KYBERNETIKA — VOLUME 35 (1999), NUMBER 4, PAGES 473–485

PARTIAL DISTURBANCE DECOUPLING PROBLEM FOR STRUCTURED TRANSFER MATRIX SYSTEMS BY MEASUREMENT FEEDBACK

ULVIYE BAŞER

Partial disturbance decoupling problems are equivalent to zeroing the first, say k Markov parameters of the closed-loop system between the disturbance and controlled output. One might consider this problem when it is not possible to zero all the Markov parameters which is known as exact disturbance decoupling. Structured transfer matrix systems are linear systems given by transfer matrices of which the infinite zero order of each nonzero entry is known, while the associated infinite gains are unknown and assumed mutually independent. The aim in this paper is to derive the necessary and sufficient conditions for the generic solvability of the partial disturbance decoupling problem for structured transfer matrix systems, by dynamic output feedback. Generic solvability here means solvability for almost all possible values for the infinite gains of the nonzero transfer matrix entries. The conditions will be stated by generic essential orders which are defined in terms of minimal weight of the matchings in a bipartite graph associated with the structured transfer matrix systems.

1. INTRODUCTION

We consider the transfer matrix system Σ as

$$z = Kd + Lu \tag{1}$$

$$y = Md + Nu \tag{2}$$

with disturbance d, control u, output z and measurement y, and K, L and M proper rational matrices and N a strictly proper matrix of suitable dimensions. On the large class of control problems related the system given above it is essential to design the closed loop transfer function between the disturbance d and the controlled output z. The early attempts along this line were devoted to zeroing the effect of disturbance on the controlled output. This problem is usually referred to as the disturbance decoupling and is abbreviated by DDP. The reader may refer to [12] for further detail.

When it is not possible to zero the effect of disturbance on the controlled output, partial disturbance decoupling problem is considered, which can be defined as zeroing the first, say k Markov parameters. This problem has been initially introduced in [6].

Structured systems are linear systems of which each of the coefficients either is fixed to zero or is an independent free parameter. Structured transfer matrix systems are linear systems given by transfer matrices for which each of the entries either is fixed to zero or has a zero at infinity of a known order, while the associated infinite gains, which are the parameters are unknown and assumed mutually independent values. Hence a structured transfer matrix system is partially given by the zero-nonzero structure of its matrices. The zero-nonzero structure can be represented by means of a bipartite graph. The edges in the graph representation of a structured transfer matrix system can be given weights equal to the infinite zero orders of the associated nonzero entries.

In this paper we examine the partial disturbance decoupling problem by measurement feedback for structured transfer matrix systems. We derive necessary and sufficient conditions for the generic solvability of the problem in terms of the generic essential orders which are calculated by the edge weights in the bipartite graph representation of the structured transfer matrix system. We also present an algorithm which is the modified version of the Algorithm 5.2 in [11] to our problem that enables us to check the generic solvability of the problem.

In the literature the infinite structure of the structured systems is presented in [1, 8] and [9] and corresponds to the sets of vertex disjoint input-output paths. The graph characterization of the generic essential orders are deduced from infinite structure of the system. DDP for structured systems are obtained in [1, 9, 10] and [4] in the state space sense. In [2] and [11] this problem are approached from a frequency domain point of view using transfer matrices. Partial DDP for structured systems by static state feedback is examined in [7] by geometric approach.

2. PROBLEM FORMULATION

In this section we formulate the problem studied here. Consider the transfer matrix system Σ given in (1). The objective for the partial DDP is to find a measurement feedback

$$u = -Cy \tag{3}$$

with a proper C such that the closed-loop matrix

$$K - L(I + CN)^{-1}CM \tag{4}$$

between z and d, which is a proper matrix has first $k+1$ Markov parameters zero. Since N is assumed as strictly proper, then $(I + CN)$ is biproper and $X := (I + CN)^{-1}C$ is proper. When X is known then C is obtained as $C = X(I - NX)^{-1}$. Thus, we can define partial DDP by a measurement feedback for a given k (PDDPM(k)) as in the following way.

Definition 2.1. (PDDPM(k)) Given the transfer matrix system Σ and a nonnegative integer k. Find a proper matrix X such that

$$K - LXM = \frac{1}{s^{k+1}} P \tag{5}$$

for some proper matrix P.

When L does not have full row rank and M full column rank, there exist biproper matrices B_1 and B_2 such that

$$B_1 L = \begin{bmatrix} \widetilde{L} \\ 0 \end{bmatrix}, \quad MB_2 = \begin{bmatrix} \widetilde{M} & 0 \end{bmatrix} \tag{6}$$

where $\widetilde{L}$ and $\widetilde{M}$ are full row rank and full column rank, respectively. Also partition $B_1 K B_2$ compatibly as

$$B_1 K B_2 = \begin{bmatrix} \widetilde{K} & K_2 \\ K_3 & K_4 \end{bmatrix}. \tag{7}$$

Let k be a positive integer such that

$$s^{k+1} \begin{bmatrix} 0 & K_2 \\ K_3 & K_4 \end{bmatrix} \tag{8}$$

is proper and let k^* denote the greatest integer among them. Then,

Theorem 2.2. PDDPM(k) is solvable for a system Σ if and only if

(i) $k \leq k^*$

(ii) PDDPM(k) is solvable for the system $\widetilde{\Sigma}$, where $\widetilde{\Sigma} = \Sigma(\widetilde{K}, \widetilde{L}, \widetilde{M}, N)$.

