Appendix to the Article "On the Inversion of Moving Averages, Linear Discrete Equalizers and "Whitening" Filters, and Series Summability"

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Some remarks to the preceding art'cle [1] are added.

Hill has been shown in [2] that

$$\text{(1)} \qquad \sum_{k=1}^{\infty} t_{nk}$$

converges for each n and

$$\lim_{n \to \infty} \sum_{k=1}^{\infty} t_{nk} = 1 \, ,$$

$$\text{(2)} \qquad \lim_{n \to \infty} \sum_{k=1}^{\infty} t_{nk}^2 = 0$$

are necessary for the transform $\mathscr{T}$ to have the Borel property, but the condition

$$\text{(3)} \qquad \sum_{k=1}^{\infty} |t_{nk}| = O(1) \quad \text{for} \quad n \to \infty$$

is not necessary. Furthermore, (1), (2), (3), are not sufficient for $\mathscr{T}$ to have the Borel property.

In this connection, we remark that the condition (74) of [1] is necessary for the validity of (2), as it is seen from the proof of Theorem 2 of [1]. But the matrix

$$\text{(4)} \qquad T = \begin{pmatrix} 1, 0, \dots \\ \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, 0, \dots \\ \vdots \end{pmatrix}$$

$\left(\text{in the } n\text{-th row, there is } n \text{ times } 1/n, \dbinom{n}{2} \text{ times } 1/n \text{ and } \dbinom{n}{2} \text{ times } -1/n\right)$ shows that in this case (74) of [1] is no more sufficient for (2).

Various further known transforms are tested as to the Borel property in [3].
The practical meaning of nonregular transforms satisfying (2) for the theory of linear discrete filters is not clear.

REFERENCES

[1] Prouza, L.: On the Inversion of Moving Averages, Linear Discrete Equalizers and „Whitening" Filters, and Series Summability. Kybernetika (1970).
[2] Hill, J. D.: The Borel property of summability methods. Pacif. J. Math. (1951), 399—409.
[3] Hill, J. D.: Remarks on the Borel property. Pacif. J. Math. (1954), 227—242.
[4] Garreau G. A.: A Note on the Summation of Sequences of 0' and 1's. Annals of Math. (1951), 1, 183—185.

VÝTAH

Dodatek k článku „O inverzi klouzavých průměrů, lineárních diskrétních vyrovnávacích a „bělících" filtrech a sumabilitě řad"

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Dodatek obsahuje některé poznámky k předešlému článku [1].

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