_A^\nu \, \mathbf{b}(k) = E^{-1}\{\mathbf{A}(k) \, L_A^{\nu-1} \, \mathbf{b}(k)\} , \quad \nu > 0 \tag{29}$$

and

$$L_A^0 \, \mathbf{b}(k) = E^{-1} \, \mathbf{b}(k) . \tag{30}$$

In the initial conditions relation (8)

$$\mathbf{P}(k) = \mathbf{I} \tag{31}$$

is the unit matrix and the matrix

$$\mathbf{R}(k) = [\mathbf{r}_0(k) \mid \mathbf{r}_1(k) \mid \ldots \mid \mathbf{r}_{s-1}(k)] \tag{32}$$

has the columns

$$\mathbf{r}_i(k) = \sum_{\nu=1}^{s-i} L_A^{\nu-1} \, \mathbf{b}_{i+\nu}(k) , \quad i = 0, 1, \ldots, s-1 . \tag{33}$$

Proof. Consider arbitrary state equations of the type (5) and (6)

$$\mathbf{x}(k+1) = \mathbf{A}^*(k) \, \mathbf{x}(k) + \boldsymbol{\beta}^*(k) \, u(k) , \tag{34}$$

$$y(k) = \mathbf{c}^*(k) \, \mathbf{x}(k) + d^*(k) \, u(k) . \tag{35}$$

By gradual application of the translation operator E^i to the equation (35) we obtain the set of equations

$$E^i \, y(k) = E^i \, \mathbf{c}^*(k) \, E^i \, \mathbf{x}(k) + E^i\{d^*(k) \, u(k)\} , \quad i = 0, 1, \ldots, s-1 . \tag{36}$$

If we substitute into the equations (36),

$$E^i \, \mathbf{x}(k) = E^{i-1} \, \mathbf{x}(k+1)$$

where $\mathbf{x}(k+1)$ is given by (34), then (36) can be written in a vector-matrix form

(37) $$\mathbf{y}(k) = \mathbf{Q}(k)\,\mathbf{x}(k) + \mathbf{R}(k)\,\mathbf{u}(k)$$

where $\mathbf{y}(k)$ and $\mathbf{u}(k)$ are given by (9) and (10) respectively.

In the equation (37)

(38) $$\mathbf{Q}(k) = [\mathbf{q}_1(k) \,\vdots\, \mathbf{q}_2(k) \,\vdots\, \ldots \,\vdots\, \mathbf{q}_s(k)]^{\mathrm{T}}$$

is the $(s \times s)$ matrix with the rows

(39) $$\mathbf{q}_1(k) = \mathbf{c}^*(k)$$

and

(40) $$\mathbf{q}_{i+1}(k) = \mathbf{q}_i(k+1)\,\mathbf{A}^*(k)\,,$$
$$i = 1, 2, \ldots, s-1\,.$$

If now $\mathbf{A}^*(k) = \mathbf{A}(k)$ and $\mathbf{c}^*(k) = \mathbf{c}$ are given by (24) and (25) respectively then

(41) $$\mathbf{Q}(k) = \mathbf{I}$$

is the unit matrix and the equation (37) has the form

(42) $$\mathbf{y}(k) = \mathbf{x}(k) + \mathbf{R}(k)\,\mathbf{u}(k)\,.$$

Comparing the structure of the equations (42) and (8) we get

(43) $$\mathbf{P}(k) = \mathbf{I}$$

as it stands in (31).

It must be shown that under conditions (24) and (25) the remaining $d(k)$, $\beta(k)$ and $\mathbf{R}(k)$ satisfy the relations (26), (27)–(30) and (32)–(33) respectively.

The scalar difference equation (1) can be written in a vector-matrix form as

(44) $$\mathbf{y}(k+1) = \mathbf{A}(k)\,\mathbf{y}(k) + \mathbf{B}(k)\,\mathbf{u}^{\dagger}(k)\,,$$

(45) $$y(k) = \mathbf{c}\,\mathbf{y}(k)$$

where $\mathbf{y}(k)$, $\mathbf{A}(k)$ and $\mathbf{c}$ are given by (9), (24) and (25) respectively,

(46) $$\mathbf{u}^{\dagger}(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+s) \end{bmatrix}$$

is an $((s+1) \times 1)$ vector and

(47) $$\mathbf{B}(k) = [\mathbf{b}_0(k) \,\vdots\, \mathbf{b}_1(k) \,\vdots\, \ldots \,\vdots\, \mathbf{b}_s(k)]$$

is the $(s \times (s+1))$ matrix with the columns $\mathbf{b}_i(k)$ given by (28).

