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Static Programming of Data Handling

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The presented method splits the process of the optimum digital data handling into two stages: in the first stage the vectors—operators are calculated, in the second one they are applied on measured data. In this way the demands to the computers can be substantially diminished.

INTRODUCTION

Many operations effected during data handling can be understood as an application of some "filters". A "signal" is induced on the input of handling process which contain a useful information, and also unwanted random disturbances. The task of the "filter" is to offer on the output the best approximation to the useful component of input signal or to the quantity obtained by the given transformation of this component. For the region of continous signals L. A. Zadeh and J. P. Ragazzini [1] have formulated and solved a problem which is an important generalization of the Wiener's problem [2]. The development of digital technique enforced, however, solution of similar problem also for discrete filters. The solution was a continuation of paper [3] by A. N. Kolmogorov and was gradually developped in many papers from which we mention: A. B. Lees [4], M. Blum [5], [7], K. R. Johnson [6], V. P. Perov [8], Ya. Z. Cypkin [9], V. V. Solodovnikov [10], P. D. Kruťko [11], L. N. Volgin [12].

In these papers time sequences of signals are discussed which are separated by regular time intervals. Methods of solution are in accordance with the regularity of the time sequence of signals. These methods, however, have a limited applicability for solution of practical problems where the handled "signal" is presented as a set of digital values of input function, given for other distribution of values of the independent variable (the variable need not mean time) within the interval of observation.

The mathematical statistics solves problems which are essentially related i.e. estimation of linear forms, handling of indirect observations, linear and nonlinear

regression, smoothing of measuring by means of given functions [13], [14]. The matrix solution which is often applied in this field is very comprehensible and allows not to limit oneself by a regular distribution of the interval of observation. The attention, however, is here paid more to the statistical properties of deviations of measured values from the smoothed curves than to the properties of functionals of those curves. Nevertheless, in problems of the Zadeh-Ragazzini type the securing of optimality of those functionals plays the essential part. Moreover, in the known problems connected with the method of least squares [13], [14] the useful random component is not considered. Therefore in paper [15] the author tried to use the matrix method for solving the generalized problem of Zadeh and Ragazzini and for a proof that the optimality of solution of this problem and the optimality of smoothed curves are in a connection. The main results of paper [15] as a theoretical basis of method of static programming are necessary to be presented in this article. The proposed procedure tends to the calculation of finished and in a certain sense unvariable matrices for effecting of the optimum digital operations and does not tend to the algorithmes for computing operation matrices within the process of data handling as we can see in paper [16] by P. E. Kalman and other papers e.g. [17]. These dynamic methods allow to solve an essentially broader range of problems, however, with much higher requirements to the computer.

GENERALIZED DISCRETE ZADEH-RAGAZZINI'S PROBLEM

We are given a set of measured values $y(t_j)$ $(j = 1, 2, \ldots, n)$ of the input variable $y(t)$ being a function of the independent variable t. Variable t may represent e.g. time or space coordinate. The distribution of the points t_j on the t-axis is not necessary to be uniform. Let us suppose the input function being a composition

$$y(t) = \sum_{i=n}^{i=m} a_i x_i(t) + \mathring{x}(t) + \bar{x}(t) \tag{1}$$

of m given nonrandom linearly independent functions $x_i(t)$, of a useful stationary random function $\mathring{x}(t)$ and of a stationary random function $\bar{x}(t)$ representing noise, measuring errors, statistical fluctuations of measured variables etc. The a_i are any unknown numbers. All functions and numbers are real. Correlation functions of $\mathring{x}$ and $\bar{x}$ are given. The ensemble average of both $\mathring{x}$ and $\bar{x}$ can be taken as zero after including their means into the sum.

The set of $y(t_j)$ can be considered as a random vector $\boldsymbol{Y}$ and the Eq. (1) can be rewritten as

$$\boldsymbol{Y}_{1,n} = \boldsymbol{A}_{1,m}\boldsymbol{X}^{\mathrm{T}}_{m,n} + \boldsymbol{x}_{1,n} + \bar{\boldsymbol{x}}_{1,n}\,. \tag{2}$$

The first index indicates the number of rows of the matrix, the second one the number of columns. The size of the matrices will be shown in necessary cases only. T denotes

the transposed matrix. The matrix $\boldsymbol{X}$ represents the given functions $x_i(t)$ by their numerical values:

$$\boldsymbol{X} = \boldsymbol{X}_{n,m} = \left\| \begin{matrix} x_1(t_1) \dots x_m(t_1) \\ x_1(t_n) \dots x_m(t_n) \end{matrix} \right\| . \tag{3}$$

The number of points (n) is supposed to be greater or equal the number of functions (m).

