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On a Classification of Context-Free Languages

JOZEF GRUSKA

The set E of strings is said to be definable (strongly definable) if there is a context-free grammar G such that E is the set of all terminal strings generated from the initial symbol (from all nonterminal symbols) of G. The classification of definable and strongly definable sets in dependence on minimal number of nonterminal symbols needed for their generation is given.

1. INTRODUCTION AND SUMMARY

It is well-known that a context-free language can be generated by grammars with a different number of nonterminal symbols (metasymbols). In this paper we shall investigate context-free languages in dependence on a minimal number of metasymbols needed for their generation. It gives a classification of languages – $\mathscr{D}_n$ will denote the class of context-free languages which may be generated by a grammar with n metasymbols but not by a grammar with a smaller number of metasymbols – and we shall investigate the properties of languages from separate classes.

In the rest of this paper context-free languages will be called – similarly as in papers [3, 4] – definable sets. Therefore, to every grammar G there is associated the definable set – $L(G)$ – defined as the set of all terminal strings which are generated from the initial symbol of G. In preceding paper [4], moreover, the set $L_s(G)$ – defined as the set of all terminal strings which are generated from all metasymbols of G – was associated to grammar G. These sets were called strongly definable. In this paper also a similar classification of strongly definable sets is given.

Strongly definable sets are investigated in detail in paper [4]. We shall also use notations and definitions of that paper. The reader should be familiar with Section 1, [4].

The technical results achieved in this paper are as follows. It is shown that for every integer n the class $\mathscr{D}_n$ is non-empty. (Section 2). (This is true even when we consider only sets in a given alphabet of just two (terminal) symbols but not in an alphabet of just one symbol (Section 3).) The sets from $\mathscr{D}_n$ are called n-definable. Some properties of n-definable sets are investigated in Section 4 and Section 5 but there are still many open problems in this area. Similar results are proved for strongly definable sets.

2. CLASSIFICATION IN ALPHABET CONSISTING OF JUST TWO SYMBOLS

For grammars with one metasymbol the concepts "definable set" and "strongly definable set" are obviously equivalent. It is natural to inquire as to whether there is a strongly definable set which is not generated by a grammar with one metasymbol. In the sequel we shall prove that such set exists and, moreover, that for every $n \geqq 1$, there is a strongly definable set which is not strongly generated by a grammar with a less number of metasymbols than n. Similar results will be proved for definable sets, sequentially definable sets*) and regular sets. All this is true even for alphabets consisting of just two terminal symbols.

Let $\mathscr{A}$ be an alphabet. Denote $\mathscr{D}_n^{\mathscr{A}}$ $(\mathscr{S}_n^{\mathscr{A}})$, $n \geqq 1$ the family of all definable (strongly definable) sets in $\mathscr{A}$ which are generated (strongly generated) by a grammar with n metasymbols but are not generated (not strongly generated) by a grammar with a smaller number of metasymbols. The sets from $\mathscr{D}_n^{\mathscr{A}}$ $(\mathscr{S}_n^{\mathscr{A}})$ will be called n-definable (n-strongly definable) in $\mathscr{A}$. By $\mathscr{D}_n$ $(\mathscr{S}_n)$ we shall denote the family of all n-definable (n-strongly definable) sets in all alphabets.

In the remaining part of this section let $\mathscr{A} = \{a, b\}$. We shall prove:

Theorem 1. $\mathscr{D}_n^{\mathscr{A}} \neq \Lambda$ *for* $n \geqq 1$

and

Theorem 2. $\mathscr{S}_n^{\mathscr{A}} \neq \Lambda$ *for* $n \geqq 1$.

Denote, for every $n \geqq 1$,

$$M_n = \{ab\}^\infty \cup \{a^2b\}^\infty \cup \ldots \cup \{a^nb\}^\infty = \bigcup_{i=1}^{n} \{a^ib\}^\infty . \tag{1}$$

At first we shall prove:

Lemma 1. $M_1 \in \mathscr{D}_1^{\mathscr{A}}$ *and* $M_n \in \mathscr{D}_{n+1}^{\mathscr{A}}$ *if* $n \geqq 1$.