Eliminating $\mathbf{y}(k)$ and $\mathbf{y}(k+1)$ from the equations (42) and (44) we can write

$$\mathbf{x}(k+1) - \mathbf{A}(k)\,\mathbf{x}(k) = \mathbf{A}(k)\,\mathbf{R}(k)\,\mathbf{u}(k) - \mathbf{R}(k+1)\,\mathbf{u}(k+1) + \mathbf{B}(k)\,\mathbf{u}^{\dagger}(k)\,. \tag{48}$$

With respect to the identity

$$\mathbf{u}^{\dagger}(k) = \begin{bmatrix} \mathbf{u}(k) \\ u(k+s) \end{bmatrix} = \begin{bmatrix} u(k) \\ \mathbf{u}(k+1) \end{bmatrix} \tag{49}$$

(48) can be written in the form

$$\mathbf{x}(k+1) - \mathbf{A}(k)\,\mathbf{x}(k) = \{[\mathbf{A}(k)\,\mathbf{R}(k) \mid \mathbf{0}] - [\mathbf{0} \mid \mathbf{R}(k+1)] + \mathbf{B}(k)\}\,\mathbf{u}^{\dagger}(k)\,. \tag{50}$$

If a matrix $\mathbf{R}(k)$ of structure as in (32) possesses now such a property that the right-hand side of (50) equals $\boldsymbol{\beta}(k)\,u(k)$, i.e.

$$\{[\mathbf{A}(k)\,\mathbf{R}(k) \mid \mathbf{0}] - [\mathbf{0} \mid \mathbf{R}(k+1)] + \mathbf{B}(k)\}\,\mathbf{u}^{\dagger}(k) = [\boldsymbol{\beta}(k) \mid \mathbf{0} \mid \dots \mid \mathbf{0}]\,\mathbf{u}^{\dagger}(k) \tag{51}$$

then a system (1) can be described by (5) and (6).

Write the equation (51) in this form:

$$\{[\mathbf{A}(k)\,\mathbf{R}(k) \mid \mathbf{0}] - [\boldsymbol{\beta}(k) \mid \mathbf{R}(k+1)] + \mathbf{B}(k)\}\,\mathbf{u}^{\dagger}(k) = \mathbf{G}(k)\,\mathbf{u}^{\dagger}(k) = \mathbf{0}\,. \tag{52}$$

Then the equation (52) is valid for an arbitrary $\mathbf{u}^{\dagger}(k)$ if

$$\mathbf{G}(k) = \mathbf{0}\,.$$

The columns of an $(s \times s+1)$ matrix

$$\mathbf{G}(k) = [\mathbf{g}_{\beta}(k) \mid \mathbf{g}_1(k) \mid \mathbf{g}_2(k) \mid \dots \mid \mathbf{g}_s(k)] \tag{53}$$

are in this case

$$\mathbf{g}_{\beta}(k) = \mathbf{A}(k)\,\mathbf{r}_0(k) - \boldsymbol{\beta}(k) + \mathbf{b}_0(k) = \mathbf{0} \tag{54}$$

and

$$\begin{aligned}
\mathbf{g}_1(k) &= \mathbf{A}(k)\,\mathbf{r}_1(k) - \mathbf{r}_0(k+1) + \mathbf{b}_1(k) = \mathbf{0}\,,\\
&\ \vdots\\
\mathbf{g}_{s-1}(k) &= \mathbf{A}(k)\,\mathbf{r}_{s-1}(k) - \mathbf{r}_{s-2}(k+1) + \mathbf{b}_{s-1}(k) = \mathbf{0}\,,\\
\mathbf{g}_s(k) &= - \mathbf{r}_{s-1}(k+1) + \mathbf{b}_s(k) = \mathbf{0}\,.
\end{aligned} \tag{55}$$

From the last equation of (55) we have

$$\mathbf{r}_{s-1}(k) = \mathbf{b}_s(k-1)\,. \tag{56}$$

If $\mathbf{r}_{s-1}(k)$ is substituted into the last but one equation (55) we have

$$\mathbf{r}_{s-2}(k) = \mathbf{A}(k-1)\,\mathbf{b}_s(k-2) + \mathbf{b}_{s-1}(k-1)\,. \tag{57}$$

By similarly repeated substitutions we obtain generally

(58)
$$\mathbf{r}_i(k) = \sum_{\nu=1}^{s-i} L_A^{\nu-1} \mathbf{b}_{i+\nu}(k), \quad i = 0, 1, \ldots, s-1;$$

where L_A^{ν} is given by (29) and (30).

