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# General Approach to Dynamic Diagnosis Procedures*

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General formulation of dynamic diagnosis concerning technical and medical problems is outlined in heuristic and mathematical terms.

## 1. HEURISTIC APPROACH

In our previous papers [1] and [2] the diagnosis of failures in simple systems was discussed. It was supposed that the changes of the system depend on repairs only, i.e. that the system does not change itself. Such diagnostic procedures can be called *static procedures*. In this paper we shall deal with the generalized model of diagnosis which reflects not only fault-finding procedures for technical systems, but also steps of medical diagnosis and therapy. In this model we suppose that the changes of the system do not depend on repairs or therapy only, but that they have also their origin in the system itself. I.e. we respect such time changes as e.g. ageing of the entire system or of its elements, development of diseases or failure-state, etc. Clearly, the diagnostic procedures for such system must respect such changes of the system, its dynamics, and they will be called *dynamic diagnostic procedures*.

Let us suppose that the given system is entirely described by its *characteristics*. These characteristics may express *microstates* (i.e. the states of individual elements of the system) and *macrostates* (i.e. parameters concerning the entire system or its parts; this group of characteristics contains such parameters as temperature, pressure, voltage, etc.). These characteristics are variable and their values depend on previous values, on type of therapy, on working conditions, etc. The set of all characteristics for given time will be called the *system-state* in this time.

Generally, the system is described by conditional probability distributions on the

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set of all characteristics expressing the probability of occurence of particular values of characteristics in dependence on previous system-states, on types of therapy and diagnosis procedures, on working conditions, etc.

The true system-state is not usually known, i.e. we do not know exactly the values of all state-characteristics, many of which can not be in many cases measured at all. It can be supposed that the true system-state, say $x \in X$ where $X$ is the set of all system-states, is in some manner transformed into some set $s \in \mathbf{S}$ where $\mathbf{S}$ is the system of all subsets of the set $X$. Let us call the set $s$ *primary information*. The transformation of $x$ into $s$ is in general a random transformation characterized by conditional probability distributions depending, roughly speaking, on previous steps of diagnosis and therapy, on professional level of the repairsman or the physician, on his knowledges of a priori probabilities of occurrence of certain diseases or failures, on statesments of the patient or on immediately observable output manifestations of the technical system. In general, the noise is present in the source of the primary informations, too. Let us remark that the primary information, the set $s$ should be as small as possible and should contain the true system-state $x$. However, in many cases the true system-state is not contained in the obtained primary information, i.e. the false primary information is obtained.

On the basis of primary information the *choice of analysis* is done. This choice depends on previous diagnosis and therapy steps, too. There are many types of analysis which can be used, e.g. measuring of some system parameters as temperature, pressure, etc., as the checking of the function of the chosen part of the technical system, X-raying of the chosen part of the patient's body, etc. Clearly, the chosen type of analysis can change the system-state.

The result of the chosen type of analysis and in the same time the primary information, the previous steps of diagnosis and therapy are inputs for information processing giving in general better information than the primary information. The ouput of this information processing will be called the *secondary information*. In the same manner as for primary information, the secondary information is the set $s \in \mathbf{S}$ where $\mathbf{S}$ is the system of all subsets of the set $X$ of all system-states. The information processing is a random transformation, too, and the secondary information, the corresponding set $s$ should be as small as possible and should contain the true system-state $x$, and this set of secondary information should be the subset of the primary information. The secondary information is the output of the diagnostic part of our model, i.e. it is the *diagnosis*.

On the basis of secondary information the *choice of therapy* is done, which depends on previous therapy and diagnosis steps. There are many types of therapy which can be used, e.g. for technical system different types of repairs of individual elements or their replacement by spare parts, etc., and different remedies etc. for medical cases. The chosen type of therapy influences upon the system i.e. it changes in general in a random manner, the system-state.

Thus, our model of dynamic diagnosis is outlined in heuristic terms. The corresponding block-diagram is shown in Fig. 1. All blocks represent random transformations and, with the only exception of the block "analysis", are supposed to have memories. All outlined block-connections have been mentioned in heuristic manner.



