Neuron Model with Electronic Treshold Circuit

Petr Hiršl

A circuit was proposed on the base of a graphically solved neuron model [1], this circuit having properties which are analogous to the properties of a motoneuron from the point of view of its external electrical behaviour. This approach permits to solve the model as a voltage-frequency converter, and vice versa.

INTRODUCTION

When designing an electronic neuron model, one starts from the graphic solution of the model [1]. One of the requirements to be met by a technical embodiment of the model is maximum plasticity of its properties permitting a ready change in the parameters and the possibility of adding further circuits for forming the paradoxical phase and adaptivity. Generally speaking, the described model represents a further development of some suggestions contained in [2], [3] and [4].

It is taken into account that a neuron is neither a digital nor an analog element but a hybrid one whose function combines both previous principles. It is obvious that a system which would represent a working model of a neuron should comprise at least two fundamental units. The first is a voltage-frequency converter simulating the function of the soma of a neuron (and of dendrites). The second unit which represents a synapse is a frequency-voltage converter, substantially an integrator formed by an $R - C$ circuit (I in Fig. 1). A more detailed analysis of the voltage integrator (I) is given in [5] (on p. 111). The axon whose function can be simply illustrated by a delay in the transmitted signal is not included in Fig. 1.

VOLTAGE — FREQUENCY CONVERTER

Let us analyse a voltage-frequency converter which simulates well neuron properties and permits also an easy change in the parameters with a view toward the closest approach to actual neuron properties. Instead of a multistable multivibrator or

blocking oscillator currently employed in neuron models there is used a Schmitt
circuit S which cannot serve as a generator of a pulse sequence without further addi-
tional circuits forming a feedback loop (shaping unit TO, inverter IN, summation
circuit $\sum$). The wave form of the feedback voltage determines directly the shape of
the refractory period. The feedback circuit determines the shape of the conversion
of the input signal $x(t)$ to the frequency of the output pulses f_y

(1)
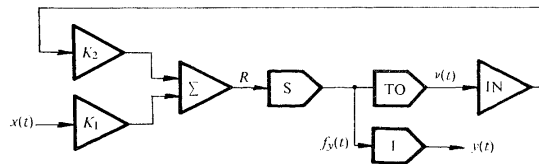$$f_y = \psi_f[\sum_i x_i(t)\, w_i] = \psi_f[x(t)]\,.$$

Fig. 1. Simple functional diagram of neuron model.

Expression (1) is analogous to expression (5) in [1].

The input signal of the converter $x(t) = \sum_i x_i(t)\, w_i$, that is the complete postsynap-
tical signal is amplified by the input circuit K_1-times and it is summed in the summa-
tion circuit with a feedback signal. The output signal R reaches the input of the
circuit S.

When meeting the condition

(2)
$$R \geqq P = hK_1$$

where P — threshold of Schmitt circut, h — threshold of neuron model, the circuit S
changes from state "O" into state "1", and its output signal is transformed by the
shaping circuit TO into a signal $v(t)$. The inverter reverses the polarity of the signal
$v(t)$ which is multiplied by a constant K_2 and then summed by the summation circuit
with the signal $x(t)$. Assuming TO to be a simple $R-C$ integrating circuit and that
the signal

$$x(t) \geqq P$$

is a short pulse, $v(t)$ in our model has an exponential rise with a time constant τ_n,
and an exponential drop with a time constant τ_s, and it is possible to meet the require-
ment

(3)
$$\tau_n \ll \tau_s\,.$$

Therefore, across the input of the Schmitt circuit there operates a signal

(4)
$$R = x(t)\, K_1 - v(t)\, K_2\,.$$

Generally, if one uses a more complicated form of the circuit TO, one can achieve that the shape of the dropping part of the signal $v(t)$ differs from an exponential curve, and thus also a change in the function $\psi_f(1)$.