Proof. The condition (i) is given in [5] . Since B_1 and B_2 are biproper and P is proper $B_1 P B_2$ is proper. So, the necessity part is obvious. For sufficiency, assume that (i) and (ii) holds. Then, there exists a proper matrix X such that it satisfies $\widetilde{K} - \widetilde{L} X \widetilde{M} = \frac{1}{s^{k+1}} \widetilde{P}$ for a proper matrix $\widetilde{P}$ and an integer $k \leq k^*$. Then, we have

$$\begin{bmatrix} \widetilde{K} & K_2 \\ K_3 & K_4 \end{bmatrix} - \begin{bmatrix} \widetilde{L} \\ 0 \end{bmatrix} X \begin{bmatrix} \widetilde{M} & 0 \end{bmatrix} = \frac{1}{s^{k+1}} \begin{bmatrix} \widetilde{P} & P_2 \\ P_3 & P_4 \end{bmatrix} \tag{9}$$

for $s^{k+1} K_2 =: P_2$, $s^{k+1} K_3 =: P_3$ and $s^{k+1} K_4 =: P_4$, which are proper. Thus, PDDPM(k) is solvable for Σ. □

As a result of the conditions of the theorem above we can assume without loss of generality that L *has full row rank* and M *has full column rank*. To reduce the number of computations we also assume the followings.

Assumption 2.3. Let L have full row rank, say r and M have full column rank, say q. Then, we can assume without loss of generality that L is replaced by one of its $r \times r$ submatrices that has a determinant of maximum degree and M is replaced by one of its $q \times q$ submatrices that has a determinant of maximum degree.

Proof. See the Appendix. □

Let $t(s)$ be a rational function written as $t(s) = \frac{n(s)}{d(s)}$, with $n(s)$ and $d(s)$ polynomials. Then, we define the degree of $t(s)$ as $\deg t(s) = \deg n(s) - \deg d(s)$, where $\deg n(s)$ and $\deg d(s)$ are the degrees in the usual sense of the polynomials $n(s)$ and $d(s)$, respectively. Note that $-\deg t(s)$ is equal to the order of the zero at infinity of $t(s)$.

The following lemma will be used in the proof of the sufficiency part of the main result.

Lemma 2.4. Let g be a proper function and k a nonnegative integer. We have the following relations between $\deg g$ and the integer $(k+1)$:

(i) If $\deg g \leq -(k+1)$, then for every proper function p, which is different from $s^{k+1}g$ we have $\deg(g - \frac{1}{s^{k+1}}p) \leq -(k+1)$

(ii) If $\deg g > -(k+1)$, then for every proper function p we have $\deg(g - \frac{1}{s^{k+1}}p) = \deg g$.

Proof. The results of this lemma can be easily seen. □

3. NOTATION AND PRELIMINARIES

We will give some notation that we use throughout this paper. A proper rational function $t(s)$ can be factorized as $t(s) = s^{-p}\lambda b(s)$, where p is the infinite zero order of $t(s)$, $b(s)$ is biproper function which is equal to 1 when s goes to ∞ and λ is real number called the infinite gain of $t(s)$. A proper transfer matrix $T(s)$ is said to be structured if each of its entries either is fixed to zero or has a zero at infinity of a known order while the associated infinite gains and biproper functions are unknown and the infinite gains are mutually independent values. A property of a structured system is said to be generic if it is true for almost all values of the parameters, where 'almost all' is to be understood as for all except for those in some proper algebraic variety of the parameter space, see [13] for further details.

We will associate with a bipartite graph $G = (U, Y, \mathcal{E})$ of the structured transfer matrix T. Here $U = \{u_1, u_2, \ldots, u_m\}$ is the set of input vertices, $Y = \{y_1, y_2, \ldots, y_p\}$ is the set of output vertices, and $\mathcal{E}$ denotes the edge set defined as $\mathcal{E}=\{(u_j, y_i) | t_{ij}(s) \neq 0\}$, where t_{ij} is the (i,j) entry of T. The edges are given weights equal to the infinite zero order of the corresponding entry in the transfer matrix T. A matching in the bipartite graph $G = (U, Y, \mathcal{E})$ is a subset $\mathcal{M}$ of the edge set $\mathcal{E}$ consisting of the edges that pairwise do not have any vertex in common. The number of edges in a matching $\mathcal{M}$ is called the order of the matching and the weight of a matching is defined to be equal to the sum of the weights of the edges of which the matching consists.

Proposition 3.1. The generic rank of T is the order of a maximum matching in G.

Note that the generic rank is the number of infinite zeros of structured system.

Proposition 3.2. The generic degree of the determinant of T equals -1 times the minimum weight of a maximum ($= t$) order matching in G from U to Y.

The generic essential orders can also be given by the edge weights in the bipartite graph,like as the infinite structure of a structured system, [3].