What would be desired to be the output of handling of input vector $\boldsymbol{Y}$ is a number z_1 being the result of linear transformation $\mathscr{L}$ of the useful input component

$$z_1 = \mathscr{L}\{\sum_{i=1}^{i=m} a_i x_i(t) + \mathring{x}(t)\}_{t=t_r} \tag{4}$$

e.g. the predicted value of a linear combination of first derivative and of the value of this component for $t = t_r$. But z_1 cannot be determined exactly because of the presence of the statistical disturbance $\bar{x}(t)$. Thus, it is necessary to limit oneself to determining the best estimate z of z_1. Here the following conditions must be respected:

1. The z is searched for as a linear estimate of the form

$$z = \boldsymbol{Y}\boldsymbol{W}, \tag{5}$$

   $\boldsymbol{W}$ being a vector (a matrix $\boldsymbol{W}_{n,1}$).

2. The z must be exactly equal to z_1 in the absence of the both $\bar{x}(t)$ and $\mathring{x}(t)$ (the estimate must be unbiased).

3. The variance of the z (ensemble averaged square of the deviation of z from the mean) must be minimum.

It can be shown that the constraint equations warranting holding of the condition 2 are of the form

$$\boldsymbol{X}^{\mathrm{T}}\boldsymbol{W} = \boldsymbol{L} \tag{6}$$

if

$$\boldsymbol{L} = \left\| \begin{matrix} \mathscr{L}\{x_1(t)\}_{t_r} \\ \vdots \\ \mathscr{L}\{x_m(t)\}_{t_r} \end{matrix} \right\|$$

where the t_r denotes the value of t for which the estimate is determined. In dependence on the choice of t_r in relation to the "observation interval" $(t_1 \div t_n)$ the operation performed can be characterised as the smoothing (interpolation) prediction (extrapolation) or filtering. The choice of the operator $\mathscr{L}$ determines what function is to be smoothed, predicted or filtered. We can understand the $\mathscr{L}$ as the symbol denoting the derivating, integrating, convolutory integrating or as the symbol for linear combination of this and similar operations. $\mathscr{L}$ may be the identical transformation also.

Thus, the vector $\boldsymbol{L}$ (7) defines the type of the estimate and the matrix equation (6) warrants the analytical properties of the digital operator $\boldsymbol{W}$.

The variance of the estimate z denoted by D_{z-z_1} can be shown as

$$D_{z-z_1} = \boldsymbol{W}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{W} - 2\boldsymbol{C}\boldsymbol{W} + d \tag{8}$$

where

$$\boldsymbol{B}_{n,n} = \overline{\mathring{\boldsymbol{x}}^{\mathrm{T}}\mathring{\boldsymbol{x}}} + \overline{\bar{\boldsymbol{x}}^{\mathrm{T}}\bar{\boldsymbol{x}}} + 2\overline{\mathring{\boldsymbol{x}}^{\mathrm{T}}\bar{\boldsymbol{x}}} \tag{9}$$

is the covariance matrix of $\mathring{\boldsymbol{x}} + \bar{\boldsymbol{x}}$, $\boldsymbol{C}$ and d being given by

$$(C_{1,n})^T = \left\| \begin{array}{c} \overline{\mathscr{L}\{\mathring{x}(t)\}_{t_r} \cdot (\mathring{x}(t_1) + \bar{x}(t_1))} \\ \vdots \\ \overline{\mathscr{L}\{\mathring{x}(t)\}_{t_r} \cdot (\mathring{x}(t_n) + \bar{x}(t_n))} \end{array} \right\|, \tag{10}$$

$$d_{1,1} = \overline{[\mathscr{L}\{\mathring{x}(t)\}_{t_r}]^2}\,. \tag{11}$$

The line above the symbols denotes the ensemble averaging. The matrix $\boldsymbol{B}$, vector $\boldsymbol{C}$ and number d can be calculated from given correlation functions.

Three conditions mentioned above define what is meant by the words "best estimate": The estimate is linear, unbiased and its variance is minimum. The linearity of the estimate (7) leads to an important advantage in regard to the effectiveness of using the digital operator $\boldsymbol{W}$ for calculation: The number of numerical operations is minimum: for consideration of each measured value y_j within the process of handling the vector $\boldsymbol{Y}$ only one multiplication and one addition is needed. Thus, the estimate has a given analytical accuracy, minimum statistical error and minimum requirements to the computer.

SOLUTION OF THE PROBLEM AND ITS PROPERTIES

The Lagrangian method gives the solution in the form of

$$\boldsymbol{W} = \boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}\boldsymbol{L} + [\boldsymbol{E} - \boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}]\,\boldsymbol{B}^{-1}\boldsymbol{C}^{\mathrm{T}} \tag{12}$$

where the $\boldsymbol{E}_{n,n}$ is the unity-matrix and $\boldsymbol{B}^{-1}$ is the invert of the $\boldsymbol{B}$. It can be shown by substitution into Eq. (6) that this equation is satisfied by $\boldsymbol{W}$ of the form (12). The minimality of the variance is proved in the Appendix. Another important properties of the solution can be pointed out:

A. Let us use the operator $\boldsymbol{W}$ for smoothing the input function $y(t)$ having the form of (1). Let us estimate the smoothed values $\tilde{y}(t_j)$ for all j. Then not only the variance of each separate $\tilde{y}(t_j)$ is minimum but also the ensemble average of the sum of squared deviations $\overline{(y(t_j) - \tilde{y}(t_j))^2}$ reaches its minimum. In the case of $\mathring{x}(t) = 0$ the sum of squared deviations is minimized, too.