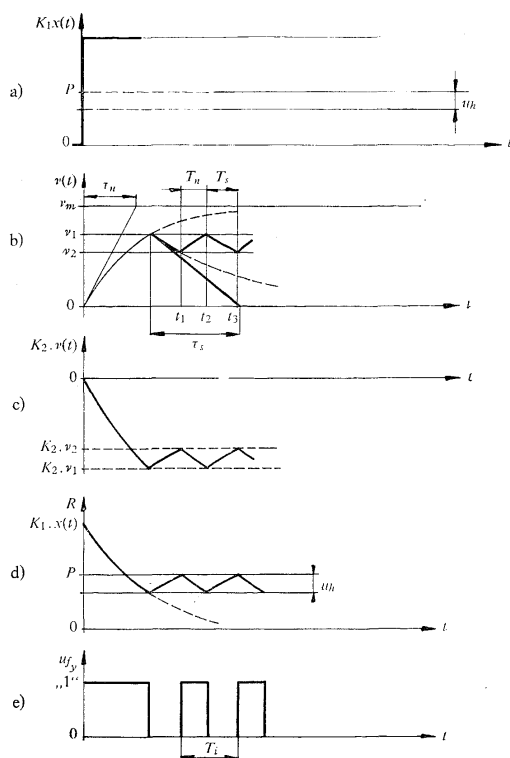In the simple case of an exponential shape of the start and drop of the signal

Fig. 2. Fig. 2a illustrates a signal $K_1\,x(t)$ and shows the threshold P and the width of the hysteresis area u_h of a Schmitt circuit. Fig. 2b shows the shape of the signal $v(t)$ which is also shown in Fig. 2c after amplification and inversion. Fig. 2d illustrates a signal R operating across the input of a Schmitt circuit. The resulting output pulses, that is the output of the circuit S are shown in Fig. 2e.

(5)
$$x(t) = 0 \quad \text{for} \quad t < 0\,,$$
$$x(t) = \text{const for} \quad t \geqq 0\,,$$

and under the condition

(6)
$$K_1\, x(t) \geqq P$$

one obtains the signal shape indicated in Fig. 2 at the various points of the converter. The Schmitt circuit is first excited if the condition (6) is met. But this causes the signal $v(t)$ to increase and leads to a decrease in the signal R down to a value

(7)
$$R = P + u_h = P - K_2(v_2 - v_1)\,.$$

At this moment the Schmitt circuit returns from state "1" into state "0", R increases with a time constant τ_s until there is again

(8)
$$R = P$$

which excites again the Schmitt circuit. Its first output pulse is longer than the following pulses (see Fig. 2e), but this difference is the smaller, the larger R than P.

The duration of a cycle of the output pulses in the steady state condition is

(9)
$$T_i = T_n + T_s\,.$$

For the rising part of the signal $v(t)$ one obtains

(10)
$$v_n(t) = v_m\big(1 - e^{-t/\tau_n}\big)\,,$$

and for the dropping part one obtains

(11)
$$v_s(t) = v_m\, e^{-t/\tau_s}$$

v_m being the possible maximum value of the signal $v(t)$ and equal to the amplitude of the output signal of the Schmitt circuit. The time T_n is determined by the distance between the intersections of the curve (10) with the values v_1 and v_2 in the rising part, and similarly, for the curve (11) also the time T_s in the dropping part of the signal $v(t)$ (Fig. 2b). For the length of the two parts of the cycle there is

(12)
$$T_n = t_2 - t_1 = \tau_n \ln \frac{v_m - v_2}{v_m - v_1}\,,$$

and

(13)
$$T_s = t_3 - t_2 = \tau_s \ln \frac{v_1}{v_2}\,.$$

With the decrease of the time constant $\tau_n \to 0$, there decreases also

(14) $T_n \to 0$.

In the extreme case, when $T_n = 0$, there is

(15) $T_i = T_s$

and relationship (1) has the shape of the curve shown by the full line in Fig. 3b. In this case the development (1) is therefore non-linear and concave. If $\tau_n \neq 0$, that
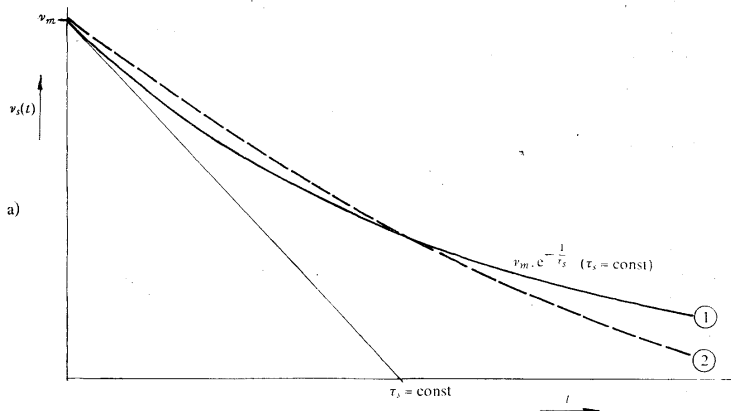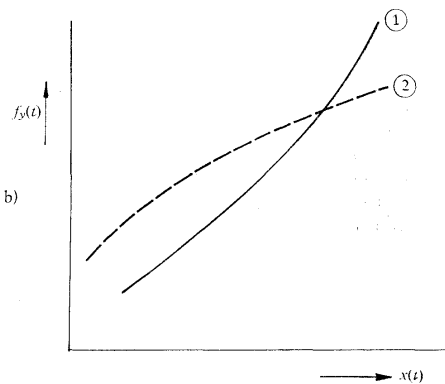