Proof. The case $n = 1$ is trivial. Now denote G^{n+1} the grammar defined by the set of rules

$$\begin{gathered} I_{n+1} \to I_i\,, \quad i = 1, 2, \ldots, n\,, \\ I_i \to I_i a^i b\,, \quad I_i \to a^i b\,, \quad i = 1, 2, \ldots, n \end{gathered} \tag{2}$$

with the initial symbol I_{n+1}. Obviously $L(G^{n+1}) = M_n$ and G^{n+1} consists of just

* For the concept "sequentially definable set" see [3].

$(n+1)$ metasymbols. Now let $G = \langle \mathscr{A}, V_n, \mathscr{R}, S \rangle$ be a grammar with a minimal number of metasymbols and such that

$$L(G) = M_n \tag{3}$$

then $G(T) \neq \Lambda$ if $T \in V_N$ because in the opposite case there would exist a grammar with a smaller number of metasymbols satisfying (3). Moreover, if $T \in V_N$, $T \neq S$ than there are t and j such that $T \to t$, $t(j) = T$ because in the opposite case T would be a reductible metasymbol (see [2]) and we could construct a grammar with a less number of metasymbols satisfying (3). Similarly we may assume that $T \to T$ for no $T \in V_N$ and that for every $T \in V_N$ there are x and j such that $S \Rightarrow x$ and $x(j) = T$.

At first we shall prove that

(4) if $T \in V_N$, $T \neq S$, then there are uniquely determined integers i_1, s, i, i_2 such that $1 \leqq i \leqq n$, $0 \leqq i_1 \leqq n$, $0 \leqq i_2 \leqq n$, $0 \leqq s \leqq 1$, $G(T) \subset a^{i_1} b^s \{a^i b\}^\infty a^{i_2}$ and either $s = 0 = i_1$ or $s = 1$.

(We shall write $i = \varphi(T)$).

Indeed, it can be readily seen that there are strings $P_1, P_2, Q_1, Q_2 \in \mathscr{A}^\infty$ such that $T \Rightarrow P_1 T Q_1$, $S \Rightarrow P_2 T Q_2$ and $P_1 Q_1 \neq \varepsilon$. Then $S \Rightarrow P_2 P_1^j T Q_1^j Q_2$ for every integer j. Let $x \in G(T)$. Then $S \Rightarrow P_2 P_1 x Q_1 Q_2 = t \in M_n$ and $t = (a^i b)^j$ for suitable i, j. Thus, there are uniquely determined integers $a_0, a_1, \bar{a}_1, \bar{a}_2, s_0, s_1, \bar{s}_1, \bar{s}_2, b_0, b_1, \bar{b}_1, b_2, \bar{b}_2, c_0, c_1, \bar{c}_1, c_2$ such that

$$P_2 = (a^i b)^{b_2} a^{c_2}, \quad P_1 = a^{a_1} b^{s_1} (a^i b)^{b_1} a^{c_1}, \quad x = a^{a_0} b^{s_0} (a^i b)^{b_0} a^{c_0},$$

$$Q_1 = a^{\bar{a}_1} b^{\bar{s}_1} (a^i b)^{\bar{b}_1} a^{\bar{c}_1}, \quad Q_2 = a^{\bar{a}_2} b^{\bar{s}_2} (a^i b)^{\bar{b}_2},$$

and, moreover,

$$\begin{aligned}
&\text{either} \quad s_1 = 0 = a_1 = c_2 \quad \text{or} \quad s_1 = 1, \quad a_1 + c_2 = i, \\
&\text{either} \quad s_0 = 0 = a_0 = c_1 \quad \text{or} \quad s_0 = 1, \quad a_0 + c_1 = i, \\
&\text{either} \quad \bar{s}_1 = 0 = \bar{a}_1 = c_0 \quad \text{or} \quad \bar{s}_1 = 1, \quad \bar{a}_1 + c_0 = i, \\
&\text{either} \quad \bar{s}_2 = 0 = \bar{a}_2 = \bar{c}_1 \quad \text{or} \quad \bar{s}_2 = 1, \quad \bar{a}_2 + \bar{c}_1 = i.
\end{aligned}$$