B. The smoothing of the smoothed function gives no changes in values $\tilde{y}(t_j)$. The analytical operation $\mathscr{L}$ performed on the smoothed function gives the same result as the direct using of the operator $\boldsymbol{W}$ calculated for the same operation $\mathscr{L}$. The practical consequence of this fact will be considered below.

C. Matrices appearing in (12) can be used not only for computing the estimate z. The estimate $\tilde{\boldsymbol{A}}$ of the unknown vector $\boldsymbol{A}$ (see Eq. (2)) can be performed as

$$\tilde{\boldsymbol{A}} = \boldsymbol{Y}\boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}. \tag{13}$$

This estimate is unbiased and the expression $(\boldsymbol{A} - \tilde{\boldsymbol{A}})(\boldsymbol{A} - \tilde{\boldsymbol{A}})^{\mathrm{T}}$ representing the sum of squared errors is minimum for the class of linear estimates of the vector $\boldsymbol{A}$. Proofs of the statements A, B, C are omitted here, they can be found in [15].

STATIC PROGRAMMING

The main idea of static programming consists in the distribution of all operations of digital data handling into two stages:

1. Calculation of optimum digital operators and of variances, resp. of covariance matrices of output quantities.
2. Application of operators.

The simplest realization of this idea is demonstrated in Fig. 1. For the sake of simplicity calculation of variance of the result is not shown in Fig. 1. The input vector is $\boldsymbol{Y}$. By means of a preliminary study of this vector or from theoretical considerations the necessary correlation functions were obtained and the matrices $\boldsymbol{B}$ and $\boldsymbol{B}^{-1}$ were calculated. Functions $x_i(t)$ being given, the matrix $\boldsymbol{X}$ is also given. The operator $\mathscr{L}$ and point t_r are determined by requirements for the type of operation and therefore the vector $\boldsymbol{L}$ is also determined. The vector $\boldsymbol{C}$ is calculated from correlation functions for the given operator $\mathscr{L}$ and point t_r. In the phase a of the first stage of static programming the matrix

$$\boldsymbol{M} = \boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1} \tag{14}$$

is calculated as well as the matrix $\boldsymbol{E} - \boldsymbol{M}$ ($\boldsymbol{E}$ being a unit matrix). In the phase b of the first stage the vectors $\boldsymbol{L}$ and $\boldsymbol{B}^{-1}\boldsymbol{C}^{\mathrm{T}}$ are calculated for the required operator $\mathscr{L}$ and point t_r and also the vector-operator $\boldsymbol{W}$ which is an output quantity of the first stage. Calculations included in the first stage are effected only once for the given conditions. In the second stage which is repeated many times for various input vectors $\boldsymbol{Y}$ only the calculation of the required output quantity by means of application of the operator $\boldsymbol{W}$ is performed, i.e. the scalar product $\boldsymbol{YW}$ is being calculated. We proceed as follows

$$(S_1) \qquad 1a—1b—2—2—2—2—2—2\ldots$$

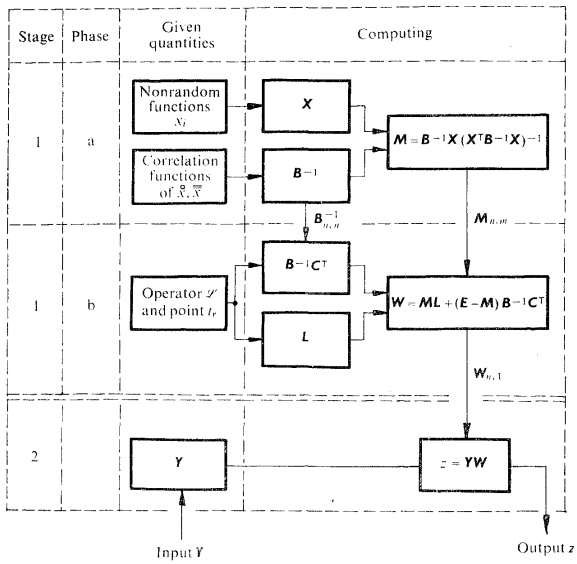

Fig. 1. Stages and phases of the first scheme of static programming.

Here the first stage can be carried out e.g. by means of an efficient medium size off-line computer and the second stage by means of a small on-line computer.