Of course, the given block-diagram can be modified in different ways according

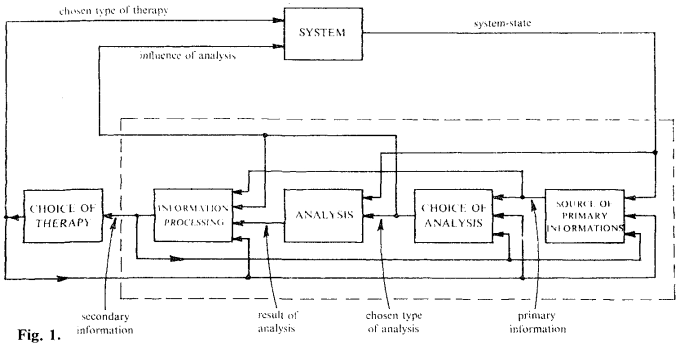


Fig. 1.

to conditions of individual cases. E.g. the connection to the "source of primary information" from the "choice of therapy" can be considered; this connection can reflect patient's aversion to some kind of remedy; similar interpretation helds for the feed-back from the "choice of analysis" to the "source of primary information", etc.

However, the given diagram is supposed to contain all principal parts of the diagnosis procedures.

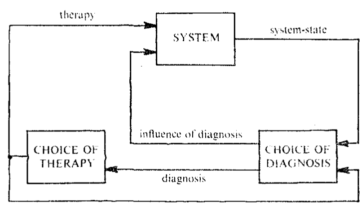


Fig. 2.

We can see that the entire block-diagram contains two decision-blocks only, namely "choice of analysis" and "choice of therapy". All other blocks represent random transformations which can not be immediately controlled at our will. In other words, the optimization of the whole dynamic diagnosis procedure depends on the control of the two decision-blocks only.

Four blocks, namely "source of primary information", "choice of analysis", "analysis", and "information processing", can be aggregated in a single block "choice of diagnosis" as shown in Fig. 1 by dashed lines. In this manner we obtain the reduced block-diagram of our model of the dynamic diagnostic procedures which is shown in Fig. 2 (or in Fig. 3 where symbols of the following sections are used). The main output of the block "choice of diagnosis" is called "diagnosis" here, instead of "secondary information" used in Fig. 1.

It should be remark that the block "choice of diagnosis" contains always two parts, one of which can be immediately controlled whereas the other one depends on its inputs without the possibility of immediate control.

## 2. MATHEMATICAL FORMULATION

Let us denote by

$X$ – the set of all system-states,

$D$ – the set of all possible analysis,

$Z$ – the set of all possible therapies, and

$\mathbf{S}$ – the system of all possible informations.

In the following we shall consider only the case when the sets $X$, $D$, $Z$ are finite and the system $\mathbf{S}$ is therefore finite too. The time will always be discrete, i.e. equal

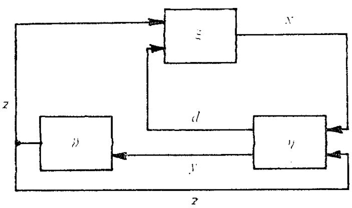


Fig. 3.

to 0, 1, 2, …, $n$. The corresponding values of signals $x$, $y$, $z$ and $d$ for given time $t$ will be denoted by $x(t)$, $y(t)$, $z(t)$ and $d(t)$.

All elements of given dynamic diagnostic diagram in Fig. 1 and Fig. 2 will be assumed to be random transformations, i.e. given by systems of conditional probabilities of outputs for given inputs and past history of signals. Therefore $\xi$ is characterized by probabilities $\xi_0, \xi_1, \ldots, \xi_n$ where

$$\xi_0(x) = P(x(0) = x)$$

and for every $i = 1, 2, \ldots, n$ and $x(0), x(1), \ldots, x(i-1)$; $d(1), \ldots, d(i-1)$; $z(0), z(1), \ldots, z(i-1)$

$$\xi_i(x; x(0), \ldots, x(i-1), d(1), \ldots, d(i-1), z(0), \ldots, z(i-1)) =$$
$$= P(x(i) = x \mid x(0), \ldots, x(i-1), d(1), \ldots, d(i-1), z(0), \ldots, z(i-1)) .$$