Fig. 3. The curves in Fig. 3a illustrate the development of (11) for $\tau_s =$ $= $ const (curve 1) and for $\tau_s \sim 1/v_s(t) +$ const (curve 2). Fig. 3b shows by the same type of line the corresponding development of (1). For $\tau_s =$ $= $ const, the shape (1) is concave, for $\tau_s \neq$ const the shape (1) is convex (,,logarithmic").

is T_n increases with an increase of $x(t)$, defferent values τ_n cause the following changes in (1).

It is possible to determine τ_n^* so that for

$$(16) \qquad\qquad\qquad \tau_n = \tau_n^*$$

the relationship (1) differs only a minimum from the linear development.

For

$$(17) \qquad\qquad\qquad \tau_n < \tau_n^*$$

(1) is concave, and for

$$(18) \qquad\qquad\qquad \tau_n > \tau_n^*$$

(1) is convex. The relationship (1) can be generally expressed as a function

$$(19) \qquad\qquad f_y = \varphi(R, \tau_s, \tau_n, P, v_1, v_2) .$$

The effect of the shape of the dropping part $v_s(t)$ on the shape (1) is much greater than the shape of the rising part $v_n(t)$. Assuming $T_n = 0$, Fig. 3b shows for the development $v_s(t)$ in Fig. 3a the relevant shapes of the relationship (1) for $v_2 - v_1 \doteq$ $\doteq 0{,}05 v_m$.

THE DESIGN OF THE MODEL

The overall diagram of the electronic model designed in accordance with the above mentioned principles is shown in Fig. 4. The input circuit A (K_1 in Fig. 1) is designed as a difference amplifier
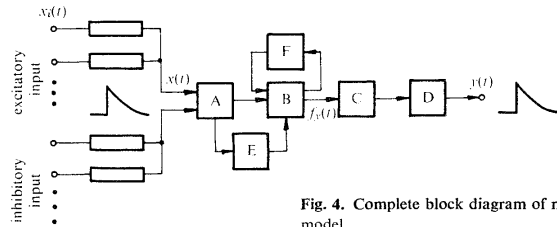

Fig. 4. Complete block diagram of neuron model.