Proposition 3.3. Let T be a structured transfer matrix with generic rank t and G be the associated bipartite graph. The ith generic (or structured) row essential order is given by;

$$t_{ie}{}^{r} = \Pi(T)_t - \Pi(T)_{t-1}^{ir}$$

where $\Pi(T)_t$ is the minimun weight of a matching of order t from U to Y and $\Pi(T)_{t-1}^{ir}$ is the minimal weight of a matching of order $t-1$ from U to $Y\backslash\{y_i\}$.

To get the generic column essential orders we proceed dually by deleting inputs instead of outputs. Also note that in case $T(s)$ is generically invertible with generic rank t then, $\det T = \sum_{j=1}^{t} t_{ij}T_{ij}$, where T_{ij} is the cofactor of (i,j)th-entry of $T(s)$, and from Propositions 3.2 and 3.3 we have that

$$t_{ie}{}^{r} = -(\deg\ \det T - \max_j \deg T_{ij})$$

ior $i, j \in \{1, 2, \dots, t\}$, (see [11]).

4. GENERIC SOLVABILITY AND ITS GRAPH INTERPRETATION

As mentioned before, we study structured transfer matrix systems in this paper. Hence we consider Σ and we assume that K, L, M and N are structured transfer matrices. Suppose that the number of infinite gains in the matrices K, L, M and N is f and each infinite gain can have any real value. We then say that:

Definition 4.1. PDDPM(k) for a structured system is generically solvable if it is solvable for all combinations of the infinite gain values except for those in some proper algebraic variety in $\boldsymbol{R}^f$.

Now, assume that L is $r \times r$ and M is $q \times q$ and both are generically invertible matrices and write $K = (k_{ab})$, $L = (l_{ia})$ and $M = (m_{bj})$. Furthermore, we denote by L_{ia} the cofactor of l_{ia} and by M_{bj} the cofactor of m_{bj}, where $i, a \in \{1, 2, \dots, r\}$ and $j, b \in \{1, 2, \dots, q\}$. Let

$$\deg\ \det L - \max_i \deg L_{ai} =: -\lambda_a$$
$$\deg\ \det M - \max_j \deg M_{jb} =: -\delta_b$$

λ_a and δ_b are the so-called essential order of row a of L and column b of M, respectively.

Let us state the main result of this paper:

Theorem 4.2. For the structured transfer matrix system Σ, under the Assumption 2.3, the PDDPM(k) is generically solvable if and only if it holds generically that

$$\deg k_{ij} \leq \max\{-(\lambda_i + \delta_j),\ -(k+1)\} \tag{10}$$

for all $i \in \{1, 2, \dots, r\}$ and $j \in \{1, 2, \dots, q\}$.

Proof. Assume that PDDPM(k) is generically solvable. Then there exists a generically proper matrix X such that it satisfies the equation (5) generically, for some generically proper matrix P. Since P is known proper matrix, L and M are generically invertible matrices then, from Proposition 3.1 in [11] the entry x_{ij} of the solution X of the equation (5) can be written explicitly as

$$x_{ij} = \frac{1}{\det L \det M} \sum_{a=1}^{r} \sum_{b=1}^{q} L_{ai}(k_{ab} - \frac{1}{s^{k+1}} p_{ab}) M_{jb} \tag{11}$$

and, from Theorem 4.1 in [11] we have generically that

$$\deg x_{ij} = \max_{a,b} \left\{ \deg L_{ai} + \deg \left(k_{ab} - \frac{1}{s^{k+1}} p_{ab} \right) + \deg M_{jb} \right\} - \deg \det L - \deg \det M \tag{12}$$

for all $i \in \{1, 2, \dots, r\}$ and $j \in \{1, 2, \dots, q\}$. If $x_{ij} \neq 0$, because of the properness of X we have that $\deg x_{ij} \leq 0$. Thus, from equation (12) we can write

$$\max_{a,b} \left\{ \deg L_{ai} + \deg \left(k_{ab} - \frac{1}{s^{k+1}} p_{ab} \right) + \deg M_{jb} \right\} \leq \deg \det L + \deg \det M \tag{13}$$

for all $i \in \{1, 2, \dots, r\}$ and $j \in \{1, 2, \dots, q\}$. Then, it holds generically that

$$\deg \left(k_{ab} - \frac{1}{s^{k+1}} p_{ab} \right) \leq (\deg \det L - \deg L_{ai}) + (\deg \det M - \deg M_{jb}) \tag{14}$$

for all $a,\ i \in \{1, 2, \dots, r\}$ and $j,\ b \in \{1, 2, \dots, q\}$ and also

$$\deg \left(k_{ab} - \frac{1}{s^{k+1}} p_{ab} \right) \leq (\deg \det L - \max_i \deg L_{ai}) + (\deg \det M - \max_j \deg M_{jb}) \tag{15}$$

for all $a \in \{1, 2, \dots, r\}$ and $b \in \{1, 2, \dots, q\}$. From definition of the essential orders, the inequality (15) can be written as

$$\deg \left(k_{ab} - \frac{1}{s^{k+1}} p_{ab} \right) \leq -(\lambda_a + \delta_b) \tag{16}$$

for all $a \in \{1, 2, \ldots, r\}$ and $b \in \{1, 2, \ldots, q\}$. On the other hand, it is clear that $\deg\left(k_{ab} - \frac{1}{s^{k+1}}p_{ab}\right) \leq \max\{\deg k_{ab}, -(k+1) + \deg p_{ab}\}$. Since p_{ab} is proper, then

$$\deg\left(k_{ab} - \frac{1}{s^{k+1}}p_{ab}\right) \leq \max\{\deg k_{ab}, -(k+1)\} \tag{17}$$