The value $x(0)$ will be called the initial system-state. The element $\eta$ will be characterized by the system of conditional probabilities $\eta_0, \eta_1, \ldots, \eta_{n-1}$ where for every $i = 0, 1, 2, \ldots, n-1$ and $y(0), \ldots, y(i-1)$, $d(1), \ldots, d(i-1)$, $x(0), x(1), \ldots, x(i)$, $z(0), \ldots, z(i-1)$

$$\eta_i(y, d; y(0), \ldots, y(i-1), d(1), \ldots, d(i-1), x(0), \ldots, x(i), z(0), \ldots, z(i-1)) =$$
$$= P(y(i) = y, d(i) = d \mid y(0), \ldots, y(i-1), d(1), \ldots, d(i-1), x(0), \ldots, x(i), z(0), \ldots$$
$$\ldots, z(i-1))$$

and the initial value of information $y(0)$ is characterized by the conditional probability

$$\eta_0(y; x(0)) = P(y(0) = y \mid x(0)) .$$

The element $\vartheta$ will be characterized by the system of conditional probabilities $\vartheta_0, \vartheta_1, \ldots, \vartheta_{n-1}$ given for every $i = 0, 1, \ldots, n-1$ and $y(0), y(1), \ldots, y(i), z(0), z(1), \ldots, z(i-1)$ by

$$\vartheta_i(z; z(0), \ldots, z(i-1), y(0), \ldots, y(i)) =$$
$$= P(z(i) = z \mid z(0), \ldots, z(i-1), y(0), \ldots, y(i)) .$$

For every realization $x(0), \ldots, x(n), d(1), \ldots, d(n-1)$ and $z(0), \ldots, z(n-1)$ the quality of the whole dynamic diagnostic procedure is characterized by the given weight function

$$W(x(0), \ldots, x(n), d(1), \ldots, d(n-1), z(0), \ldots, z(n-1)) \geqq 0 .$$

We shall say that the procedure characterized by $\xi$, $\eta^*$ and $\vartheta^*$ is optimum when for these $\eta^*$ and $\vartheta^*$ the expected value of $W$, i.e. the risk, is minimum, i.e.

$$E_{\eta^*\vartheta^*}[W] = \min_{\eta, \vartheta} E_{\eta\vartheta}[W]$$

holds, where the expected value is taken according to the probability measure on $X^n \times D^{n-1} \times Z^n$ given by $\xi$, $\eta$ and $\vartheta$.

The determination of the optimum $\eta^*$ and $\vartheta^*$ in concrete cases is very difficult and tedious. In the following we shall limit ourselves on a special case of this general one.

## 3. SPECIAL CASE

In this section we shall study a special case of the block diagram shown in Fig. 3, where the block $\eta$ is given in the form shown in Fig. 4.

The element $\sigma$ in this case is supposed to be characterized by the system of conditional probabilities $\sigma_0, \sigma_1, \ldots, \sigma_{n-1}$ where for every $i = 0, 1, \ldots, n-1$ and $y(0), \ldots$

$\ldots, y(i-1), d(1), \ldots, d(i), x(0), \ldots, x(i), z(0), \ldots, z(i-1)$

$$\sigma_0(y; x(0)) = P(y(0) = y \mid x(0)),$$

$$\sigma_i(y; y(0), \ldots, y(i-1), x(0), \ldots, x(i), d(1), \ldots, d(i), z(0), \ldots, z(i-1)) =$$
$$= P(y(i) = y \mid y(0), \ldots, y(i-1), x(0), \ldots, x(i), d(1), \ldots, d(i), z(0), \ldots, z(i-1)).$$

The element $\delta$ is characterized by the system of conditional probabilities $\delta_1, \delta_2, \ldots$

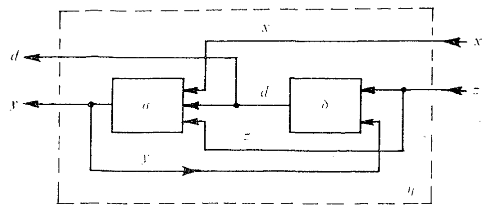


Fig. 4.

$\ldots, \delta_{n-1}$ where for every $i = 1, 2, \ldots, n-1$, $d(1), \ldots, d(i-1)$ and $z(0), \ldots, z(i-1)$, $y(0), \ldots, y(i-1)$

$$\delta_i(d; d(1), \ldots, d(i-1), z(0), \ldots, z(i-1), y(0), \ldots, y(i-1)) =$$
$$= P(d(i) = d \mid d(1), \ldots, d(i-1), z(0), \ldots, z(i-1), y(0), \ldots, y(i-1)).$$

Then obviously for every $i = 1, 2, \ldots, n-1$ and $x(0), \ldots, x(i)$, $z(0), \ldots, z(i-1)$, $d(1), \ldots, d(i-1)$ and $y(0), \ldots, y(i-1)$

$$\eta_i(y, d; y(0), \ldots, y(i-1), d(1), \ldots, d(i-1), x(0), \ldots, x(i), z(0), \ldots, z(i-1)) =$$
$$= \delta_i(d; d(1), \ldots, d(i-1), z(0), \ldots, z(i-1), y(0), \ldots, y(i-1)) .$$
$$. \sigma_i(y; y(0), \ldots, y(i-1), x(0), \ldots, x(i), d(1), \ldots, d(i), z(0), \ldots, z(i-1)).$$

The optimization of $\eta$ corresponds now to the optimization of $\delta$, because the element $\sigma$ is constant, given a priori and therefore uncontrollable.