which permits to apply the same signal polarity both for the excitory and inhibitory inputs. The circuit B is the voltage-frequency converter proper. The axon function is simulated by the circuit C (monostable multivibrator) with an adjustable value of the delay t_z. The circuit D is a synapsis model marked in Fig. 1 as integrator I. The output signal of the model has therefore the shape of a postsynaptical signal. The circuit E simulating the paradoxical phase is formed by a non-linear element with signal inversion. The circuit F simulates adaptivity. A detailed circuit arrangement of the model is shown in Fig. 5.
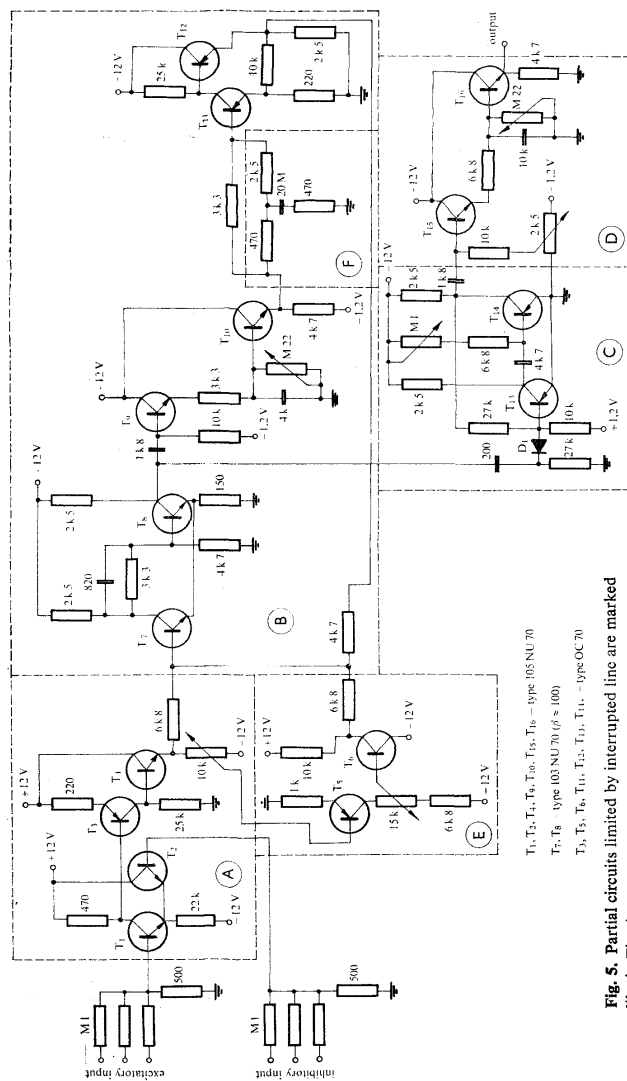
Fig. 5. Partial circuits limited by interrupted line are marked like in Fig. 4.
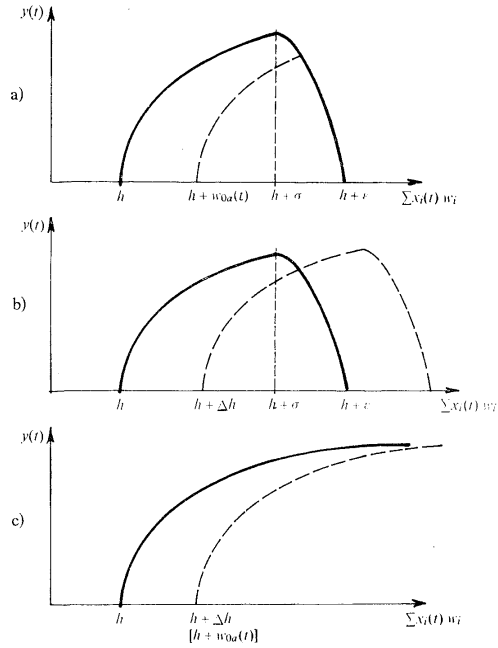
Fig. 6. Effect of adaptive component of threshold $w_{0a}(t)$ on neuron characteristic $y(t) = \psi \sum_i x_i(t) \, w_i$.
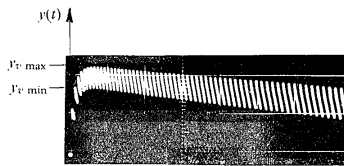

Fig. 7. Typical output signal of neuron model in Fig. 5.