Then the validity of (33) is evident.

From the equation (54) we get in a similar way

(59)
$$\boldsymbol{\beta}(k) = E \sum_{\nu=1}^{s} L_A^{\nu} \mathbf{b}_{\nu}(k)$$

and the validity of (27) is proved.

The matrix $\mathbf{R}(k)$ has the following triangular structure

(60)
$$\mathbf{R}(k) = \begin{bmatrix} b_s(k-s) & 0 & \ldots & 0 & 0 \\ & b_s(k-s+1) & \ldots & 0 & 0 \\ & & \ddots & \vdots & \vdots \\ & \times & & b_s(k-2) & 0 \\ & & & & b_s(k-1) \end{bmatrix}.$$

Then we obtain from (42) and (45)

(61)
$$y(k) = \mathbf{c}\,\mathbf{x}(k) + \mathbf{c}\,\mathbf{R}(k)\,\mathbf{u}(k) = \mathbf{c}\,\mathbf{x}(k) + b_s(k-s)\,u(k)$$

and (26) is correct.

If the coefficients of the system scalar equation (1) are

(62)
$$b_i(k) = 0 \quad \text{for all} \quad i > p\,;\ p < s$$

then

(63)
$$\mathbf{r}_i(k) = \mathbf{0} \quad \text{for all} \quad i \geqq p$$

and

(64)
$$d(k) = 0\,.$$

EXAMPLE

Let us determine the state equations for the system described by scalar difference equation

$$y(k+2) + k\,y(k+1) + e^{-k}\,y(k) = u(k) + e^{-k}\,u(k+1) + u(k+2)\,.$$

Obviously

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \quad \mathbf{y}(k) = \begin{bmatrix} y(k) \\ y(k+1) \end{bmatrix}, \quad \mathbf{u}(k) = \begin{bmatrix} u(k) \\ u(k+1) \end{bmatrix}$$

in this case.

First method

According to (15) and (16) we shall determine at first

$$\alpha_0(k) = e^{-k}, \quad \beta_0(k) = 1,$$
$$\alpha_1(k) = k - 1, \quad \beta_1(k) = e^{-(k-1)},$$
$$\alpha_2(k) = 1, \quad \beta_2(k) = 1.$$

Substituting into (11), (12), (13) and (14) we can write (5) and (6) as

$$\mathbf{x}(k+1) = \begin{bmatrix} 1-k & 1 \\ -e^{-k} & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 - k + e^{1-k} \\ 1 - k \end{bmatrix} u(k)$$

and

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k) + u(k).$$

The output initial conditions $y(k_0)$ and $y(k_0 + 1)$ are transformed into

$$\mathbf{x}(k_0) = \begin{bmatrix} 1 & 0 \\ k_0 - 1 & 1 \end{bmatrix} \mathbf{y}(k_0) - \begin{bmatrix} 1 & 0 \\ e^{1-k_0} & 1 \end{bmatrix} \mathbf{u}(k_0).$$

Second method

By substitution into (24)–(27) with respect to (28)–(30) we obtain the state equations (5) and (6)

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -e^{-k} & -k \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 - k + e^{1-k} \\ 1 - ke^{1-k} - e^{-k} + k(k-1) \end{bmatrix} u(k)$$

and

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k) + u(k).$$

In accordance with (31)–(33) the relation (8) can be written as

$$\mathbf{x}(k_0) = \mathbf{y}(k_0) - \begin{bmatrix} 1 & 0 \\ 1 - k_0 + e^{1-k_0} & 1 \end{bmatrix} \mathbf{u}(k_0).$$

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Formulace stavových rovnic lineárního nestacionárního diskrétního systému

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Práce pojednává o vektorově-maticovém popisu lineárního diskrétně pracujícího systému s časově proměnnými parametry. Jsou zde uvedeny dvě kompaktní metody převodu skalární diferenční rovnice systému na rovnice stavové včetně odpovídající transformace počátečních podmínek.

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