The method according to Fig. 1 is advantageous for one or for a few single operations of a given type on many input vectors. Two important cases of the second stage

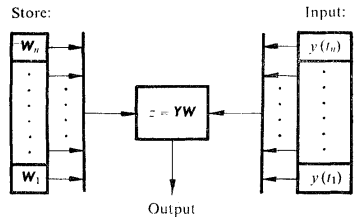

Fig. 2. Second stage of static programming for nonstationary space estimation.

of static programming can be mentioned here. In the first case, demonstrated in Fig. 2, all values of components of the vector $\boldsymbol{Y}$ are taken in the same instant e.g. the independent variable t is a space coordinate. In the computer store the vector $\boldsymbol{W}$

calculated in the first stage is stored. The process represented by components of the vector $\boldsymbol{Y}$ can be nonstationary. In every instant of estimating z the computer takes the operator $\boldsymbol{W}$ off the store and all components of the vector $\boldsymbol{Y}$ off the input to effect the scalar product $\boldsymbol{YW}$. The vector $\boldsymbol{Y}$ has not to be stored after finishing the operation.

The vector $\boldsymbol{Y}$, however, can be formed by time consequence of values of the input function, the independent variable t being time (Fig. 3). In the store of computer also the vector $\boldsymbol{Y}$ is stored formed by the last n measured values and supplemented by the newest measured value. The oldest value is forgotten in every further step.

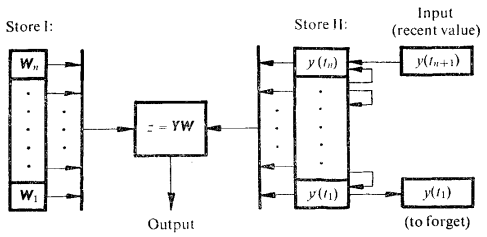

Fig. 3. Second stage of static programming for time estimating.

If more operations on the same vector $\boldsymbol{Y}$ are required (e.g. k) it is necessary to calculate in the first stage vectors – operators for using in the second stage i.e. the matrix operator $\boldsymbol{W}_{n,k}$. If the existence of useful random component $\mathring{x}(t)$ is not presumed and when it is necessary to perform a great number of various operations on every vector $\boldsymbol{Y}$ then two stages of static programming accord. to Fig. 4 are more convenient. Calculation of covariance matrices of output quantities is again omitted in Fig. 4. The handling proceeds according to the following scheme

(S_2) 1a—1b – 2a—2b—2a—2b—2a—2b—2a—2b ...

This case differs from using k vectors $\boldsymbol{W}$ accord. to Fig. 1 only by the requirements to the storing of matrices passing from the first stage into the second one: according to Fig. 1 it would be necessary to store a matrix with nk elements, accord. to Fig. 2 to store one matrix with nm elements and one with mk elements. The method accord. Fig. 4 enables to condensate the input data for storing and later handling. Then

(S_3) 1a—2a—S—2a—S—2a—S—2a—S—2a—S ...

(S denotes the operation of storing the vector $\boldsymbol{A}$). The later handling proceeds accord. to the scheme

(S_4) 1b—E—2b—E—2b—E—2b—E—2b ...

(E denotes extraction of the vector $\boldsymbol{A}$ off the store). A direct storing of p vectors $\boldsymbol{Y}_{1,n}$ would require pn cells, a condensed storing needs pm cells. (Often $n \gg m$.) Moreover, there is a possibility in later handling to change the type of operations and repeat the chain (S_4). Furthermore, there are some important cases, e.g. harmonic analysis, where the chain (S_4) need not exist at all and the handling proceeds according to (S_2) only.

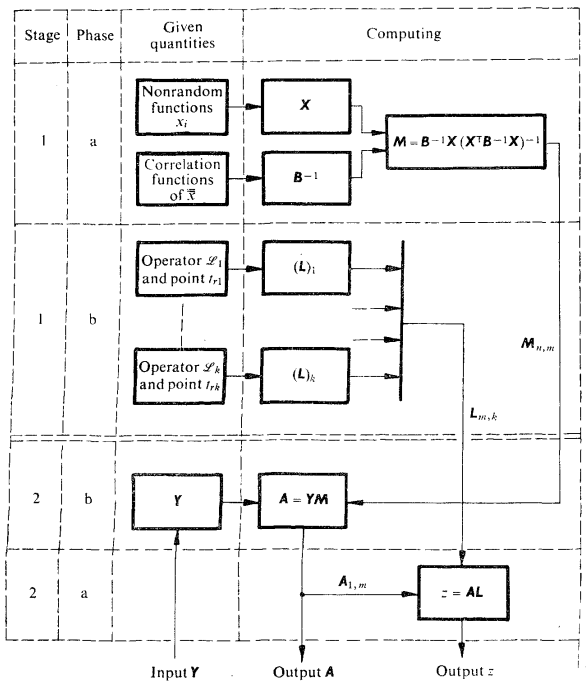

Fig. 4. Stages and phases of the second scheme of static programming.

It is necessary to mention that in practical using the chains (S_1) up to (S_4) can be variously modified and that the static programming can be realised in a less pure form than it is presented here. It depends both on the complication of solved problems

and on the necessity to calculate in some cases not only the estimates of the wanted quantities but also their errors.