Different conceptions of the adaptive circuit F from the point of view of the results of its operation are possible. In [2], the signal which increases the threshold by a component $w_{0a}(t)$ is derived from excitatory input signals so that for a constant input signal $x(t) =$ const, the output frequency $f_y(t)$ decreases gradually to zero. In our case adaptivity is controlled by a signal derived from the output signal of the circuit B which results in a gradual approach of $f_y(t)$ toward
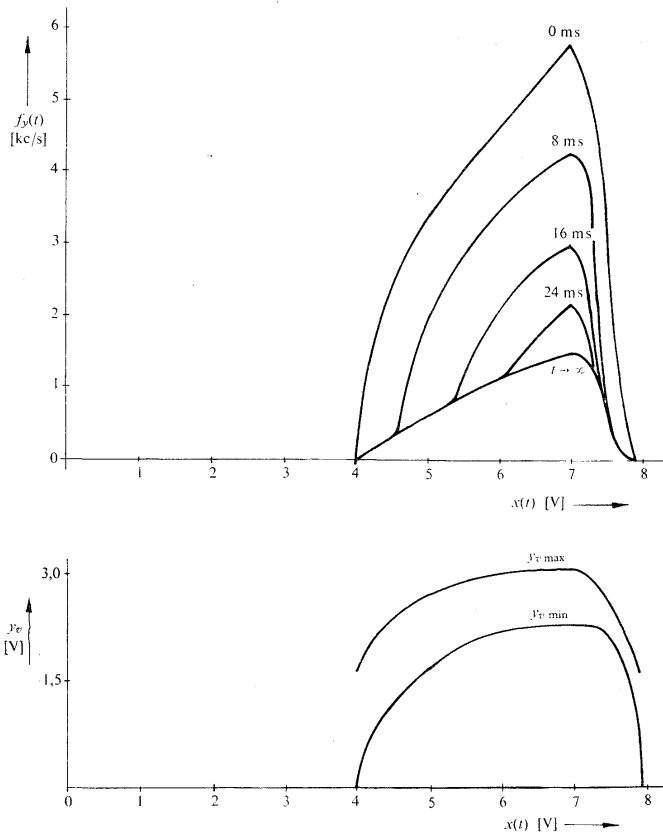
Fig. 8. Shape of dependence $f_y(t) = \psi_f[\sum_i x_i(t)\, w_i]$ (fig. a) and shape of dependence $y(t) = \psi[\sum_i x_i(t)\, w_i]$ (fig. b).

a certain minimum non-zero value. The adaptive circuit can be looked upon as a negative feedback circuit with a time constant τ_a which is relatively large with respect to the time constant τ of the postsynaptical signal [6].

If the upper limit of the active region is determined by the existence of the paradoxical phase, the width of the active region σ [1] decreases (Fig. 6a) if in Fig. 4 the instantenous values of the

threshold $w_0(t)$ is increased by a component $w_{0a}(t)$. But if the feedback circuit is changed so that it does not feed back the signal into the input of the circuit B (as in Fig. 4), but directly into the circuit A, one obtains a slightly different result. The value threshold will effectively increase by a component Δh which is a function of the output signal $f_y(t)$. But the component Δh increases also the upper limit of the active region so that the width of the active region σ remains unchanged (Fig. 6b). If the neuron has no paradoxical phase, it can be readily found that the effect of both methods of the adaptive feedback is equivalent (Fig. 6c).

Fig. 7 shows a typical output signal of our neuron model for the output signal $x(t)$ in the shape of a step function (5). The dependence (1) for various amplitudes $x(t)$ of the shape (5) has been studied and shown in Fig. 8a, time being the parameter. For a time $t = 0$, $f_{y\max} \doteq 6$ kc/s at the beginning, and it decrease gradually approximately linear to a steady value for $t \to \infty$. The dependence of the output signal y_v expressed in voltage is shown in Fig. 8b (for measured signal — see Fig. 7).

NEURON CIRCUITS WITH POSITIVE FEEDBACK

In conclusion, note the circuit arrangement of a neuron model with external positive feedback in accordance with Fig. 9, the value of the weight of the feedback being $w_z > 0$.

In the first place let us use a neuron model without adaptivity and paradoxical phase in the circuit arrangement in accordance with Fig. 9. Across its input there operates a total signal

$$(20) \qquad x(t) = x_i(t)\, w_i + y(t)\, w_z\,,$$

there being $y(t) > 0$, that is the neuron model is fired under the condition

$$(21) \qquad x_i(t)\, w_i \geqq h\,.$$

Generally, if signals affect several inputs simultaneously, there is

$$(22) \qquad x(t) = \sum_i x_i(t)\, w_i + y(t)\, w_z\,.$$

The general condition for refiring within a time $t > 0$ is

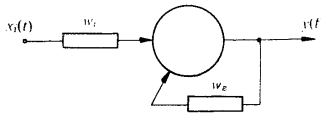$$(23) \qquad x(t) \geqq h + w_{0r}(t)\,,$$

Fig. 9. Circuit arrangement with feedback.

where $w_{0r}(t)$ is the refractory component of the threshold.

The signal $y(t)$ has a delay t_z with respect to $x_i(t)$ and an exponential drop with a synaptical time constant τ which affects the fulfilment of the condition (23).

A detailed analysis would show that the least favourable conditions for a subsequent firing occurs with the arrival of the first pulse of the signal $y(t)$ because the feedback signal is yet small. Only if the neuron is refired, firing conditions improve further because the amplitude of the signal $y(t)$ increases gradually (see Fig. 5 in [1]).