Now, consider the case

$$\deg\left(k_{ab} - \frac{1}{s^{k+1}}p_{ab}\right) = \max\{\deg k_{ab}, -(k+1)\}. \tag{18}$$

If $\max\{\deg k_{ab}, -(k+1)\} = -(k+1)$ we have that

$$\deg k_{ab} \leq -(k+1). \tag{19}$$

So, from (16), (18) and (19) the following inequalities hold for all $a \in \{1, 2, \ldots, r\}$ and $b \in \{1, 2, \ldots, q\}$.

$$\deg k_{ab} \leq -(k+1) \leq -(\lambda_a + \delta_b) \tag{20}$$

When $\max\{\deg k_{ab}, -(k+1)\} = \deg k_{ab}$, we have $-(k+1) \leq \deg k_{ab}$. Then, by (18) and (16) we have

$$-(k+1) \leq \deg k_{ab} \leq -(\lambda_a + \delta_b) \tag{21}$$

for all $a \in \{1, 2, \ldots, r\}$ and $b \in \{1, 2, \ldots, q\}$. The other possibility for the inequality (17) is

$$\deg\left(k_{ab} - \frac{1}{s^{k+1}}p_{ab}\right) < \max\{\deg k_{ab}, -(k+1)\}. \tag{22}$$

This would happen only if $\deg k_{ab} = \deg \frac{1}{s^{k+1}}p_{ab} = \deg p_{ab} - (k+1)$. Since p_{ab} is proper, then $\deg p_{ab} - (k+1) \leq -(k+1)$ and the inequality (19) holds in this case too. Hence, by the inequalities (19), (20) and (21) we deduce:

$$\deg k_{ab} \leq \max\{-(\lambda_a + \delta_b), -(k+1)\}. \tag{23}$$

Conversely, assume that the inequality (10) holds. If we define x_{ij} which is of the form (11), for a proper p_{ab} satisfying the inequality (16) we would obtain $\deg x_{ij} \leq 0$. Consequently, $X = (x_{ij})$ solves the equation (5), for $P = (p_{ab})$. Hence, to prove the sufficiency we should define p_{ab} satisfying (16). Thus, to define p_{ab} satisfying (16), first let $\max\{-(\lambda_a+\delta_b), -(k+1)\} = -(k+1)$. Then, we have $-(\lambda_a+\delta_b) \leq -(k+1)$ and, by assumption we have also $\deg k_{ab} \leq -(k+1)$. Then, from the Lemma 2.4 (i) there exists a proper p_{ab} such that $\deg(k_{ab} - \frac{1}{s^{k+1}}p_{ab}) \leq -(k+1)$. Thus, let us define p_{ab} as in the following form:

$$s^{k+1}(k_{ab} - s^{-\nu}\gamma b_{ab}) =: p_{ab} \tag{24}$$

$$\nu \geq (\lambda_a + \delta_b) \tag{25}$$

where b_{ab} is an arbitrary biproper rational function and γ is an infinite gain of $s^{-\nu}\gamma b_{ab}$. We should note that we can also define p_{ab} as

$$s^{k+1}k_{ab} =: p_{ab}. \tag{26}$$

Then, we obtain $\left(k_{ab} - \frac{1}{s^{k+1}}p_{ab}\right) = s^{-\nu}\gamma b_{ab}$ or zero. In both case p_{ab} is proper. If x_{ij} is defined as (11) by means of p_{ab} given in (24) or (26), because of the choice of ν it will be proper or it will be zero. Now, let $\max\{-(\lambda_a+\delta_b),\ -(k+1)\} = -(\lambda_a+\delta_b)$. Then, we have $\deg k_{ab} \leq -(\lambda_a+\delta_b)$. In this case (16) is satisfied for $p_{ab}=0$. Besides that from the assumption we have $-(k+1) \leq -(\lambda_a+\delta_b)$. Then, there are two possibilities. These are $\deg k_{ab} \leq -(k+1)$ and $-(k+1) < \deg k_{ab}$. The first case is already examined above. When $-(k+1) < \deg k_{ab}$, from the definition of the degree of a rational function, for every proper p_{ab} we have

$$\deg\left(k_{ab} - \frac{1}{s^{k+1}}p_{ab}\right) = \deg k_{ab}. \tag{27}$$

Thus, in this case (16) is satisfied for an arbitrary proper p_{ab}. Consequently, when $\deg k_{ab} \leq -(\lambda_a+\delta_b)$ there exists a proper p_{ab} satisfying the inequality (16). □

Remark 4.3. The condition $\deg k_{ab} \leq -(\lambda_a+\delta_b)$ is also necessary and sufficient for the solution of exact disturbance decoupling problem for structured transfer matrix systems (see Corollary 4.5 in [11]). The solvability condition for the partial disturbance decoupling is weaker than this condition. As it can be shown from the sufficiency proof of Theorem 4.2 even if the exact disturbance decoupling problem for a structured transfer matrix system is not solvable, i. e. $-(\lambda_a+\delta_b) < \deg k_{ab}$ by the suitable definition of proper matrix P it is possible to solve PDDPM(k). (See the sufficiency part of the theorem above when $-(\lambda_a+\delta_b) < \deg k_{ab} \leq -(k+1)$).