SOME EXAMPLES OF USING THE STATIC PROGRAMMING

Following examples can be mentioned from the field of using the computers for control of reactors and nuclear power stations and for handling the data in nuclear engineering:

From the temperature of various fuel elements of a reactor it is necessary to calculate in regular time intervals the mean temperature, i.e. to integrate the smoothed curve. An on-line computer can carry out this operation according to Fig. 2. According to the same scheme e.g. time behaviour of thermal gradient in the most stressed spots of a pressure vessel or of cooling piping can be followed. By double using the scheme accord. to Fig. 2 we can obtain the local value of buckling as the negative ratio of the local value of the second derivative and of the local value of the curve representing a smoothed space distribution of neutron flux.

Examples for the scheme represented by Fig. 3 are as follows:

Calculation of the predicted value of a controled quantity for using in the optimum control.

Calculation of the exact value of the instantaneous period of a reactor as a ratio of an instantaneous value and of an instantaneous value of the first derivative of the smoothed reactor power function.

Calculation of the predicted value of temperature of important elements of a reactor for alarm or scram signalisation.

Calculation of the instantaneous value of reactivity as a convolution integral from the neutron flux.

For functionning accord. to Fig. 3 the sampled interval (i.e. the interval between two values of an independent variable t) is accepted as a constant one. For proceeding accord. to Fig. 2 the distribution of the points t_j is determined by constructional aspects (by the possibility of situating the detectors) and often it is not regular. This must be respected when calculating the matrix $\boldsymbol{X}$.

As an example of the method of calculations accord. to Fig. 4, we can mention a digital registration of the space distribution of neutron flux for a later further handling: for the phase a of the first stage a system of linearly independent functions $x_i(t)$ and thus also the matrix $\boldsymbol{X}$ is chosen so that a small number of functions $x_i(t)$ could characterize the approximated curve with sufficient accuracy. In the phase a of the second stage an analysis of measured curves as a linear combination of given vectors of the type $\boldsymbol{A}\boldsymbol{X}^{\mathrm{T}}$, calculation and storing of the vector $\boldsymbol{A}$ accord. to the chain (S_3) is effected in necessary time intervals. In a later phase of handling the accumulated data accord. to the chain (S_4) it is possible e.g. to carry out an analysis of time depend-

ence of some functionals from measured curves. Another example of a supplementary handling is an analysis of time dependence of components of the vector $\boldsymbol{A}$. In this case, e.g. the i-th components of these vectors form a new input vector $(\boldsymbol{Y_A})_i$ which can be further analysed according to any of the mentioned schemes after a new choosing of suitable matrix $\boldsymbol{X}$ and after absolving the calculations of the new first stage. Thus we can effect e.g. a time interpolation of components of the vectors $\boldsymbol{A}$ for the moment for which the measuring was not carried out and obtain the time-interpolated distribution curve. The advantage of this method is the reduction of the influence of random errors, storing the condensed data and little extra demand on the on-line computer.

One Example in More Details: Optimum Digital Harmonic Analysis

To get a real idea about the importance of redundance (being represented here by the difference $n - m$, by using the n measured values for calculation of the m unknowns) for reduction of errors of the result and for illustration of the effectivity of using the optimum digital operators, let us introduce a simple example of using the static programming for a harmonic analysis.

The method of reactor oscillator is one of the most important methods for experimental determination of dynamic properties of a reactor as well as of the cooling systems of the reactor. Evaluation of results of this measuring meets with following difficulties (particularly in the power range):

a) The measured quantity is disturbed by statistic fluctuations.
b) For practical reasons the amplitude of periodical disturbance must be small, compared with the mean value.
c) It is difficult to warrant that the mean values of all measured quantities are being constant throughout the whole duration of the experiment.
d) It is necessary to handle a great deal of experimental data for an analysis of the response not only of the reactor but also of a great number of further elements of a nuclear power plant at different frequencies.

These difficulties can be essentially moderated by using the static programming according to Fig. 4, chain (S_3).

An example: the vector $\boldsymbol{Y}$ is formed by values of one of the investigated quantities taken in the moments t_1 up to t_N. Functions $x_i(t)$ will be chosen as follows:

$$x_1(t) = \sin(t), \quad x_4(t) = \cos(t), \quad x_7(t) = 1,$$
$$x_2(t) = \sin(2t), \quad x_5(t) = \cos(2t), \quad x_8(t) = t,$$
$$x_3(t) = \sin(3t), \quad x_6(t) = \cos(3t), \quad x_9(t) = t^2.$$

Let the matrix $\boldsymbol{B}$ be a diagonal matrix with the equal diagonal elements S_Y. The useful random component is identically zero. Let the points t_j be equally distributed

on the interval from t_1 up to $t_N = t_1 + 2\pi$. The interval 2π is therefore divided into $N - 1$ equal intervals. By calculation of the phase a of the first stage one obtains the matrix of the operators $\boldsymbol{M}$ for calculation of the vectors $\boldsymbol{A}$. Its first six components form in this case the best estimates of amplitudes of harmonic components and the three remaining components are estimates of coefficients of the quadratic polynomial.