For a neuron with adaptivity, condition (21) is again valid for the first firing, but in the adaptive component of the threshold $w_{0a}(t)$ apears in expression (23)

$$(24) \qquad\qquad x(t) \geqq h + w_{0r}(t) + w_{0a}(t)$$

which worsens firing conditions. At the beginning, when $w_{0a}(t)$ has a small value, the same conclusions are practically valid as for neurons without adaptivity. But as $w_{0a}(t)$ increases, excitation conditions gradually deteriorate. In the case of a small input signal $\sum_i x_i(t)\, w_i$ and a small value w_z and t_z there is readily created the con-
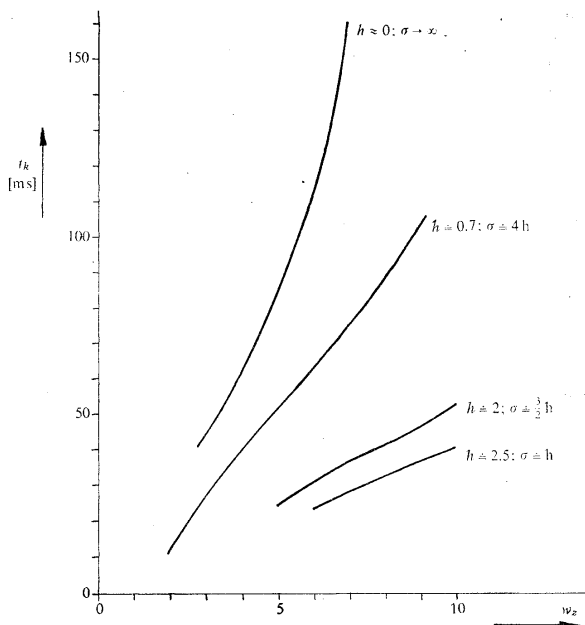

Fig. 10. Dependence of time duration of fired state of model t_k on magnitude of weight w_z of feedback for different values of threshold h.

dition in which

$$(25) \qquad\qquad x(t) < h + w_{0r}(t) + w_{0a}(t)$$

and oscillations disappear. The creation of this condition is also assisted by the
existence of the paradoxical phase because for larger values $x(t)$, as $x(t)$ increases,
$y(t)$ decreases.

Consequently, upon meeting requirement (24), the circuit falls into oscillations
and it remains exited for a time t_k. Thus one obtains a storage in which the excitation
state information is temporarily stored. The results of measuring the time t_k in the
circuit arrangement according to Fig. 9 using a neuron model with adaptivity and
paradoxical phase (see the characteristic in Fig. 8) are shown in Fig. 10 .The time t_k
is measured as a function of the magnitude of the feedback weight w_z, and the value
of the threshold h is the parameter at $\sigma = $ const, $t_z = $ const and $\tau = $ const. The
circuit was brought into the excited state by a short reactangular pulse. It is obvious
from the results that a large values w_z is necessary to maintain oscillations for large
values of the threshold h, because oscillations are quickly suppressed. But if one
reduces the threshold $h \rightarrow 0$, a relatively small value $w_z \doteq 7$ is sufficient for keeping
up stable oscillations.

It may be questioned whether in the nervous system the weights of the connections
can achieve such large values that the individual neurons can be set into oscillations,
at least as long as it may be correctly assumed that neuron firing requires cooperation
of at least 10 synapses [7]. However, if there exists in some neurons feedback of the
mentioned type, circuits are formed which can perform the function of a storage
of short duration. The storage time can be affected both by the time of operation
and by the magnitude of the signals which caused the oscillations, and by the action
of inhibitory signals. Excitatory signals cause a lengthening and inhibitory signals
cause a shortening of the time t_k.

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VÝTAH

Model neuronu s elektronickým prahovým obvodem

PETR HIRŠL

Tato práce je logickým pokračováním práce [1], neboť návrh elektronického modelu vychází z výsledků v ní dosažených. Jsou uvedeny základní vlastnosti elektronického modelu s adaptací a paradoxní fází, pojatého jako obvod s impulsní hustotní modulací a integrátorem. Závěrem jsou uvedeny podmínky pro vznik a udržení kmitů u neuronu s vnější kladnou zpětnou vazbou a výsledky, dosažené s modelem v tomto zapojení.

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