Let $P_1 \neq \varepsilon$ (the case $Q_1 \neq \varepsilon$, $P_1 = \varepsilon$ is investigated similarly). Then there is a j such that $P_2 P_1^j x Q_1^j Q_2 \in M_n$ and the length of P_1^j is more than $2n + 1$. Hence we may assume without loss of generality that $b_1 \geqq 2$; it means that for every $x_0 \in G(T)$ we have $P_2 P_1 x_0 Q_1 Q_2 = (a^i b)^{j_0}$ for a suitable j_0. Hence $x_0 = a^{a_0} b^{s_0} (a^i b)^k a^{c_0}$ for a suitable k, whereby $a_0 = i - c_1$, $s_0 = 1$ if $c_1 \neq 0$ and $a_0 = 0 = s_0$ otherwise, $c_0 = i - \bar{a}_1$ if $\bar{s}_1 \neq 1$ and $c_0 = 0$ if $\bar{s}_1 = 1$. This completes the proof of (4).

Now suppose that there is a t such that $S \Rightarrow t$ and $t(k) = S$ for some k. Then there are strings $P, Q \in \mathscr{A}^\infty$ such that $PQ \neq \varepsilon$ and $S \Rightarrow PSQ$. That is $S \Rightarrow P^{3n+4} S Q^{3n+4}$, too. Assume that $P \neq \varepsilon$. (The case $P = \varepsilon \neq Q$ is investigated similarly). Then there are integers i, j and strings P_1, $Q_1 \in \mathscr{A}^\infty$ such that $P^{3n+4} = P_1(a^ib)^j Q_1$, $j > 3$. Since $L(G) = M_n$, there is $i_1 \neq i$ such that $(a^ib)^3 \in L(G)$. But then $(a^ib)^j$, $(a^{i_1}b)^3$ are substrings of the string $P^{3n+4}(a^{i_1}b)^3 Q^{3n+4}$ which contradicts the definition of M_n. Hence $S \Rightarrow t$ implies $t(j) = S$ for no j.

Now let n_0 be the number of metasymbols of G. Obviously $1 \leqq n_0 \leqq n + 1$ and suppose that $n_0 \leqq n$. Then there is an i such that $1 \leqq i \leqq n$ and $\varphi(T) = i$ for no $T \in V_N - \{S\}$. If $S \to t$ and $t(k) \in V_N$ for some k then, with regard to the definition of M_n, $\{x; t \Rightarrow x \in \mathscr{A}^\infty\} \subset \{a^{\varphi(t(k))}b\}^\infty$. Hence the set $\{x; x \in \{a^ib\}^\infty, S \to x\}$ is finite which contradicts the definition of M_n. Thus, $n_0 = n + 1$. This completes the proof of Lemma 1.

Lemma 2. *Denote* $N_2 = \{a\}^\infty \cup \{b\}$. *Then* $N_2 \in D_2^{\mathscr{A}}$.

Proof. Let G be a grammar defined by rules

$$B \to A\,, \quad B \to b\,, \quad A \to Aa\,, \quad A \to a\,, \tag{5}$$

with the initial symbol B. Then $L(G) = N_2$. Now suppose on the contrary that there is a grammar G_1 with one metasymbol S such that $L(G_1) = N_2$. Then $S \to b$ has to be the rule of G_1. Moreover, there is a t such that $S \to t$, $t = PSQ$ where $PQ \in \in \{\mathscr{A} \cup \{S\}\}^\infty$. Thus, there is a string x such that $S \Rightarrow x \in \mathscr{A}^\infty$; x has at least two symbols and $x(i) = b$ for some i; this contradicts the definition of N_2 and therefore $N_2 \in D_2^{\mathscr{A}}$. This completes the proof of Lemma 2.