Calculation of the phase a of the first stage for the considered case was effected for various values of N and the obtained operators were used in the stage 2a for handling of measured values of frequency transfer functions of real parts of a steam generator of a nuclear power plant. It will be of interest to mention that in the given case for $N = 31$ the computer Elliot 803 B needs several hundred times more time for calculation of the stage 1a than for calculation of the stage 2a which needs a few seconds for one vector $\boldsymbol{Y}$. Codes for both stages of calculation were written in ALGOL 60. Using of the machine code for calculation of the stage 2a would not represent any problem and could further increase the effectivity of using the considered method.

For the considered case (after calculation of the matrix $\boldsymbol{M}$) it is easily possible to determine variances of every component of the vector $\boldsymbol{A}$ by effecting the product of the number $S_{\boldsymbol{Y}}$ and of the sum of squares of elements of the respective column of the matrix $\boldsymbol{M}$. (The number $S_{\boldsymbol{Y}}$ represents variance of the measured values y_j.) Variance of the amplitude of the k-th harmonic will be determined as the sum of variance of sine and cosine amplitudes of this harmonic. Variances of amplitudes and coefficients of the polynominal calculated for several values of N are demonstrated in Tab. I. When $N = 9$ the redundance equals zero and variances of the output values are very high. With the increase of N the variances quickly decrease.

Table I.

Variance of amplitudes of the harmonics and of the coefficients of the polynomial at optimum digital harmonic analysis

n — number of points t_j on the investigated interval 2π,
S_{ak} — variance of the amplitude of k-th harmonic,
S_{bk} — variance of k-th coefficient of the polynomial $\sum_{k=0}^{2} b_k t_k$,
S_y — variance of components of the input vector $\boldsymbol{Y}$ i. e. of the measured values.

n	S_{a1}	S_{a2}	S_{a3}	S_{b0}	S_{b1}	S_{b2}
9	$22{\cdot}934 S_y$	$2{\cdot}500\ S_y$	$1{\cdot}19073 S_y$	$178{\cdot}53\ S_y$	$46{\cdot}906\ \ S_y$	$4{\cdot}6875 \cdot 10^{-1} S_y$
15	$8{\cdot}381 S_y$	$0{\cdot}9262 S_y$	$0{\cdot}45224 S_y$	$42{\cdot}379 S_y$	$4{\cdot}783\ \ S_y$	$1{\cdot}8646 \cdot 10^{-2} S_y$
31	$5{\cdot}422 S_y$	$0{\cdot}5217 S_y$	$0{\cdot}22766 S_y$	$19{\cdot}825 S_y$	$0{\cdot}61347\ S_y$	$5{\cdot}9759 \cdot 10^{-4} S_y$
61	$3{\cdot}439 S_y$	$0{\cdot}3046 S_y$	$0{\cdot}12219 S_y$	$10{\cdot}784 S_y$	$0{\cdot}093382 S_y$	$2{\cdot}4229 \cdot 10^{-5} S_y$

The calculation effected for $N = 31$, for the same polynomial component and for the same distribution of the points t_j on the interval of 2π shows how the increase of the number of analysed harmonics (H) lowers the accuracy of determining the amplitude of basic harmonic and of coefficients of polynomial component. Results are summarized in Tab. II.

Table II.

Variance of amplitude of the first harmonic and of the coefficients of the polynomial at optimum digital harmonic analysis at $n = 31$

H — number of harmonics considered in the analysis. Remaining symbols are the same as as in Table I.

H	S_{a1}	S_{b0}	S_{b1}	S_{b2}
1	$0{\cdot}8581 S_y$	$2{\cdot}5603 S_y$	$0{\cdot}074339 S_y$	$7{\cdot}1726 \cdot 10^{-5} S_y$
3	$5{\cdot}422\ S_y$	$19{\cdot}825\ S_y$	$0{\cdot}61347\ S_y$	$5{\cdot}9759 \cdot 10^{-4} S_y$
5	$15{\cdot}836\ S_y$	$59{\cdot}991\ S_y$	$1{\cdot}8716\ S_y$	$1{\cdot}8259 \cdot 10^{-3} S_y$

ACCURACY OF OUTPUT QUANTITIES

Error of the output quantities may be caused by four reasons:

1. Representation of the nonrandom component of the input signal is not satisfactory, the chosen functions $x_i(t)$ do not enable a sufficiently accurate approximation to this component.
2. The output quantities contain a disturbing random component which is introduced into the input of the process together with the useful component.
3. Representation of the input and output values and of components of vectors — operators as well as operations with those members are not sufficiently accurate.
4. Correlation functions which were supposed by us do not sufficiently meet the real properties of the input random components.

Reason of the type 4 causes that at a certain "analytical" accuracy of output quantities the variance of this quantity is greater than it could be if the information of the input random components were more complete. Sometimes, however, even a rather nonaccurate knowledge of properties of the input random components is no hindrance for practical using of results. Beside that, knowledge of statistical properties of input data can be supplemented and made more accurate within its handling and the digital operators can be additionally corrected by means of a new calculation 1a.