Remark 4.4. As we explained before λ_i and δ_j are the structured essential orders of row i of L and column j of M, respectively. But, the proper rational functions λ_i and γ_j in the second condition of the Theorem 4.1 in [5] correspond to infinite zero of ith row of L and infinite zero of jth column of M, respectively. Since the degrees of the infinite zeros and essential orders are different, solvability conditions of the partial disturbance decoupling problem for structured transfer matrix systems are also different from the solvability conditions of the partial disturbance decoupling problem for the usual transfer matrix systems.

Remark 4.5. Internal stability for the transfer matrix systems Σ is equivalent to the stability of X, if the system matrices are taken as stable, [5]. But, even if for those systems the stability of X do not guarantee the stability of C.

In the following we will express the results given in Theorem 4.2 in graph theory terminology: Let us denote $T = \begin{pmatrix} K & L \\ M & N \end{pmatrix}$ and assume that the dimension of K, L, M and N are $\widetilde{r}\times\widetilde{q}$, $\widetilde{r}\times m$, $p\times\widetilde{q}$ and $p\times m$, respectively. Then, we can represent the degree structure of T by means of a bipartite graph $G=(V,W,\mathcal{E})$, where V and W are the vertex sets, $\mathcal{E}$ is the edge set. For the vertex sets, we have $V = D\cup U$ and $W = Z\cup Y$, where $D=\{d_1,d_2,\ldots,d_{\tilde{q}}\}$, $U=\{u_1,u_2,\ldots,u_m\}$, $Z=\{z_1,z_2,\ldots,z_{\tilde{r}}\}$ and $Y=\{y_1,y_2,\ldots,y_p\}$. The edge set $\mathcal{E}=\{(d_j,z_i)|k_{ij}(s)\neq 0\}\cup\{(u_j,z_i)|l_{ij}(s)\neq 0\}\cup\{(d_j,y_i)|m_{ij}(s)\neq 0\}\cup\{(u_j,y_i)|n_{ij}(s)\neq 0\}$.

Let r and q be the generic ranks of L and M, respectively. Let ${l_i}_e{}^r$ be the ith generic row essential order of the subsystem from control inputs U to regulated outputs Z and ${m_j}_e{}^c$ be the jth generic column essential order of the subsystem from disturbance D to measured outputs Y. Then,

$$\begin{aligned} {l_i}_e{}^r &= \Pi(L)_r - \Pi(L)_{r-1}^{ir} \\ {m_j}_e{}^c &= \Pi(M)_q - \Pi(M)_{q-1}^{jc} \end{aligned}$$

where $\Pi(L)_r$ is minimum weight of a matching of order r from U to Z. $\Pi(L)_{r-1}^{ir}$ is the minimum weight of a matching of order $r-1$ from U to $Z\backslash\{z_i\}$, $\Pi(M)_q$ is the minimum weight of a matching of order q from D to Y and finally $\Pi(M)_{q-1}^{jc}$ is the minimum weight of a matching of order $q-1$ from $D\backslash\{d_j\}$ to Y.

Let us state the graph theoretic interpretation of the main result of this paper:

Theorem 4.6. For the structured transfer matrix system Σ under the Assumption 2.3, the PDDPM(k) is generically solvable if and only if the weight of the edge from jth disturbance to ith regulated output greater than or equal to the minimum of $({l_i}_e{}^r + {m_j}_e{}^c)$ and $(k+1)$, for all $i \in \{1, 2, \dots, r\}$ and $j \in \{1, 2, \dots, q\}$.

5. ALGORITHM

1 Compute the maximum order of a matching in G from U to Z, say r.

2 – Compute the maximum order of a matching in G from D to Y, say q.

3 – Compute the minimum weight of a order r matching in G from U to Z, say μ.

4 – Compute the minimum weight of a order q matching in G from D to Y, say ν.

5 – Compute a size r-matching from U to Z with weight equal to μ. Assume that the matching links $u_1, u_2, \dots, u_r$ to $z_1, z_2, \dots, z_r$. Denote by $\widetilde{L}_1$ the square matrix made up of the first r rows and columns of L.

6 – Compute a size q-matching from D to Y with weight equal to ν. Assume that the matching links $d_1, d_2, \dots, d_q$ to $y_1, y_2, \dots, y_q$. Denote by $\widetilde{M}_1$ the square matrix made up of the first q rows and columns of M.