Theorem 1 is now an immediate consequence of Lemma 1 and Lemma 2.

To prove Theorem 2 we proceed as follows: If a set Z is strongly generated by a grammar G with n metasymbols then obviously there is a grammar G_1 with $n + 1$ metasymbols such that $L(G_1) = Z$. The set M_n (see (8)) is strongly generated by the grammars with rules:

$$S_i \to S_i a^i b\,, \quad S_i \to a^i b\,, \quad i = 1, 2, \ldots, n$$

which has n metasymbols. If M_n would be strongly generated by a grammar G_1 with a smaller number of metasymbols than n, then, as we have just shown, there would exist a grammar G_2 consisting of $j \leqq n$ metasymbols and such that $L(G_2) = = M_n$, contrary to our previous result $M_n \in \mathscr{D}_{n+1}^{\mathscr{A}}$. Hence $M_n \in S_n^{\mathscr{A}}$. This completes the proof of Theorem 2.

We have actually proved much more. By (2), the set M_n, $n > 1$ is sequentially definable and, moreover, regular and is generated by a sequential grammar with $n + 1$ metasymbols but not by a sequential grammar (and also non-self-embedding grammar) with a less number of metasymbols. Similarly the set $N_2(M_1)$ is regular and is generated by a non-self-embedding grammar with two (one) metasymbols. Hence the same result as in Theorem 1 is valid for a family of sequentially definable sets (even when we consider only sequential grammars) and also for a family of regular sets.

By a method similar to that given in this section it can be shown that if $n > 1$, $1 \leqq i_1 < i_2 < \ldots < i_n$ and

(6) $$M_n(i_1, \ldots, i_{n-1}) = \{a^{i_1}b\}^\infty \cup \ldots \{a^{i_{n-1}}b\}^\infty$$

then

(7) $$M_n(i_1, \ldots, i_{n-1}) \in \mathscr{D}_n^{\mathscr{A}}(\in \mathscr{S}_{n-1}^{\mathscr{A}}) .$$

Moreover, if Q_{i_n} is a finite non-empty subset of $\{a^{i_n}b\}^\infty$ and

$$N_n(i_1, \ldots, i_n) = M_n(i_1, \ldots, i_{n-1}) \cup Q_{i_n}$$

then

(8) $$N_n(i_1, \ldots, i_n) \in \mathscr{D}_n^{\mathscr{A}}(\in \mathscr{S}_n^{\mathscr{A}}) .$$

Finally, there holds

(9) $$P_n(i_1, \ldots, i_{n-1}) = \{x;\, x = c^j y c^j, j \geqq 0, y \in M_n(i_1, \ldots, i_{n-1}), c \text{ is a letter}\} \in \mathscr{D}_n$$

3. CLASSIFICATION IN AN ALPHABET CONSISTING OF JUST ONE SYMBOL

Now we shall investigate a similar problem as in the previous section but for the case that the alphabet consists of just one symbol.

Theorem 3. *Every definable (strongly definable) set in the alphabet consisting of just one symbol is generated (strongly generated) by a grammar with two metasymbols. Moreover, there is a definable (strongly definable) set which cannot be generated (strongly generated) by a grammar with one metasymbol.*

Proof. Let M be a (strongly) definable set in $\mathscr{A} = \{a\}$. By Corrollary 2, [3], $\{n;\, a^n \in M\}$ is an ultimately periodic set of integers, i.e. if $\{x_n\}_{n \geqq 1}$ is the sequence of its elements ordered by magnitude and the sequence $\{y_n\}_{n \geqq 1}$ is defined by $y_1 = x_1$, $y_{i+1} = x_{i+1} - x_i$, $i = 1, 2, \ldots$, then there are n_0 and p such that $y_{m+p} = y_m$ for all $m \geqq n_0$. Let G be the grammar defined by rules:

(10)
$$\begin{aligned} S &\to a^{x_i}, \quad i = 1, 2, \ldots, n_0 ,\\ S &\to S_1 ,\\ S_1 &\to a^{x_i}, \quad i = n_0 + 1, \ldots, n_0 + p ,\\ S_1 &\to S_1 a^j \end{aligned}$$

where

$$j = \sum_{k=n_0+1}^{n_0+p} y_k = x_{n_0+p} - x_{n_0} .$$

Then $L(G) = M = L_s(G)$ and the first assertion of Theorem 3 is proved. Now we prove that the set $R = \{a^3\}^\infty \cup \{a^2\}$ is not (strongly) generated by a grammar with one metasymbol.

Assume on the contrary that there is a grammar G_1 with one metasymbol S_0 such that $L(G_1) = R$. Since R is an infinite set, there are t and j such that $S_0 \to t$, $t(j) = S_0$. We may assume that $t = S_0^{r_0} a^{r_1}$ where $r_0 \geqq 1$, $r_1 \geqq 0$, $r_0 + r_1 > 1$. (Since G_1 consists of just one symbol, ordering of symbols in t is obviously irrevelant). As the strings a^2 and a^3 belong to R, we have $t \Rightarrow a^{3(r_0 - 1) + 3 + r_1} \in R = L(G_1)$, $t \Rightarrow$ $\Rightarrow a^{3(r_0 - 1) + 2 + r_1} \in R$, whereby $3(r_0 - 1) + 2 + r_1 = 3r_0 + r_1 - 1 \geqq 3$ contrary to the definition of R. Hence the assumption that G_1 has only one metasymbol yields a contradiction and the proof of Theorem 3 is completed.

Remark. In an alphabet consisting of just one symbol the concepts: definable set, sequentially definable set and regular set are, by [3], equivalent. Therefore, Theorem 3 is valid if we replace the word "definable" by the words "sequentially definable" or "regular".

4. CLOSURE PROPERTIES OF n-(STRONGLY) DEFINABLE SETS

Let $n \geqq 1$ be an integer and U_1, U_2 some n-(strongly) definable sets. What can we say about $U_1 \cap U_2$, $\overline{U}_1$ (complement)? Are $U_1 \cap U_2$, $\overline{U}_1$ definable sets? As to the intersection we have

Lemma 3. *For every n there are U_1, $U_2 \in \mathscr{D}_n$ ($\in \mathscr{S}_n$) such that $U_1 \cap U_2$ is not a definable set.*

Proof. We shall not carry out the proof in detail, only the main idea will be sketched. Consider grammars G_1 and G_2 defined as in (11) and (12)

$$A \to aaAc\,, \quad A \to bAc\,, \quad A \to bc\,, \tag{11}$$

$$A \to aAcc\,, \quad A \to aAb\,, \quad A \to ab\,. \tag{12}$$

Then $L_s(G_1) = L(G_1) \in \mathscr{D}_1 = \mathscr{S}_1$, $L_s(G_2) = L(G_2) \in \mathscr{D}_1 = \mathscr{S}_1$ and $L(G_1) \cap L(G_2) =$ $= L_s(G_1) \cap L_s(G_2)$ is not definable (see [3], p. 381) and hence neither strongly definable. Now let $n > 1$ and $1 \leqq i_1 < i_2 < \ldots < i_{2n-2}$. By using a similar method as in Section 2 we can show that if $Q_{i_{n-1}}$ ($Q_{i_{2n-2}}$) is a finite subset of $\{e^{i_{n-1}}d\}^\infty$ ($\{e^{i_{2n-2}}d\}^\infty$) then

$$U_1 = L(G_1) \cup \{e^{i_1}d\}^\infty \cup \ldots \cup \{e^{i_{n-2}}d\}^\infty \cup Q_{i_{n-1}} \in \mathscr{D}_n \, (\in \mathscr{S}_n)\,,$$

$$U_2 = L(G_2) \cup \{e^{i_n}d\}^\infty \cup \ldots \cup \{e^{i_{2n-3}}d\}^\infty \cup Q_{i_{2n-2}} \in \mathscr{D}_n \, (\in \mathscr{S}_n)\,.$$

But then $U_1 \cap U_2 = L(G_1) \cap L(G_2)$ and hence $U_1 \cap U_2$ is not definable.