Reason mentioned under 3 is essentially of technical character. For the second stage of the calculation i.e. for the proper ensemble data handling the usual accuracy

of an on-line computer will be sufficient, requirements to the accuracy of its input equipment can be even lower because of the utilization of redundance. On the other hand, calculations of the first stage require an accuracy which is usually necessary for effecting of scientific calculations. These calculations, however, even because of speed and because of requirements to the store are usually necessary to be effected by means of a medium-size off-line computer warranting the demanded accuracy.

Two components thus remain under 1 and 2 i.e. the analytical and the statistical component. Unfortunately, decrease of the influence of one of those components leads to an increase of the other one, the quantity of the input information remaining constant. The increase of quantity of the input information (the inrease of n) enables to decrease the result error, however, either for the price of increasing the demands to technical equipment or for the price of time retardation of the output information or for the price of both those complications. Therefore it is necessary to consider even the calculation of variations resp. of covariance matrices of output quantities as an important component of the first stage of static programming. This calculation makes it possible to estimate still before the beginning of the second stage the statistical component of errors of results (supposing a covariance matrix $\boldsymbol{B}$ of the input vector $\boldsymbol{Y}$). The calculation of the variance D_{z-z_t} of the output quantity z for the scheme according to Fig. 1 is effected according to Eq. (8). One can easily see that the covariance matrix $\boldsymbol{K_A}$ of the vector $\boldsymbol{A}$ which is the output quantity of the phase 2a (proceeding accord. to Fig. 4) is given by the expression

$$\boldsymbol{K_A} = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1} \tag{15}$$

and the covariance matrix $\boldsymbol{K_Z}$ of the vector $\boldsymbol{Z}$ which is the output quantity of the phase 2b (accord. to Fig. 4) is given by the expression

$$\boldsymbol{K_Z} = \boldsymbol{L}^{\mathrm{T}}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}\,\boldsymbol{L} \tag{16}$$

where $\boldsymbol{L}_{m,k}$ is the matrix composed of the vectors $\boldsymbol{L}$ calculated for each individual operation. We see that for calculation of variances or of covariance matrices we need (beside the given quantities) the matrices which are calculated in the first stage as well.

The question of suitability of the choice of type of the functions $x_i(t)$ and of its number m is more complicated and cannot be definitely solved in a general case without a deeper investigation of properties of the input vectors $\boldsymbol{Y}$ although it is often possible to give the matrix $\boldsymbol{X}$ only by intuition or apriori knowledge of character of the vector $\boldsymbol{Y}$ and although the analytical error can be roughly estimated under certain prepositions about properties of the nonrandom part of the input signal. The verification of suitability of choice of the matrix $\boldsymbol{X}$ is a task of mathematical statistics, it requires a testing of the hypothesis that the quantities $y_j - \tilde{y}_j$ (the errors of smoothing) have really a random character. Even for calculation of the smoothed vector $\boldsymbol{Y}$ and thus also for the testing of this hypothesis one can use matrices calculated in the first stage. This test, being not comprised in the mentioned schemes,

would be effected, if necessary, between the stages 1 and 2 to get a warranty of the correctness of the method. First then many repeating of the stage 2 would follow.

CONCLUSION

Method of static programming consists in splitting of the handling process into two stages. In the first stage, the digital operators are prepared, in the second stage the operators are applied in the calculation of the scalar product into which the vector-operator as well as the vector of measured values enter. The method allows to perform operations of an mass data handling most economically and with best results from the point of view of using the input informations.

APPENDIX

Let us consider a vector $\hat{\boldsymbol{W}}_{n,1}$ giving a linear estimate

$$\hat{z} = \boldsymbol{Y}\hat{\boldsymbol{W}} \tag{A1}$$

which is unbiased:

$$\boldsymbol{X}^{\mathrm{T}}\hat{\boldsymbol{W}} = \boldsymbol{L}\,. \tag{A2}$$

The variance $D_{\hat{z}-z_1}$ of z is given by the same formula (8) as the D_{z-z_1}:

$$D_{\hat{z}-z_1} = \hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{B}\hat{\boldsymbol{W}} - 2\boldsymbol{C}\hat{\boldsymbol{W}} + d\,. \tag{A3}$$

After substitution of (12) into (8) and after subtraction of the equation (8) from (A3) we have

$$\begin{aligned} D_{\hat{z}-z_1} - D_{z-z_1} = {} & \hat{\boldsymbol{W}}^{\mathrm{T}}\boldsymbol{B}[\boldsymbol{E} - \boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}]\,\hat{\boldsymbol{W}} - \\ & - 2\boldsymbol{C}[\boldsymbol{E} - \boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}]\,\hat{\boldsymbol{W}} + \\ & + \boldsymbol{C}[\boldsymbol{E} - \boldsymbol{B}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}]\,\boldsymbol{B}^{-1}\boldsymbol{C}^{\mathrm{T}}\,. \end{aligned} \tag{A4}$$

$\boldsymbol{B}$ being a covariance matrix is invertible and symmetrical, it can be therefore represented by the product of a nonsingular matrix $\boldsymbol{S}$ and of its transpose:

$$\boldsymbol{B} = \boldsymbol{S}\boldsymbol{S}^{\mathrm{T}} \tag{A5}$$

or

$$\boldsymbol{B}^{-1} = (\boldsymbol{S}^{\mathrm{T}})^{-1}\,\boldsymbol{S}^{-1}\,. \tag{A6}$$