Note that the system $\widetilde{\Sigma}_1$ with transfer matrices $\widetilde{K}_1$, $\widetilde{L}_1$, $\widetilde{M}_1$ and $\widetilde{N}_1$ satisfies the Assumption 2.3. Here $\widetilde{K}_1$ denotes the matrix made up the first r rows and q columns of K, and $\widetilde{N}_1$ denotes the matrix made up the first q rows and r columns of N.

7 – If $r \neq \widetilde{r}$ and $q \neq \widetilde{q}$, then let k^* be the minimal weight of an order one matching in G from D to Z. If $k > k^*$ then, PDDPM(k) is not generically solvable and we can stop checking its solvability here. Otherwise, we have to continue.

We must note that when $r \neq \widetilde{r}$ and $q \neq \widetilde{q}$ PDDPM(k) for original system with transfer matrices K, L, M and N is generically solvable iff (i) $k \leq k^*$ and (ii) it is generically solvable for the system generated by $\widetilde{K}_1$, $\widetilde{L}_1$, $\widetilde{M}_1$ and $\widetilde{N}_1$. Thus, 8 – Compute the row essential orders of $\widetilde{L}_1$ and column essential orders of $\widetilde{M}_1$ which are the same with the row and column essential orders of L and M, respectively when L is of full row rank and M is of full column rank.

9 – Apply Theorem 4.6 to the system $\widetilde{\Sigma}_1$.

Example. The example below is taken from [11]. We consider a structured transfer matrix system Σ. Assume that the infinite structure of the matrices K, L, M and N are given in the following matrices;

$$\Delta_K = \begin{bmatrix} 2 & \cdot \\ \cdot & 4 \end{bmatrix},\ \Delta_L = \begin{bmatrix} 1 & \cdot \\ 1 & 2 \end{bmatrix},\ \Delta_M = \begin{bmatrix} 1 & 3 \\ \cdot & 2 \end{bmatrix},\ \Delta_N = \begin{bmatrix} \cdot & 2 \\ 1 & \cdot \end{bmatrix}. \tag{28}$$

The dot $\cdot$ in an infinite zero order matrix corresponds to an entry that is identically equal to zero. Infinite zero order matrices, Δ_K, Δ_L, Δ_M and Δ_N correspond to the matrices K, L, M and N, respectively which are is of the form;

$$\begin{aligned} K(s) &= \begin{bmatrix} s^{-2}\kappa_{11}\alpha_{11}(s) & 0 \\ 0 & s^{-4}\kappa_{22}\alpha_{22}(s) \end{bmatrix}, \\ L(s) &= \begin{bmatrix} s^{-1}\lambda_{11}\beta_{11}(s) & 0 \\ s^{-1}\lambda_{21}\beta_{21}(s) & s^{-2}\lambda_{22}\beta_{22}(s) \end{bmatrix}, \end{aligned} \tag{29}$$

$$\begin{aligned} M(s) &= \begin{bmatrix} s^{-1}\tau_{11}\delta_{11}(s) & s^{-3}\tau_{12}\delta_{12}(s) \\ 0 & s^{-2}\tau_{22}\delta_{22}(s) \end{bmatrix}, \\ N(s) &= \begin{bmatrix} 0 & s^{-2}\xi_{12}\eta_{11}(s) \\ s^{-1}\xi_{21}\eta_{21}(s) & 0 \end{bmatrix}, \end{aligned} \tag{30}$$

where κ_{11}, κ_{22}, λ_{11}, λ_{21}, λ_{22}, τ_{11}, τ_{12}, τ_{22}, ξ_{12} and ξ_{21} are the unknown infinite gains and $\alpha_{11}(s)$, $\alpha_{22}(s)$, $\beta_{11}(s)$, $\beta_{21}(s)$, $\beta_{22}(s)$, $\delta_{11}(s)$, $\delta_{12}(s)$, $\delta_{22}(s)$, $\eta_{12}(s)$ and $\eta_{21}(s)$ are unknown biproper rational functions. The system with the above infinite zero order matrices can be represented by means of a bipartite graph as depicted in Figure 1. It is immediate from Figure 1 that the maximum order of a matching in G from U to Z is equal to 2. In fact, there is only one such matching and its weight is equal to 3. Also, the maximum order matching in G from D to Y is equal to 2. Again, there is only one matching of order 2 with weight 3. So, in terms of Algorithm we have $r = 2$, $q = 2$, $\mu = 3$ and $\nu = 3$. Since the number of elements of Z equals r and the number of elements of D equals q, we can skip steps 5, 6 and 7 and compute row and column essential orders. From Figure 1, we calculate $\Pi(L)_2 = 3$, $\Pi(L)_1^{1r} = 1$, $\Pi(L)_1^{2r} = 1$ $\Pi(M)_2 = 3$, $\Pi(M)_1^{1c} = 2$, $\Pi(M)_1^{2c} = 1$ and then, we obtain ${l_{1e}}^r = 3 - 1 = 2$, ${l_{2e}}^r = 3 - 1 = 2$ and ${m_{1e}}^c = 3 - 2 = 1$, ${m_{2e}}^c = 3 - 1 = 2$. The weights of the edges (d_1, z_1) and (d_2, z_2) are 2 and 4, respectively. So, $\deg k_{11} = -2$ and $\deg k_{22} = -4$. Correspondingly, ${l_{1e}}^r + {m_{1e}}^c = 3$ and ${l_{2e}}^r + {m_{2e}}^c = 4$. Since $-\deg k_{11} \leq {l_{1e}}^r + {m_{1e}}^c$, then from Corollary 4.5 in [11] DDP for structured transfer matrix system is not solvable. When $k = 1$, The weights of the edges (d_1, z_1) and (d_2, z_2) are greater than or equal to $\min\{({l_{1e}}^r + {m_{1e}}^c),\ (k+1)\} = \min\{3, 2\} = 2$ and $\min\{({l_{2e}}^r + {m_{2e}}^c),\ (k+1)\} = \min\{4, 2\} = 2$, respectively. As a result, from