It is an open question whether for every i there is a $U \in \mathscr{D}_i$ ($\in \mathscr{S}_i$) such that the complement of U is not definable (although, by Theorem 2, [4] there exists some i with this property).

Let $n \geqq 1$ and $U_1, U_2 \in \mathcal{D}_n (\in \mathcal{S}_n)$. It is readily seen that $U_1 \cup U_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_k)$ for some $j \leqq 2n + 1$ $(k \leqq 2n)$. It is not difficult to prove with respect to (7), (8) and (9) that for each $j \in \langle n, 2n + 1 \rangle$ $(k \in \langle n, 2n \rangle)$ there are $U_1, U_2 \in \mathcal{D}_n$ $(\in \mathcal{S}_n)$ such that $U_1 \neq U_2$ and $U_1 \cup U_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_k)$. Indeed, let $1 \leqq i_1 < i_2 < \ldots < i_{2n+1}$. If $j \in$ $\in \langle n, 2n - 1 \rangle$ and $U_1 = M_n(i_1, \ldots, i_{n-1}) \cup Q^{(1)}$, $U_2 = M_n(i_{j-n+1}, \ldots, i_{j-1}) \cup Q^{(2)}$, where $Q^{(1)}, Q^{(2)}$ are different non-empty finite subsets of $\{a^{i_{2n+1}}b\}^\infty$, then (see (8)), $U_1, U_2 \in \mathcal{D}_n$ $(\in \mathcal{S}_n)$ and $U_1 \cup U_2 = M_j(i_1, \ldots, i_{j-1}) \cup Q_1 \cup Q_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_j)$. If $j =$ $= 2n$ and $U_3 = P_n(i_{j-n+1}, \ldots, i_{j-1})$, then $U_3 \in \mathcal{D}_n$ (see (9)) and $U_1 \cup U_3 \in \mathcal{D}_{2n}$. If $U_4 = P_n(i_1, \ldots, i_{n-1})$, then $U_4 \in \mathcal{D}_n$ and $U_3 \cup U_4 \in \mathcal{D}_{2n+1}$. The proof of this assertions is not difficult but cumbersome and ·it is therefore omitted. Moreover $U_1 = M_{n+1}(i_1, \ldots, i_n) \in \mathcal{S}_n$, $U_2 = M_{n+1}(i_{n+1}, \ldots, i_{2n}) \in \mathcal{S}_n$, and $U_1 \cup U_2 =$ $= M_{2n+1}(i_1, \ldots, i_{2n}) \in \mathcal{S}_{2n}$.

Like in the proof of Lemma 2 we can show that the set $U_1 = \{a^n b^m; n \geqq m > 1\} \cup$ $\cup \{ab^2\} \in \mathcal{D}_2 (\in \mathcal{S}_2)$ and also $U_2 = \{a^n b^m; m \geqq n > 1\} \cup \{a^2 b\} \in \mathcal{D}_2 (\in \mathcal{S}_2)$. But $U_1 \cup U_2 \in \mathcal{D}_1$ $(\in \mathcal{S}_1)$. Therefore there are n, j and U_1, U_2 such that $j < n$, $U_1, U_2 \in$ $\in \mathcal{D}_n (\in \mathcal{S}_n)$ and $U_1 \cup U_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_j)$. But it is an open question whether for arbitrary $n > j \geqq 1$ there are $U_1, U_2 \in \mathcal{D}_n$ $(\in \mathcal{S}_n)$ such that $U_1 \cup U_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_j)$.