Using (A5) and (A6) we can rewrite the equation (A4) as the quadratic form

$$\begin{aligned} & D_{\hat{z}-z_1} - D_{z-z_1} = \\ & [\boldsymbol{W}^{\mathrm{T}}\boldsymbol{S} - \boldsymbol{C}\boldsymbol{S}^{\mathrm{T}}]\,[\boldsymbol{E} - \boldsymbol{S}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})\,\boldsymbol{X}^{\mathrm{T}}\boldsymbol{S}^{\mathrm{T}-1}]\,[\boldsymbol{W}^{\mathrm{T}}\boldsymbol{S} - \boldsymbol{C}\boldsymbol{S}^{\mathrm{T}}]^{\mathrm{T}}\,. \end{aligned} \tag{A7}$$

The matrix

$$\boldsymbol{R}_{n,n} = \boldsymbol{E} = \boldsymbol{S}^{-1}\boldsymbol{X}(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}^{-1}\boldsymbol{X})\,\boldsymbol{X}\boldsymbol{S}^{\mathrm{T}-1} \tag{A8}$$

being a symmetrical matrix can be represented as

$$\boldsymbol{R} = \boldsymbol{F}_0^{\mathrm{T}}\boldsymbol{M}_{\mathrm{D}}\boldsymbol{F}_0 \tag{A9}$$

where $(\boldsymbol{M}_{\mathrm{D}})_{n,n}$ is a matrix of diagonal form and $\boldsymbol{F}_0$ is an orthogonal matrix:

$$\boldsymbol{F}_0\boldsymbol{F}_0^{\mathrm{T}} = \boldsymbol{E}_{n,n}\,. \tag{A10}$$

We can see from (A8) that

$$\boldsymbol{R}\boldsymbol{R} = \boldsymbol{R} \tag{A11}$$

but using (A9) and (A10) we have also

$$\boldsymbol{M}_{\mathrm{D}}\boldsymbol{M}_{\mathrm{D}} = \boldsymbol{M}_{\mathrm{D}}\,. \tag{A12}$$

This may be if and only if the diagonal elements of the diagonal matrix $\boldsymbol{M}_{\mathrm{D}}$ (the real numbers m_j) satisfy the equation

$$m_j^2 = m_j \tag{A13}$$

that is they are unity or zero.

Thus, the quadratic form (A7) can be represented by

$$D_{\hat{z}-z_1} - D_{z-z_1} = \sum_{j=1}^{j=n} m_j^2 p_j^2 \tag{A14}$$

where the p_j denotes the j-th component of the vector $\boldsymbol{F}_0[\boldsymbol{W}^{\mathrm{T}}\boldsymbol{S} - \boldsymbol{C}\boldsymbol{S}^{T}]^{\mathrm{T}}$. Hence

$$D_{\hat{z}-z_1} - D_{z-z_1} \geqq 0\,. \tag{A15}$$

The vector $\boldsymbol{W}$ gives the estimate z the variance D_{z-z_1} of which is not greater than the variance of any unbiased linear estimate which fact had to be proved.

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VÝTAH

Statické programování pro zpracování dat

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Článek se zabývá numerickým zpracováním dat, jehož cílem je získání nejlepšího odhadu hodnoty lineárního funkcionálu od vstupní funkce, dané číselnými hodnotami v libovolně rozdělených bodech nezávisle proměnné. Vstupní funkce je neznámou lineární kombinací daných nenáhodných funkcí a stacionární náhodné funkce. Do procesu zpracování vzstupuje vstupní funkce v součtu s rušivou stacionární náhodnou funkcí. Korelační funkce obou náhodných funkcí jsou dány. Popisovaná metoda statického programování vychází z maticového řešení zobecněné diskrétní analogie úlohy Zadeha a Ragazziniho. Rozděluje proces zpracování na dvě etapy. V první etapě se pro dané podmínky vypočtou číslicové operátory použitelné pro jakákoliv konkrétní data. Ve druhé etapě se provádí vlastní zpracování dat aplikací těchto číslicových operátorů vždy jako skalární součin vektoru-operátoru a vektoru, tvořeného souborem dat.

Často se požaduje mnohonásobné opakování procesu zpracování dat získaných za týchž podmínek. V takovém případě se mnohonásobně opakuje pouze druhá etapa, vyžadující minimální počet numerických operací. Zpracování dat podle uvedené metody je v takovém případě optimální nejen z hlediska disperse výsledku (která je minimální), ale i z hlediska ekonomického. Jednoduchost druhé etapy zpracování dat dovoluje podstatně snížit nároky na počítač (rychlost i paměť), který má pracovat v reálném čase. V článku jsou uvedeny některé aplikace metody na úlohy, vyskytující se v jaderné technice a použití pro optimální číslicovou harmonicko-polynomickou analýzu.

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