Theorem 4.6 we can say that PDDPM(1) is generically solvable. Thus, to exhibit the solution let us define $P(s)$ as

$$P(s) = \begin{bmatrix} \kappa_{11}(1 - \frac{1}{s})\alpha_{11}(s) & 0 \\ 0 & s^{-2}\mu\gamma(s) \end{bmatrix} \tag{31}$$

where μ is the infinite gain of the corresponding entry of $P(s)$ and $\gamma(s)$ is an arbitrary biproper rational function. We should note that $\mu\gamma(s)$ is a known proper function. Since $\deg k_{22} = -({l_{2e}}^r + {m_{2e}}^c) = -4$ exact disturbance decoupling is possible on this channel. So, we can choose $\mu\gamma(s) = 0$, for the simplicity in calculation. But, to reduce the degree of k_{11} we have to choose p_{11} which is of the same infinite gain with k_{11} and satisfying the degree condition $\deg k_{11} = \deg \frac{1}{s^2} p_{11}$. Then, the solution $X(s)$ of $K - LXM = \frac{1}{s^2} P$, for a given $P(s)$ in (31) is

$$X(s) = \begin{bmatrix} \frac{\kappa_{11}\alpha_{11}(s)}{s\lambda_{11}\tau_{11}\beta_{11}(s)\delta_{11}(s)} & \frac{-\kappa_{11}\alpha_{11}(s)\tau_{12}\delta_{12}(s)}{s^2\lambda_{11}\tau_{11}\tau_{22}\beta_{11}(s)\delta_{11}(s)\delta_{22}(s)} \\ \frac{-\lambda_{21}\beta_{21}(s)\kappa_{11}\alpha_{11}(s)}{\lambda_{11}\tau_{11}\lambda_{22}\beta_{11}(s)\delta_{11}(s)\beta_{22}(s)} & \frac{\lambda_{21}\beta_{21}(s)\kappa_{11}\alpha_{11}(s)\tau_{12}\delta_{12}(s) + s\kappa_{22}\alpha_{22}(s)\lambda_{11}\beta_{11}(s)\tau_{11}\delta_{11}(s)}{s\lambda_{11}\tau_{11}\beta_{11}(s)\delta_{11}(s)\lambda_{22}\tau_{22}\beta_{22}(s)\delta_{22}(s)} \end{bmatrix} \tag{32}$$

which is generically proper. Since $N(s)$ in (30) is strictly proper $(I - N(s)X(s))$ is biproper and it has a proper inverse. Hence, the compensator $C(s)$ can be solved by straightforward calculations from the equation $C(s) = X(s)(I - N(s)X(s))^{-1}$. $C(s)$ is not presented here since, because of the parameters the expression would be messy. Consequently, PDDPM(1) is generically solvable for a system given in this example.

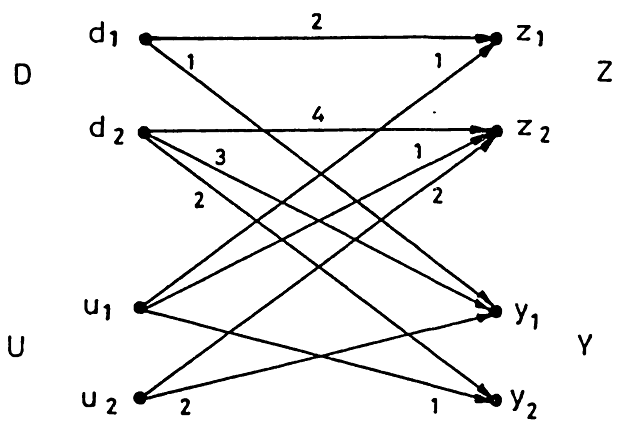

Fig. 1.

6. CONCLUSIONS

We examine the partial disturbance decoupling problem for structured transfer matrix systems by dynamic output feedback and we obtain the necessary and sufficient conditions for the generic solvability of this problem. The conditions are stated by the generic essential orders defined in terms of minimal weights of the matchings in a bipartite graph associated with structured transfer matrix systems.

Stability issues are not considered in this paper. This is a subject of the further research.