As to intersection, it is easy to prove that for every n and j such that $n \geqq j \geqq 1$ there are different sets $U_1, U_2 \in \mathcal{D}_n$ $(\in \mathcal{S}_n)$ such that $U_1 \cap U_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_j)$. Indeed, let $1 \leqq i_1 < i_2 < \ldots < i_{2n}$ and $U_1 = M_n(i_1, \ldots, i_{n-1}) \cup Q_1$, $U_2 = M_n(i_{n-j+1}, \ldots$ $\ldots, i_{2n-j-1}) \cup Q_2$ where $Q_1 \subset Q_2$ and Q_1, Q_2 are non-empty different finite subsets of $\{a^{i_{2n}}b\}^\infty$. Then $U_1 \cap U_2 = M_{j-1}(i_{n-j+1}, \ldots, i_{n-1}) \cup Q_1 \in \mathcal{D}_j$ $(\in \mathcal{S}_j)$ and $U_1, U_2 \in$ $\in \mathcal{D}_n$ $(\in \mathcal{S}_n)$.

It is an open question whether for every n and $j > n$ there are $U_1, U_2 \in \mathcal{D}_n$ $(\in \mathcal{S}_n)$ such that $U_1 \cap U_2 \in \mathcal{D}_j$ $(\in \mathcal{S}_j)$.

It is readily seen that if $M \in \mathcal{D}_n$ then $M^\infty \in \mathcal{D}_k$ for a $k \leqq n$. (If $G = \langle V_T, V_N, \mathcal{R}, S \rangle$ is a grammar such that $L(G) = M$, and if we put $G_1 = \langle V_T, V_N, \mathcal{R} \cup \{S \to SS\}, S \rangle$, then $L(G_1) = M^\infty$). Hence if $M \in \mathcal{D}_1$, then also $M^\infty \in \mathcal{D}_1$. Moreover, for every $n > 1$ there is a $M \in \mathcal{D}_n$ such that $M^\infty \in \mathcal{D}_n$. Indeed, the set $M_n(1, \ldots, n-1) \in \mathcal{D}_n$ (see (7)) and $M_n^\infty \in \mathcal{D}_n$, too. The proof of this assertion and of the assertion given below is omitted because it is similar to the proof of Lemma 1. It is an open question whether for every $1 \leqq j \leqq n$ there is a $U \in \mathcal{D}_n$ such that $U^\infty \in \mathcal{D}_j$.

If $M \in \mathcal{S}_n$, $n > 1$ then (see Section 3, [4]), $M^\infty \in \mathcal{S}_k$ for a $k \leqq n + 1$. The set $M_{n+1}(1, \ldots, n)$ defined by (6) belongs to $\mathcal{S}_n$ and $M_{n+1}^\infty \in \mathcal{S}_{n+1}$, too. But it is also an open question whether for every $1 \leqq j \leqq n$ there is a $U \in \mathcal{S}_n$ such that $U^\infty \in \mathcal{S}_j$.

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VÝŤAH

O jednej klasifikácii bezkontextových jazykov

JOZEF GRUSKA

Množina reťazcov E sa nazýva definovateľnou (silne definovateľnou) ak existuje bezkontextová gramatika G taká, že E je množinou všetkých terminálnych reťazcov odvodených z daného neterminálneho symbola (zo všetkých neterminálnych symbolov) gramatiky G.

V práci je dané rozdelenie definovateľných množín (tj. bezkontextových jazykov) a silne definovateľných množín do tried podľa minimálneho počtu neterminálnych symbolov potrebných k ich odvodeniu. Vyšetrujú sa niektoré vlastnosti množín z jednotlivých tried.

RNDr. Jozef Gruska CSc., Matematický ústav SAV, Bratislava, ul. Obrancov mieru 41.