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On a Certain Event Recognizable in Real Time

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In the article, an event B which is recognizable in real time by an automaton with three tapes is given. On the other hand, the event $A = B^{-1}$ which consists of the converse words of those from B is known not to be recognizable in real time by any automaton with finite number of tapes (even by any automaton with finite number of multidimensional tapes).

We shall consider the class of automata with finite number of internal states, input and output, and finite number of unbounded tapes. The input and output alphabets as well as the alphabets on the tapes are finite. We suppose that the automata have free input, i.e., that the input sequence of length n is read over just within n tacts for any n. Otherwise, the activity of the automaton at a given tact is defined as usual, i.e., according to the input symbol, the internal state and the symbols scanned on the tapes, the automaton performs the following acts: writes new symbols on active (just scanned) squares of the tapes, moves the heads (to the right, left, or no move), transfers to a new internal state and gives an output symbol. Further, we suppose that the automaton is in a quite definite internal state before starting its activity.

An event over the alphabet Σ is a subset of the set Σ^∞ where the symbol Σ^∞ denotes the set of all finite words consisting of symbols of the alphabet Σ. If $a \in \Sigma^\infty$, $a = \sigma_1\sigma_2 \dots \sigma_r$, then a^{-1} means the converse word, $a^{-1} = \sigma_r\sigma_{r-1} \dots \sigma_1$.

The automata of the above type can serve for recognization events. If the output alphabet Π of an automaton J consists of two symbols, $\Pi = \{0, 1\}$, then the automaton J with the input alphabet Σ defines an event $A \subset \Sigma^\infty$ in the following manner: a word $a \in \Sigma^\infty$ of length n belongs to A iff the automaton J, being fed with the input word a, gives the symbol 1 at tact n.

Definition. We say that *an event A over alphabet Σ is recognizable in real time if an automaton J of the above type with the input alphabet Σ exists that recognizes the event A.*

Hartmanis and Stearns have shown in [2] a simple event that is not recognizable in real time. Their event A is defined as follows. Let $\Sigma = \{0; 1; *\}$, $U = \{0; 1\}$. Let A be the set of the words w of the form

$$w = u_1 * u_2 * \ldots * u_n * v \,, \tag{1}$$

where $n \geqq 1$, $u_i \in U^\infty$, $i = 1, 2, \ldots, n$, $v \in U^\infty$, and an index i, $1 \leqq i \leqq n$ exists that

$$u_i = v^{-1} \,. \tag{2}$$

Let event B be the event that consists just of such words w that $w^{-1} \in A$ (we write $A = B^{-1}$). It is clear that the event B consists of the words of the form

$$w = v * u_1 * u_2 * \ldots * u_n \,, \tag{3}$$

where $n \geqq 1$, $u_i \in U^\infty$, $i = 1, 2, \ldots, n$, $v \in U^\infty$, and an index i, $1 \leqq i \leqq n$ exists that (2) holds.

Bečvář in [1] states the problem whether the event B is recognizable in real time. The following theorem solves the problem positively.

Theorem. *The event B is recognizable in real time.*

Proof. We shall show that an automaton J with three tapes (T_1, T_2, T_3) exists that recognizes the event B in real time. Only substantial features of the activity of the automaton J will be given but it will be clear that a detailed construction of J is possible. We shall divide the activity of the automaton J into several stages.

1. The automaton is fed with the word v.

The word v is written down on the tape T_1 into squares number 1, 2, ..., four adjacent symbols of the word v into one square. Thus, the head on T_1 is moving to the right one square in each fourth tact. The first square (the left end of v) is marked. It is clear that the number α of squares needed for copying of the word v equals

$$\alpha = \left[\frac{|v| + 3}{4}\right], \tag{4}$$

where $|v|$ denotes the length of v and braces denote integral part. The tapes T_2, T_3 are not used during this stage.

2. The automaton is fed with the first symbol $*$.

In this tact, the right end of the word v on T_1 is marked. During following tacts, the automaton is fed with the symbols of the word u_1. Now, the head on T_1 is moving back to the left (one square in each fourth tact) and compares the scanned symbols on T_1 with those of u_1 on the input, examining whether (2) holds for $i = 1$, i.e., whether $u_1 = v^{-1}$.

Once established that $u_1 \neq v^{-1}$, the head on T_1 stays until the tact when the next symbol $*$ appears on the input. If $u_1 = v^{-1}$, the word $v * u_1 = v * v^{-1}$ is accepted. If this word is followed by the symbol $*$, any prolongation of the word is accepted. But, if a symbol $\sigma \in U$ follows, the word $v * v^{-1}\sigma$ is rejected, the automaton leaves the head of T_1 on its place, rejects any prolongation of the word $v * v^{-1}\sigma$ and awaits the next symbol $*$ on input. The tapes T_1, T_2 are not used in this stage.