## 4. APPLICATION TO FAULT-FINDING PROCEDURES

In previous papers [1] and [2] the determination of the optimum searching procedure for finding all defective elements of a non-operating system was solved. We shall show now the application of the general model discussed in previous sections to this case.

First of all we shall consider that $X$ is the set of all $N$-tuples of zeros and ones, i.e. 

$$X = \{0, 1\}^N ,$$
$$Z = \{0, 1, 2, \ldots, N\}$$

and

$$D = \{0, 1, 2, \ldots, N\}$$

Further let $\mathbf{S}$ be the system of all subsets of the set $X$.

When $x \in X$, then $x = (x^1, x^2, \ldots, x^N)$ and $x^j = 0$ when the $j$-th element is good,

$x^j = 1$ when the $j$-th element is defective.

$z = j$ means the repair of the $j$-th element ($z = 0$ – no repair is made),

$d = j$ means the analysis of the $j$-th element or the measurement immediately behind the $j$-th element ($d = 0$ denotes the measurement of the whole system).

For every $A \in \mathbf{S}$ and $d \in D$ let us denote by $\mathbf{A}_d$ the partition of $A$ into $n(d)$ disjoint sets $\{A_1(d), \ldots, A_{n(d)}(d)\}$. If $x \in A$ then there exists such an index $j$ that $x \in A_j(d)$.

Further let us suppose that $x(0)$ is the random variable with probability distribution $\xi_0$ and for every $i = 1, 2, \ldots, n$

$$x(i) = x(i-1) \quad \text{when} \quad z(i-1) = 0$$

and

$$x(i) = (x^1(i-1), \ldots, x^{j-1}(i-1), 0, x^{j+1}(i-1), \ldots, x^N(i-1)) \quad \text{for} \quad z(i-1) = j .$$

The random variable $y(0)$ is an element of $\mathbf{S}$ and for every $i = 1, 2, \ldots, n$

$$y(i) = A_j(d(i)) ,$$

where $A_j(d(i))$ is such an element of the partition $\mathbf{A}_{d(i)}$ of $y(i-1)$ that $x(i)$ is its element.

We shall suppose that

$$z(i) = j \quad \text{when} \quad y(i) \subset \{x : x^j = 1\} ,$$
$$= 0 \quad \text{for other cases} .$$

Further we shall suppose that the weight function $W$ is given by the relation

$$W(x(0), \ldots, x(n), d(1), \ldots, d(n-1), z(0), \ldots, z(n-1)) = \sum_{i=1}^{n-1} w(d(i)) .$$

Then the optimum diagnostic procedure is characterized by the system of conditional probabilities, $\delta_1, \delta_2, \ldots, \delta_n$ for which the expected value of $W$ is minimum. However, this corresponds to the consideration given in [2] and it can be shown that the optimum diagnostic procedure is regular and homogeneous and given by the function $\Delta$ transforming $\boldsymbol{S}$ into $D$ for which the corresponding risk $\varrho$ fulfills the Bellman equation

$$\varrho(y_0) = \min_{d \in D} \left[ w(d) + \sum_y \varrho(y) \, P(y) \mid y_0) \right],$$

where the sum is taken over the partition of $y_0$ given by $d$. Then $\Delta(y_0) = d_0$ for which

$$\varrho(y_0) = w(d_0) + \sum_{y \in y_0(d_0)} \varrho(y) \, P(y \mid y_0) \, .$$

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VÝTAH

## Obecný přístup k dynamickým diagnostickým procedurám

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Dynamickými diagnostickými procedurami se v tomto článku chápou takové procedury, při nichž se uvažuje vývoj sledovaného systému i během provádění analýz a zásahů. Obecné schéma (obr. 1) je vysvětleno intuitivně a je dán jeho matematický popis vycházející z náhodných transformací. Je ukázán přechod na statický případ vyhledávání poruch v systému uvedený v [2].

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