APPENDIX

P r o o f of the Assumption 2.3: Suppose that L has full row rank, say r and M has full column rank, say q. Assume that they are not square. Also assume that after some column and row permutations we have

$$L = \begin{bmatrix} \widetilde{L}_1 & \widetilde{L}_2 \end{bmatrix}, \quad M = \begin{bmatrix} \widetilde{M}_1 \\ \widetilde{M}_2 \end{bmatrix} \tag{33}$$

with $\widetilde{L}_1$ being $r \times r$ and $\deg(\det \widetilde{L}_1)$ equal to the maximum degree of any rth-order minor of L and $\widetilde{M}_1$ being $q \times q$ and $\deg(\det \widetilde{M}_1)$ equal to the maximum degree of any qth-order minor of M. Accordingly, denote $K =: \widetilde{K}_1$ and $N =: \widetilde{N}_1$. Hence, there exist biproper matrices B_1 and B_2 such that

$$LB_1 = \begin{bmatrix} \widetilde{L}_1 & 0 \end{bmatrix}, \quad B_2 M = \begin{bmatrix} \widetilde{M}_1 \\ 0 \end{bmatrix}. \tag{34}$$

In order to show the result, we should prove the following claim that is "the solvability of PDDPM(k) for Σ is equivalent to the solvability of PDDPM(k) for $\widetilde{\Sigma}_1$, where $\widetilde{\Sigma}_1 = \widetilde{\Sigma}_1(\widetilde{K}_1, \widetilde{L}_1, \widetilde{M}_1, \widetilde{N}_1)$". To prove the necessity part assume that PDDPM(k) is solvable for Σ then there exists a proper X which satisfies the equation (5), for some proper matrix P. Then, by (34) we can define

$$\widetilde{X} := \begin{bmatrix} I & 0 \end{bmatrix} B_1^{-1} X B_2^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{35}$$

which is proper. Then, we have

$$\widetilde{K}_1 - \widetilde{L}_1 \widetilde{X} \widetilde{M}_1 = \frac{1}{s^{k+1}} P \tag{36}$$

hence PDDPM(k) is solvable for $\widetilde{\Sigma}_1$.

Conversely, assume PDDPM(k), solvable for $\widetilde{\Sigma}_1$. This implies the existence of a proper matrix $\widetilde{X}$ such that (36) holds, for some proper P. Let us define

$$X = B_1 \begin{bmatrix} \widetilde{X} & 0 \\ 0 & 0 \end{bmatrix} B_2 \tag{37}$$

which is also proper and satisfies $\widetilde{K}_1 - \begin{bmatrix} \widetilde{L}_1 & 0 \end{bmatrix} B_1^{-1} X B_2^{-1} \begin{bmatrix} \widetilde{M}_1 \\ 0 \end{bmatrix} = \frac{1}{s^{k+1}} P$. Together with (34) we will obtain (5). □

(Received February 25, 1998.)

REFERENCES

[1] C. Commault, J. M. Dion and A. Perez: Disturbance rejection for structured systems. IEEE Trans. Automat. Control *AC-36* (1991), 884-887.

[2] C. Commault, J. M. Dion and M. Benahcene: Output feedback disturbance decoupling graph interpretation for structured systems. Automatica *6* (1993), 1463–1472.

[3] C. Commault, J. M. Dion and M. Benahcene: Decoupling and minimality of structured systems in a transfer matrix framework. In: Proc. IFAC Conference System Structure and Control, Nantes 1995, pp. 84–89.

[4] C. Commault, J. M. Dion and V. Hovelaque: A geometric approach for structured systems: Application to disturbance decoupling. Automatica *3* (1997), 403–409.

[5] V. Eldem, H. Özbay, H. Selbuz and K. Özçaldiran: Partial disturbance rejection with internal stability and H_∞ norm bound. SIAM J. Control Optim. *36* (1998), 1, 180–192.

[6] E. Emre, and L. H. Silverman: Partial model matching of linear systems. IEEE Trans. Automat. Control *AC-25* (1980), 280-281.

[7] M. Malabre and J. C. M. Garcia: The partial model matching or partial disturbance rejection problem: geometric and structural solutions. IEEE Trans. Automat. Control *40* (1995), 2, 356–360.

[8] K. J. Reinschke: Multivariable Control: A Graph Theoretic Approach. (Lecture Notes in Control and Information Sciences 108.) Springer–Verlag, Berlin 1988.

[9] J. W. van der Woude: On the structure at infinity of a structured system. Linear Algebra Appl. *148* (1991), 145–169.

[10] J. W. van der Woude: Disturbance decoupling by measurement feedback for structured systems: a graph theoretic approach. In: Proc. 2nd European Control Conf. ECC'93, Groningen 1993, pp. 1132–1137.

[11] J. W. van der Woude: Disturbance decoupling by measurement feedback for structured transfer matrix systems. Automatica *32* (1996), 357–363.

[12] J. C. Willems and C. Commault: Disturbance decoupling by measurement feedback with internal stability or pole placement. SIAM J. Control Optim. *19* (1981), 490–504.

[13] W. M. Wonham: Linear Multivariable Control: A Geometric Approach. Third edition. Springer–Verlag, New York 1985.

Assoc. Prof. Ulviye Başer, Department of Mathematics, Faculty of Sciences and Letters, Istanbul Technical University, Maslak, Istanbul 80626. Turkey.
e-mail: baser@sariyer.cc.itu.edu.tr