3. The automaton is fed with the next symbol $*$.

Suppose, the head on T_1 is scanning square number k, $1 \leqq k \leqq \alpha$. On the tapes T_2, T_3, there can be any words at this moment. During following tacts, the automaton is fed with the symbols of the word u_2. Now, the head on T_1 goes to the right, one square per tact, until the right end of the word v is reached. It is clear that the number of tacts needed for it equals

$$\beta = \alpha - k \,. \tag{5}$$

During this stage, having marked the first square, the head on T_2 moves to the right one square in each even tact, and clears the scanned squares, coding in this way the number β.

On the tape T_3, the symbols of the word u_2 are written down from the left to the right.

4. Suppose, the head on T_1 has reached at the previous tact the right end of the word v. Now, in each even tact, the heads on T_1 and T_2 go to the left copying the read symbols (quadruples) from T_1 on T_2 without clearing the tape T_1. On T_3, next input symbols of u_2 are written down from the left to the right. This stage lasts β tacts and is over as soon as the head on T_2 reaches the square that was marked at the beginning of the third stage.

Having finished this stage, the situation on tapes is following (details about exact positions of the heads are omitted):

a) The head of T_1 stays in distance $[\beta/2]$ squares from the right end of the word v.

b) On the tape T_2, the final segment of v is written in $[\beta/2]$ squares.

c) On the tape T_3, the initial segment of u_2 of length 2β is stored (from the left to the right), the head being on the right end of the segment.

The situation on tapes at the end of stage four is illustrated on Fig. 1.

5. During this stage, the relation (2) is examined for $i = 2$, i.e., the relation

$$u_2 = v^{-1} \,. \tag{6}$$

The head on T_2 goes to the right one square in each fourth tact, the head on T_3, moves to the left and the relation (6) is examined for the initial segment of u_2 (of length 2β).

The head of T_1 goes to the left one square in each fourth tact and compares the symbols on T_1 with those on input examining (6) for the remaining part of the word u_2. It can be seen from (4) and (5) that the automaton J is able to establish validity of (6) at the moment when u_2 reaches the same length as v.

As soon as it is for the first time established that $u_2 \neq v^{-1}$, the heads on all tapes stay on their places until the automaton J is fed with another symbol $*$. If $u_2 = v^{-1}$,

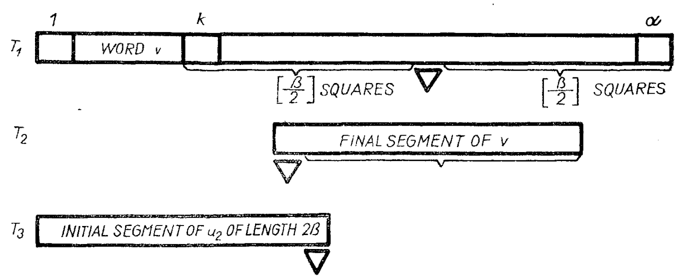

Fig. 1.

J accepts the input word. If $u_2 = v^{-1}$ is followed by the symbol $*$, the automaton J accepts any prolongation of the input word. If $u_2 = v^{-1}$ is followed by a symbol from U, the automaton rejects any prolongation of the word, the heads stay on their places and another symbol $*$ is awaited on input.

6. The automaton is fed with another symbol $*$ and the previous input word was not accepted. Then, the whole process beginning from the third stage is repeated where, of course, the word u_2 is replaced by u_3, respectively by u_4, etc. If the symbol $*$ appears on input during the stage three, four or five, the automaton J follows immediately the instructions of the stage three.

It is clear that the described automaton J recognizes the event B in real time. The theorem is proved.

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REFERENCES

[1] Bečvář, J.: Real-Time and Complexity Problems in Automata Theory. Kybernetika *1* (1965), 6, 475–498.

[2] Hartmanis, J., Stearns, R. E.: Computational complexity of recursive sequences. In: Proc. of the Fifth Annual Symposium on Switching Circuit Theory and Logical Design, held at Princeton University, 1964, 82–90.

O jistém jevu rozeznatelném v reálném čase

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V článku je dokázána existence jevu A rozeznatelného v reálném čase, přičemž jev B obsahující právě obrácená slova jevu A není rozeznatelný v reálném čase.

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